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Erratum

Correction to "Fixed Points of Maps of a Nonaspherical Wedge"

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In the original paper, it was assumed that a selfmap of $X = P \vee C$, the wedge of a real projective space P and a circle C, is homotopic to a map that takes P to itself. An example is presented of a selfmap of X that fails to have this property. However, all the results of the paper are correct for maps of the pair (X, P).

Let $X = P \vee C$ be the wedge of the real projective plane P and the circle C. As the example below demonstrates, the statement on page 3 of [1] "Given a map $f : X \to X$ we may deform f by a homotopy so that f_P, its restriction to P, maps P to itself." is incorrect. If, instead of an arbitrary self-map of X, we consider a map of pairs $f : (X, P) \to (X, P)$, the map can be put in the *standard form* defined on that page and then all the results of the paper are correct for such maps of pairs.

To describe the example, represent points x of the unit 2-sphere S^2 by spherical coordinates $x = (r = 1, \theta, \phi)$ where r denotes the radius, θ the elevation and ϕ the azimuth. Let $S^2 = D_+^2 \cup A_+ \cup E \cup A_- \cup D_-^2$ where x is in D_+^2, A_+, E, A_- or D_-^2, if $\pi/3 < \theta \leq \pi/2, \pi/6 < \theta \leq \pi/3, -\pi/6 \leq \theta \leq \pi/6, -\pi/3 \leq \theta < -\pi/6$ or $-\pi/2 \leq \theta < -\pi/3$, respectively. Let $Y = S_+^2 \cup I_+ \cup S^2 \cup I_- \cup S_-^2$, where $S_\pm^2$ are the 2-spheres of radius one in $\mathbb{R}^3$ with centers, in cartesian coordinates, at $(\pm 2, 0, \pm 2)$, I_+ denotes the points $(t, 0, 1)$ for $0 \leq t \leq 2$ and I_- the points $(t, 0, -1)$ for $-2 \leq t \leq 0$. Define $\widetilde{f}_P : S^2 \to Y$ in the following manner. For $x = (1, \theta, \phi) \in A_\pm$, let

$$\widetilde{f}_P(x) = \widetilde{f}_P(1, \theta, \phi) = \left(\frac{12\theta}{\pi} - 2, 0, \pm 1\right) \in \mathbb{R}^3 \tag{1}$$

in cartesian coordinates. For $(1,\theta,\phi) \in E$, set $\tilde{f}_P(1,\theta,\phi) = (1,3\theta,\phi)$. Let $\rho_\pm = (1,\pm\pi/2,0) \in S^2$ be the poles and define $K_\pm : D^2_\pm \to S^2 - \rho_\mp$ by

$$K_\pm(x) = K_\pm(1,\theta,\phi) = \left(1, 6\theta \mp \frac{5\pi}{2}, \phi\right). \tag{2}$$

Returning to cartesian coordinates, define $T_\pm : S^2 \to S^2_\pm$ by

$$T_\pm(x_1,x_2,x_3) = (x_1 \pm 2, x_2, x_3 \pm 2). \tag{3}$$

We complete the definition of $\tilde{f}_P : S^2 \to Y$ by setting $\tilde{f}_P(x) = T_\pm K_\pm$ for $x \in D^2_\pm$. Note that $(\tilde{f}_P)_* : H_2(S^2,\mathbb{Z}/2\mathbb{Z}) \to H_2(Y,\mathbb{Z}/2\mathbb{Z})$ such that $(\tilde{f}_P)_*(1) = (1,1,1)$. We may embed Y in the universal covering space $p : \tilde{X} \to X$ because $\tilde{X}$ is an infinite tree with a 2-sphere replacing each vertex in such a way that two edges are attached at each of two antipodal points. The embedding induces a monomorphism of homology. The map $\tilde{f}_P$ has been defined so that if $x, -x$ are antipodal points of S^2, then $p\tilde{f}_P(x) = p\tilde{f}_P(-x)$ and therefore $\tilde{f}_P$ induces a map $f_P : P \to X$. If f_P were homotopic to a map $g_P : P \to P \subseteq X$, then the homotopy would lift to cover g_P by a map $\tilde{g}_P : S^2 \to \tilde{X}$ which sends S^2 to a single 2-sphere in $\tilde{X}$. Therefore the image of $(\tilde{g}_P)_* : H_2(S^2,\mathbb{Z}/2\mathbb{Z}) \to H_2(\tilde{X},\mathbb{Z}/2\mathbb{Z})$ would be either trivial or a single generator of $H_2(\tilde{X},\mathbb{Z}/2\mathbb{Z})$. On the other hand, the image of $(\tilde{f}_P)_*$ in $H_2(\tilde{X},\mathbb{Z}/2\mathbb{Z})$ is nontrivial for three generators, so no such homotopy can exist. Therefore, if $f : X \to X = P \vee C$ is a map whose restriction to P is the map f_P defined above, then it cannot be homotoped to a map that takes P to itself.

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References

[1] S. W. Kim, R. F. Brown, A. Ericksen, N. Khamsemanan, and K. Merrill, “Fixed points of maps of a nonaspherical wedge,” *Fixed Point Theory and Applications*, vol. 2099, Article ID 531037, 18 pages, 2009.