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Mizoguchi-Takahashi-type theorems in tvs-cone metric spaces

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Abstract

In this paper, the concepts of a set-valued contraction of Mizoguchi-Takahashi type in the context of topological vector space (tvs)-cone metric spaces are introduced and a fixed point theorem in the context of tvs-cone metric spaces with respect to a solid cone is proved. We obtained results which extend and generalize the main results of S. H. Cho with J. S. Bae, Mizoguchi with Takahashi and S. B. Nadler Jr. Two examples are given to illustrate the usability of our results.

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Introduction and preliminaries

Huang and Zhang introduced in [1] the concept of cone metric spaces as a generalization of metric spaces. They have replaced the real numbers (as the co-domain of a metric) by an ordered Banach space. They described the convergence in cone metric spaces, introduced their completeness and proved some fixed point theorems for contractive mappings on cone metric spaces. The concept of cone metric space in the sense of Huang-Zhang is characterized by Al-Rawashdeh, Shatanawi and Khandaqji in [2]. Indeed (X, d) is a cone metric space if and only if (X, d^E) is an E-metric space, where E is a normed ordered space, with $Int(E^+) \neq \varnothing$ ([2], Theorem 3.8). Recently in [3-28] many authors proved fixed point theorems in cone metric spaces.

Du in [13] introduced the concept of topological vector space (tvs)-cone metric and tvs-cone metric space to improve and extend the concept of cone metric space in the sense of Huang and Zhang [1]. In [7,9,13,14] the authors tried to generalize this approach using cones in tvs instead of Banach spaces. However, it should be noted that an old result shows that if the underlying cone of an ordered tvs is solid and normal, then such tvs must be an ordered normed space. Thus, proper generalizations when passing from norm-valued cone metric spaces to tvs-valued cone metric spaces can be obtained only in the case of nonnormal cones (for more details see [14]).

We recall some definitions and results from [14,15], which will be needed in the sequel.

Let E be a tvs with its zero vector θ. A nonempty subset P of E is called a convex cone if $P + P \subseteq P$ and $\lambda P \subseteq P$ for $\lambda \geq 0$. A convex cone P is said to be pointed (or proper) if $P \cap (-P) = \{\theta\}$; and P is a normal (or saturated) if E has a base of neighborhoods of zero consists of order-convex subsets. For a given cone $P \subseteq E$, we define a partial ordering $\preccurlyeq$ with respect to P by $x \preccurlyeq y$ if and only if $y - x \in P$; $x \prec y$ will stand for $x \preccurlyeq y$ and $x \neq y$, while $x \ll y$ stand for $y - x \in \text{int}P$, where int$P$ denotes the interior of P. The cone P is said to be solid if it has a nonempty interior.

In the sequel, E will be a locally convex Hausdorff tvs with its zero vector θ, P is a proper, closed and convex pointed cone in E with $\text{int}P \neq \emptyset$ and $\preccurlyeq$ denotes the induced partial ordering with respect to P .

Definition 1.1. [7,13,14] Let X be a nonempty set and (E, P) be an ordered tvs. A vector-valued function $d : X \times X \to E$ is said to be a tvs-cone metric, if the following conditions hold:

(C_1) $\theta \preccurlyeq d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;

(C_2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(C_3) $d(x, z) \preccurlyeq d(x, y) + d(y, z)$ for all $x, y, z \in X$.

The pair (X, d) is then called a tvs-cone metric space.

Remark **1.2**. The concept of a cone metric space [1] (E is a real Banach space and $d : X \times X \to E$ satisfies (C_1), (C_2) and (C_3)) is more general than that of a metric space, because each metric space is a cone metric space, where $E = \mathbb{R}$ and $P = [0, +\infty)$ (see [1, Example 1]). Clearly, a cone metric space in the sense of Huang and Zhang is a special case of tvs-cone metric spaces when (X, d) is tvs-cone metric space with respect to a normal cone P.

Definition 1.3. [7,13,14] Let (X, d) be a tvs-cone metric space, $x \in X$ and let $\{x_n\}$ be a sequence in X. Then

(i) $\{x_n\}$ tvs-cone converges to x whenever for every $c \in E$ with $\theta \ll c$ there is a natural number n_0 such that $d(x_n, x) \ll c$, for all $n \geq n_0$. We denote this by cone$-\lim_{n\to\infty} x_n = x$;

(ii) $\{x_n\}$ is a tvs-cone Cauchy sequence whenever for every $c \in E$ with $\theta \ll c$ there is a natural number n_0 such that $d(x_n, x_m) \ll c$, for all $n, m \geq n_0$;

(iii) (X, d) is tvs-cone complete if every tvs-cone Cauchy sequence in X is tvs-cone convergent.

Let (X, d) be a tvs-cone metric space. The following properties are often used, particularly in the case when the underlying cone is nonnormal. The only assumption is that the cone P has a nonempty interior (i.e. P is a solid). For more details about these properties see [14] and [15].

(p_1) If $u \preccurlyeq v$ and $v \ll w$, then $u \ll w$.

(p_2) If $u \ll v$ and $v \preccurlyeq w$, then $u \ll w$.

(p_3) If $u \ll v$ and $v \ll w$, then $u \ll w$.

(p_4) If $\theta \preccurlyeq u \ll c$ for each $c \in \text{int}P$, then $u = \theta$.

(p_5) If $a \preccurlyeq b + c$, for each $c \in \text{int}P$, then $a \preccurlyeq b$.

(p_6) If E is a *tvs* cone metric space with a cone P, and if $a \preccurlyeq \lambda a$, where $a \in P$ and $0 \leq \lambda < 1$, then $a = \theta$.

(p_7) If $c \in \text{int}P$, $a_n \in E$ and $a_n \to \theta$ in locally convex Hausdorff tvs E, then there exists an n_0 such that, for all $n > n_0$, we have $a_n \ll c$.

In [11], the concept of a set-valued contraction of Mizoguchi-Takahashi type was introduced and a fixed point theorem in setting of a normal cone was proved. In this article, we prove the same theorem in the setting of a tvs-cone metric space. We generalize results of [11], by omitting the assumption of normality in the results, that is the normality of P is not a necessary. We use only the definition of convergence in terms of the relation "≪". The only assumption is that the interior of the cone P in locally convex Hausdorff tvs E is nonempty, so we neither use continuity of the vector metric d, nor Sandwich Theorem. In such a way, we generalize results of [11,29,30].

Main results

Let E be a locally convex Hausdorff tvs with its zero vector θ, P a proper, closed and convex pointed cone in E with int$P \neq \varnothing$ and $\preccurlyeq$ be a partial ordering with respect to P. Let (X, d) be a tvs-cone metric space with a solid cone P and let $\mathcal{A}$ be a collection of nonempty subsets of X. According to [11], we denote

$$s(p) = \{q \in E : p \preccurlyeq q\}$$

for $p \in E$, and

$$s(a, B) = \bigcup_{b \in B} s(d(a, b))$$

for $a \in X$ and $B \in \mathcal{A}$. For $A, B \in \mathcal{A}$, we denote

$$s(A, B) = \left(\bigcap_{a \in A} s(a, B)\right) \cap \left(\bigcap_{b \in B} s(b, A)\right).$$

The following lemma will be used to prove Theorem 2.3.

Lemma 2.1. *Let (X, d) be a tvs-cone metric space with a solid cone P in ordered locally convex space E, and let $\mathcal{A}$ be a collection of nonempty subsets of X. Then we have:*

(1) *For all $p, q \in E$. If $p \preccurlyeq q$, then $s(q) \subset s(p)$.*
(2) *For all $x \in X$ and $A \in \mathcal{A}$. If $\theta \in s(x, A)$, then $x \in A$.*
(3) *For all $q \in P$ and $A, B \in \mathcal{A}$ and $a \in A$. If $q \in s(A, B)$, then $q \in s(a, B)$.*
(4) *For all $q \in P$ and $A, B \in \mathcal{A}$. Then $q \in s(A, B)$ if and only if there exist $a \in A$ and $b \in B$ such that $d(a, b) \preccurlyeq q$.*

Remark **2.2**. Let (X, d) be a tvs-cone metric space. If $E = \mathbb{R}$ and $P = [0, +\infty)$, then (X, d) is a metric space. Moreover, for $A, B \in CB(X)$, $H(A, B) = \inf s(A, B)$ is the Hausdorff distance induced by d. Also, $s(\{x\}, \{y\}) = s(d(x, y))$, for all $x, y \in X$.

Now let us prove the following main results of this article.

Theorem 2.3. *Let (X, d) be a tvs-cone complete metric space with a solid cone $P, \mathcal{A} \neq \emptyset$ be a collection of nonempty closed subsets of x and $T : X \to \mathcal{A}$ be a multi-valued map. If there exists a function $\phi : P \to [0, 1)$ such that*

$$\overline{\lim_{n\to\infty}}\, \varphi(c_n) < 1 \tag{1}$$

for any decreasing sequence $\{c_n\}$ in P, and if

$$\varphi(d(x, y))\, d(x, y) \in s(Tx, Ty) \tag{2}$$

for all $x, y \in X$ $(x \neq y)$, then T has a fixed point in X.

Proof. Let $x_0 \in X$, $x_1 \in Tx_0$ and assume $x_0 \notin Tx_0$. Then from (2), we have

$$\varphi(d(x_0, x_1))\, d(x_0, x_1) \in s(Tx_0, Tx_1).$$

Now by Lemma 2.1(3), we have $\phi(d(x_0, x_1))d(x_0, x_1) \in s(x_1, Tx_1)$. By definition, we can take $x_2 \in Tx_1$ such that $\phi(d(x_0, x_1))d(x_0, x_1) \in s(d(x_1, x_2))$. So, $d(x_1, x_2) \preccurlyeq \phi(d(x_0, x_1))d(x_0, x_1)$.

Inductively, we can construct a sequence $\{x_n\}$ in X such that for all $n \in \mathbb{N}$, we have

$$d(x_n, x_{n+1}) \preccurlyeq \varphi(d(x_{n-1}, x_n))\, d\,(x_{n-1}, x_n), x_{n+1} \in Tx_n. \tag{3}$$

If $x_n = x_{n+1}$ for some $n \in \mathbb{N}$, then T has a fixed point.

We may assume that $x_n \neq x_{n+1}$, for all $n \in \mathbb{N}$. From (3), $\{d(x_n, x_{n+1})\}$ s a decreasing sequence in P. Hence, from (1), there exists $r \in 0, 1)$ such that

$$\overline{\lim_{n\to\infty}}\, \varphi(d(x_n, x_{n+1})) = r.$$

Thus, for any $\lambda \in (r, 1)$, there exists $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, we have $\phi(d(x_n, x_{n+1})) < \lambda$. Then we get, for $n \geq n_0$,

$$d(x_n, x_{n+1}) \preccurlyeq \varphi(d(x_{n-1}, x_n))\, d\,(x_{n-1}, x_n) \prec \lambda d(x_{n-1}, x_n) \prec \lambda^{n-n_0} d(x_{n_0}, x_{n_0+1}).$$

For $m > n \geq n_0$, we have

$$\begin{aligned} d(x_n, x_m) &\preccurlyeq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d\,(x_{m-1}, x_m) \\ &\preccurlyeq (\lambda^{n-n_0} + \lambda^{n+1-n_0} + \cdots + \lambda^{m-1-n_0})\, d\,(x_{n_0}, x_{n_0+1}) \\ &\preccurlyeq \lambda^n \left[\frac{1}{\lambda^{n_0}(1-\lambda)}\right] d(x_{n_0}, x_{n_0+1}). \end{aligned}$$

Since $\lambda^n \to 0$ as $n \to \infty$, we obtain that $\lambda^n \left[\frac{1}{\lambda^{n_0}(1-\lambda)}\right] d(x_{n_0}, x_{n_0+1}) \to \theta$ in the locally convex space E, as $n \to \infty$. Now, according to (p_7) and (p_1), we can conclude that for every $c \in E$ with $\theta \ll c$ there is a natural number n_1 such that $d\,(x_n, x_m) \ll c$ for all $m, n \geq \max\,\{n_0, n_1\}$, so $\{x_n\}$ is a tvs-cone Cauchy sequence. Since (X, d) is tvs-cone complete, then $\{x_n\}$ is tvs-cone convergent in X and cone-$\lim_{n\to\infty} x_n = x$, that is, for every $c \in E$ with $\theta \ll c$, there is a natural number k such that $d(x_n, x) \ll c$ for all $n \geq k$.

We now show that $x \in Tx$. Indeed, from (2) we have $\phi\,(d\,(x_n, x))\, d\,(x_n, x) \in s\,(Tx_n, Tx) = s\,(x_{n+1}, Tx)$ for $n \in \mathbb{N}$. By Lemma 2.1(3), there exists $y_n \in Tx$ such that

$$\varphi(d(x_n, x))\, d\,(x_n, x) \in s(d(x_{n+1}, y_n)).$$

Hence, $d\,(x_{n+1}, y_n) \preccurlyeq \phi\,(d\,(x_n, x))\, d\,(x_n, x) \preccurlyeq d\,(x_n, x)$. Moreover, for a given $c \in intP$, we have

$$\begin{aligned} d(x, y_n) &\preccurlyeq d(x, x_{n+1}) + d\,(x_{n+1}, y_n) \\ &\preccurlyeq d\,(x, x_{n+1}) + d(x_n, x) \\ &\ll \frac{c}{2} + \frac{c}{2} = c, \text{for } n \geq k = k(c). \end{aligned}$$

Hence, according to Definition 1.3(i), we have that cone-$\lim_{n\to\infty} y_n = x$. As Tx is closed, then $x \in Tx$, hence x is a fixed point of T and this ends the proof. □

The next example shows that Theorem 2.3 is a proper generalization of the main result from [11]. Indeed, as in Example 2.4, the cone P is nonnormal, so Theorem 2.1 of [11] is not applicable.

Example **2.4**. Let $X = [0, 1]$ and $E = C[0, 1]$ be endowed with the strongest locally convex topology $\tau(E, E^*)$, and let $P = \{x \in E :, x(t) \geq 0, t\ 0[1]\}$. Then the cone P is a normal cone with respect to the norm of the space E with a coefficient of normality $K = 1$. Also, P is a solid cone. Since $\tau(E, E^*)$ is stronger than a norm-topology, then the interior of P is nonempty with respect to $\tau(E, E^*)$. Thus, P is $\tau(E, E^*)$-solid. This cone is nonnormal with respect to the topology $\tau(E, E^*)$. Indeed, if it were normal, then according to Theorem 2.1 of [14], the space $(E, \tau\ (E, E\ ))$ would be normed, which is impossible as an infinite-dimensional space with the strongest locally convex topology cannot be metrizable (see [31]). Let us define the tvs-cone metric with $d : X \times X \to E$, by $d(x, y)(t) := |x - y|e^t$. Let $\mathcal{A}$ be a family of nonempty closed subsets of X of the form $\mathcal{A} = \{[0, x]; x \in X\}$. Consider a mapping $T : X \to \mathcal{A}$ defined by $T(x) = [0, \frac{x}{3}]$. Let $\varphi(c) = \frac{1}{2}$ for all $c \in P$. Obviously, the hypothesis (1) is satisfied. We now show that (2) is also a satisfied. Moreover, for $x, y ⌊ X (x \neq y)$ we have,

$$\begin{aligned}
\varphi(d(x, y))\, d\,(x, y) &\in s(Tx, Ty) \Leftrightarrow \frac{1}{2}d(x, y) \in s(Tx, Ty) \\
&\Leftrightarrow \frac{1}{2}d(x, y) \in \left(\bigcap_{a\in T}\bigcup_{xb\in Ty} s(d(a, b))\right) \cap \left(\bigcap_{b\in Ty}\bigcup_{a\ \in Tx} s(d(a, b))\right) \\
&\Leftrightarrow (\exists a \in Tx)\ (\exists b \in Ty)\ \frac{1}{2}d(x, y) \in s(d(a, b)) \\
&\Leftrightarrow d(a, b) \preccurlyeq \frac{1}{2}d(x, y).
\end{aligned}$$

Now, taking $a = \frac{1}{3}x$, and $b = \frac{1}{3}y$, we obtain that the hypothesis (2) is satisfied. Hence using Theorem 2.3, it follows that T has a fixed point.

Example **2.5**. Let $E = C^1_{\mathbb{R}}[0, 1]$ with a norm $||u|| = ||u||_\infty + ||u'||_\infty$, $u \in E$ and let $P = \{u \in E : u(t) \geq 0, t \in 0[1]\}$. It is well known that this cone is solid but it is not normal. Now consider the space $E = C^1_{\mathbb{R}}[0, 1]$ endowed with the strongest locally convex topology t^*. Then P is also t^* -solid (it has nonempty t^*-interior), but not t^* -normal. (For more details, see [31], Example 2.2).

Let $X = \{a, b, c\}$ and define a tvs-cone metric $d : X \times X \to P$ by

$$\begin{aligned}
d(a, b)(t) &:= 2 + 3t,\ d\,(b, c)(t) := 5 - 3t,\ d(a, c)(t) := 3, \\
d(x, y) &= d\,(y, x) \text{ and } d(x, x) = \theta \text{ for } x, y \in X.
\end{aligned}$$

Then (X, d) is a complete tvs-cone metric space over the nonnormal cone P . Now, consider the mapping $T : X \to X$ which is given by $Ta = \{a, b\}$, $Tb = \{a, c\}$ and $Tc = \{a, b, c\}$. Let $\varphi(c) = \frac{1}{2}$, for all $c \in P$. It is clear that the hypothesis (1) is satisfied. So let us prove that (2) is also satisfied, that is $\frac{1}{2}d(x, y) \in s(Tx, Ty)$, for all$x, y \in X$ $(x \neq y)$. Now, we have the following:

$1^0\ \frac{1}{2}d(a, b) \in s(Ta, Tb) = s(\{a, b\}, \quad \{a, c\}) \Leftrightarrow \exists a_1 \in Ta, \exists b_1 \in Tb$ such that $d(a_1, b_1) \preccurlyeq \frac{1}{2}d(a, b)$ Take $a_1 = b_1 = a$;

$2^0\ \frac{1}{2}d(a, c) \in s(Ta, Tc) = s(\{a, b\}, \quad \{a, b, c\}) \Leftrightarrow \exists a_2 \in Ta, \exists b_2 \in Tc$ such that $d(a_2, b_2) \preccurlyeq \frac{1}{2}d(a, b)$. Take $a_2 = b_2 = a$;

$3^0 \frac{1}{2}d(a,\ b) \in s(Tb,\ Tc) = s(\{a,\ c\},\quad \{a,\ b,\ c\}) \Leftrightarrow \exists a_3 \in Tb, \exists b_3 \in Tc\, such \quad that$ $d(a_3, b_3) \preccurlyeq \frac{1}{2}d(a,b)$. Take $a_3 = b_3 = a$.

Therefore, all conditions of Theorem 2.3 are satisfied and hence T has a fixed point. Precisely, $x = a$ and $x = c$ are the fixed points of T.

Finally, we finish our paper by introducing the following consequence corollaries of our main theorem. let (X, d) be a given metric space, and let us define the following:

- $CB\,(X) = \{A : A$ is a nonempty closed and bounded subset of $X\}$,
- $D\,(a, B) = \inf\,\{d\,(a, b)\colon b \in B \bullet X\}$, for $a \in X$,
- $H\,(A, B) = \max\,\{\sup\,\{D\,(a, B)\colon a \in A\}\,,\ \sup\,\{D\,(b, A)\colon b \in B\}\}$.

It is clear that H is a metric on $CB\,(X)$, which is called the Hausdorff-Pompeu metric induced by d. A set-valued mapping $T : X \to CB(X)$ is said to be a multi-valued contraction mapping, if there exists a fixed real number λ, $0 \le \lambda < 1$ such that,

$$H(Tx,\ Ty) \le \lambda d(x,\ y),$$

for all $x, y \in X$. A point $x \in X$ is called a fixed point of T , if $x \in Tx$. Then as a consequence of Theorem 2.3 and in particular by taking $E = \mathbb{R}$, $P = [0, +\infty)$, $\mathcal{A} = CB(X)$,$\phi(c) = \lambda$, for all $c \in P$, we obtain the following corollary.

Corollary 2.6. (Nadler [30]) *Let (X, d) be a complete metric space and let $T : X \to CB(X)$ be a multi-valued contraction mapping. Then T has a fixed point.*

Also, according to Remark 2.2, we obtain the following corollary.

Corollary 2.7. (Mizoguchi-Takahashi [29]) *Let (X, d) be a complete metric space and let $T : X \to 2^X$ be a multi-valued mapping such that, Tx is a closed bounded subset of X, for all $x \in X$. If there exists a function $\phi : [0, +\infty) \to [0, 1)$ such that,*

$$\overline{\lim_{r \to t+}}\, \varphi(r) < 1 \ for\ all\ t \in [0,\ +\infty)$$

and if

$$H(Tx,\ Ty)\ \le\ \varphi(d(x,\ y))d(x,\ y),$$

for all $x, y \in X (x \ne y)$, then T has a fixed point in X.

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Authors' contributions
All the authors contributed equally. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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