Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH   **Open Access**

Common fixed point theorems for dominating and weak annihilator mappings in ordered metric spaces

Wasfi Shatanawi[1] and Mihai Postolache[2*]

*Correspondence:
mihai@mathem.pub.ro
[2]Faculty of Applied Sciences,
University Politehnica of Bucharest,
313 Splaiul Independenţei,
Bucharest, 060042, Romania
Full list of author information is
available at the end of the article

Abstract

Very recently, Haghi *et al.* (Topol. Appl. 160:450-454, 2013) proved that some fixed point theorems in partial metric spaces can be obtained from metric spaces. In this paper, we prove some common fixed point theorems for four mappings f, g, S and T satisfying a nonlinear contraction in ordered metric spaces, where the mappings f and g are dominating and weak annihilators of the mappings T and S, respectively. We utilize the techniques of Haghi *et al.* to derive our main result, which is a generalization of the result of Shobkolaei *et al.* (Appl. Math. Comput. 219:443-452, 2012). Also, we introduce an example to support the usability of our results.

MSC: Primary 54H25; secondary 47H10

Keywords: partial metric space; common fixed point; altering distance function; almost contraction; ordered metric space

1 Introduction

One of the most important problems in mathematical analysis is to establish existence and uniqueness theorems for some integral and differential equations. Fixed point theory, in ordered metric spaces, plays a major role in solving such kind of problems. The first result in this direction was obtained by Ran and Reurings [1]. This one was extended for nondecreasing mappings by Nieto and Lopez [2, 3]. Meanwhile, Agarwal *et al.* [5] and O'Regan and Petruşel [4] studied some results for generalized contractions in ordered metric spaces. Then, many authors obtained fixed point results in ordered metric spaces. For some works in ordered metric spaces, we refer the reader to [6–19].

Berinde [20–24] initiated the concept of almost contraction and studied existence fixed point results for almost contraction in complete metric spaces. Later, many authors studied different types of almost contractions and studied fixed point results; for example, see [25–31].

In 1994 Matthews [32] introduced the concept of partial metric spaces and proved the Banach contraction principle in these spaces. Then, many authors obtained interesting results in partial metric spaces [33, 36–38]. Very recently, Haghi *et al.* [33] proved that some fixed point theorems in partial metric spaces can be obtained from metric spaces.

The purpose of this paper is to prove some common fixed point theorems for four mappings f, g, S and T satisfying a nonlinear contraction in ordered metric spaces, where the

Springer

mappings f and g are dominating and weak annihilators of the mappings T and S, respectively. We utilize the results of Haghi *et al.* [33] to derive Theorem 2.1.

2 Previous notations and results

In the sequel, we have to recall previous notations and results.

Let f and g be self-mappings of a set X. If $w = fx = gx$ for some $x \in X$, then x is called a *coincidence point* of f and g, and w is called a *point of coincidence* of f and g. Two self-mappings f and g are said to be *weakly compatible* if they commute at their coincidence point, that is, if $fx = gx$, then $fgx = gfx$. For details, please see [34].

Now, consider $(X, \preceq)$ to be a partially ordered set. According to [35], a mapping f is called *weak annihilator* of g if $fgx \preceq x$ for all $x \in X$ and f is called *dominating* if $x \preceq fx$ for all $x \in X$.

Recently, Shobkolaei *et al.* [38] initiated the concept of almost generalized (S, T)-contractive condition in a partial metric space and studied some common fixed point results in partial metric spaces.

Definition 2.1 [38] Let f, g, S and T be self-maps on a partial metric space (X, p). Then f and g are said to satisfy *almost generalized (S, T)-contractive condition* if there exists $\delta \in [0, 1)$ such that

$$p(fx, gy) \leq \delta \max \left\{ p(Sx, Ty), p(fx, Sx), p(gy, Ty), \frac{p(Sx, gy) + p(fx, Ty)}{2} \right\} \tag{2.1}$$

for all $x, y \in X$.

Theorem 2.1 [38] *Let $(X, p, \preceq)$ be a complete ordered partial metric space. Let $f, g, T, S : X \to X$ be mappings with $fX \subseteq TX$ and $gX \subseteq SX$, and the dominating mappings f and g are weak annihilators of T and S, respectively. Suppose that f and g satisfy almost generalized (S, T)-contractive condition* (2.1) *for any two comparable elements $x, y \in X$. If for a nondecreasing sequence $\{x_n\}$ with $x_n \preceq y_n$ for all n but finitely many and $y_n \to u$ implies that $x_n \preceq u$, and furthermore*

(1) *$\{f, S\}$ and $\{g, T\}$ are weakly compatible;*

(2) *one of fX, gX, SX and TX is a closed subspace of X,*

then f, g, S and T have a common fixed point.

Moreover, the set of common fixed points of f, g, S and T is well ordered if and only if f, g, S and T have one and only one common fixed point.

3 Main result

Let Ψ denote all functions $\psi : [0, +\infty) \to [0, +\infty)$ such that

(1) ψ is continuous and nondecreasing;

(2) $\psi(t) = 0$ if and only if $t = 0$.

Also, let Φ denote all functions $\phi : [0, +\infty) \times [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ such that

(1) ϕ is continuous;

(2) $\phi(t, s, u) = 0$ if and only if $u = s = t = 0$.

If $\psi \in \Psi$, then ψ is called an *altering distance function* (see [39]).

Now, we introduce our definition.

Definition 3.1 Let f, g, S and T be self-mappings on a metric space (X, d). Then f and g are said to *satisfy the almost nonlinear (S, T, L, ψ, ϕ)-contractive condition* if there exist $\psi \in \Psi$, $\phi \in \Phi$ and $L \in [0, +\infty)$ such that

$$\psi\big(d(fx, gy)\big) \leq \psi\left(\max\left\{d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2}\big(d(Sx, gy) + d(fx, Ty)\big)\right\}\right)$$

$$- \phi\big(d(Sx, Ty), d(Sx, gy), d(fx, Ty)\big)$$

$$+ L \min\big\{d(Sx, Ty), d(Sx, gy), d(fx, Ty)\big\} \tag{3.1}$$

for all $x, y \in X$.

Now, let $(X, d, \preceq)$ be an ordered metric space. We say that X *satisfies the property (π)* if the following statement holds true.

(π) If $\{x_n\}$ is a nondecreasing sequence in X and $\{y_n\}$ is a sequence in X such that $x_n \preceq y_n$ for all n but finitely many and $y_n \to u$, then $x_n \preceq u$ for all n but finitely many.

In the rest of this paper, $\mathbb{N}$ stands for the set of nonnegative integer numbers.

Theorem 3.1 Let $(X, d, \preceq)$ be a complete ordered metric space. Let $f, g, T, S : X \to X$ be four mappings such that f and g satisfy the nonlinear (S, T, L, ψ, ϕ)-contractive condition (3.1) for any two comparable elements $x, y \in X$. Suppose that f, g, S and T satisfy the following conditions:
  (1) $fX \subseteq TX$;
  (2) $gX \subseteq SX$;
  (3) $La - \phi(a, a, a) < 0$ for all $a > 0$;
  (4) f is dominating and weak annihilator of T;
  (5) g is dominating and weak annihilator of S;
  (6) $\{f, S\}$ and $\{g, T\}$ are weakly compatible;
  (7) one of fX, gX, SX and TX is a closed subspace of X.
If X has the property (π), then f, g, S and T have a common fixed point.

Proof Let $x_0 \in X$. Since $fX \subseteq TX$, we choose $x_1 \in X$ such that $fx_0 = Tx_1$. Also, since $gX \subseteq SX$, we choose $x_2 \in X$ such that $gx_1 = Sx_2$. Continuing this process, we can construct a sequence $\{y_n\}$ in X such $y_{2n} := fx_{2n} = Tx_{2n+1}$ and $y_{2n+1} := gx_{2n+1} = Sx_{2n+2}$. Since f is dominating and weak annihilator of T and g is dominating and weak annihilator of S,

$$x_{2n} \preceq fx_{2n} = Tx_{2n+1} \preceq fTx_{2n+1} \preceq x_{2n+1} \preceq gx_{2n+1} = Sx_{2n+2} \preceq gSx_{2n+2} \preceq x_{2n+2}.$$

Thus, for all $n \geq 0$, we have $x_n \preceq x_{n+1}$.

Suppose $y_{2n} = y_{2n+1}$ for some $n \in \mathbb{N}$. We claim that $y_{2n+1} = y_{2n+2}$. Since x_{2n+2} and x_{2n+3} are comparable, we have

$$\psi\big(d(y_{2n+2}, y_{2n+1})\big)$$

$$= \psi\big(d(fx_{2n+2}, gx_{2n+1})\big)$$

$$\leq \psi\left(\max\left\{d(Sx_{2n+2}, Tx_{2n+1}), d(fx_{2n+2}, Sx_{2n+2}), d(gx_{2n+1}, Tx_{2n+1}),\right.\right.$$

$$\frac{1}{2}\big(d(Sx_{2n+2}, gx_{2n+1}) + d(fx_{2n+2}, Tx_{2n+1})\big)\Big\}\Big)$$

$$- \phi\big(d(Sx_{2n+2}, Tx_{2n+1}), d(Sx_{2n+2}, gx_{2n+1}), d(fx_{2n+2}, Tx_{2n+1})\big)$$

$$+ L\min\big\{d(Sx_{2n+2}, Tx_{2n+1}), d(Sx_{2n+2}, gx_{2n+1}), d(fx_{2n+2}, Tx_{2n+1})\big\}$$

$$= \psi\Big(\max\Big\{d(y_{2n+1}, y_{2n}), d(y_{2n+2}, y_{2n+1}), d(y_{2n+1}, y_{2n}),$$

$$\frac{1}{2}\big(d(y_{2n+1}, y_{2n+1}) + d(y_{2n+2}, y_{2n})\big)\Big\}\Big) - \phi\big(d(y_{2n+1}, y_{2n}), d(y_{2n+1}, y_{2n+1}), d(y_{2n+2}, y_{2n})\big)$$

$$+ L\min\big\{d(y_{2n+1}, y_{2n}), d(y_{2n+1}, y_{2n+1}), d(y_{2n+2}, y_{2n})\big\}$$

$$= \psi\Big(\max\Big\{0, d(y_{2n+2}, y_{2n+1}), \frac{1}{2}\big(d(y_{2n+2}, y_{2n+1})\big)\Big\}\Big) - \phi\big(0, 0, d(y_{2n+2}, y_{2n+1})\big)$$

$$= \psi\big(d(y_{2n+2}, y_{2n+1})\big) - \phi\big(0, 0, d(y_{2n+2}, y_{2n+1})\big)$$

$$\le \psi\big(d(y_{2n+2}, y_{2n+1})\big).$$

Therefore, $\phi(0, 0, d(y_{2n+2}, y_{2n+1})) = 0$ and hence $y_{2n+2} = y_{2n+1}$. Similarly, we may show that $y_{2n+3} = y_{2n+2}$. Thus $\{y_n\}$ is a constant sequence in X, hence it is a Cauchy sequence in (X, d).

Suppose $y_{2n} \ne y_{2n+1}$ for all $n \in \mathbb{N}$. Given $n \in \mathbb{N}$. If n is even, then $n = 2t$ for some $t \in \mathbb{N}$. Since x_{2t} and x_{2t+1} are comparable, we have

$$\psi\big(d(y_n, y_{n+1})\big)$$

$$= \psi\big(d(y_{2t}, y_{2t+1})\big)$$

$$= \psi\big(d(fx_{2t}, gx_{2t+1})\big)$$

$$\le \psi\Big(\max\Big\{d(Sx_{2t}, Tx_{2t+1}), d(fx_{2t}, Sx_{2t}), d(gx_{2t+1}, Tx_{2t+1}),$$

$$\frac{1}{2}\big(d(Sx_{2t}, gx_{2t+1}) + d(fx_{2t}, Tx_{2t+1})\big)\Big\}\Big)$$

$$- \phi\big(d(Sx_{2t}, Tx_{2t+1}), d(Sx_{2t}, gx_{2t+1}), d(fx_{2t}, Tx_{2t+1})\big)$$

$$+ L\min\big\{d(Sx_{2t}, Tx_{2t+1}), d(Sx_{2t}, gx_{2t+1}), d(fx_{2t}, Tx_{2t+1})\big\}$$

$$= \psi\Big(\max\Big\{d(y_{2t-1}, y_{2t}), d(y_{2t}, y_{2t+1}), \frac{1}{2}\big(d(y_{2t-1}, y_{2t+1})\big)\Big\}\Big)$$

$$- \phi\big(d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1}), 0\big)$$

$$\le \psi\Big(\max\Big\{d(y_{2t-1}, y_{2t}), d(y_{2t}, y_{2t+1}), \frac{1}{2}\big(d(y_{2t-1}, y_{2t}) + d(y_{2t}, y_{2t+1})\big)\Big\}\Big)$$

$$- \phi\big(d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1}), 0\big)$$

$$= \psi\big(\max\big\{d(y_{2t-1}, y_{2t}), d(y_{2t}, y_{2t+1})\big\}\big) - \phi\big(d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1}), 0\big)$$

$$\le \psi\big(\max\big\{d(y_{2t-1}, y_{2t}), d(y_{2t}, y_{2t+1})\big\}\big).$$

If

$$\max\big\{d(y_{2t-1}, y_{2t}), d(y_{2t}, y_{2t+1})\big\} = d(y_{2t}, y_{2t+1}),$$

then

$$\phi\big(d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1}), 0\big) = 0,$$

and hence $d(y_{2t-1}, y_{2t}) = d(y_{2t-1}, y_{2t+1}) = 0$. Thus $y_{2t-1} = y_{2t}$, a contradiction.

Thus,

$$\max\big\{d(y_{2t-1}, y_{2t}), d(y_{2t}, y_{2t+1})\big\} = d(y_{2t-1}, y_{2t}). \tag{3.2}$$

Therefore,

$$\psi\big(d(y_{2t}, y_{2t+1})\big) \leq \psi\big(d(y_{2t-1}, y_{2t})\big) - \phi\big(d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1}), 0\big). \tag{3.3}$$

If n is odd, then $n = 2t + 1$ for some $t \in \mathbb{N}$. Since x_{2t+2} and x_{2t+1} are comparable, we have

$$\psi(d(y_n, y_{n+1}))$$
$$= \psi\big(d(y_{2t+2}, y_{2t+1})\big)$$
$$= \psi\big(d(fx_{2t+2}, gx_{2t+1})\big)$$
$$\leq \psi\bigg(\max\bigg\{d(Sx_{2t+2}, Tx_{2t+1}), d(fx_{2t+2}, Sx_{2t+2}), d(gx_{2t+1}, Tx_{2t+1}),$$
$$\frac{1}{2}\big(d(Sx_{2t+2}, gx_{2t+1}) + d(fx_{2t+2}, Tx_{2t+1})\big)\bigg\}\bigg)$$
$$- \phi\big(d(Sx_{2t+2}, Tx_{2t+1}), d(Sx_{2t+2}, gx_{2t+1}), d(fx_{2t+2}, Tx_{2t+1})\big)$$
$$+ L\min\big\{d(Sx_{2t+2}, Tx_{2t+1}), d(Sx_{2t+2}, gx_{2t+1}), d(fx_{2t+2}, Tx_{2t+1})\big\}$$
$$= \psi\bigg(\max\bigg\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1}), \frac{1}{2}\big(d(y_{2t+2}, y_{2t})\big)\bigg\}\bigg)$$
$$- \phi\big(d(y_{2t+1}, y_{2t}), 0, d(y_{2t+2}, y_{2t})\big)$$
$$\leq \psi\bigg(\max\bigg\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1}), \frac{1}{2}\big(d(y_{2t+2}, y_{2t+1}) + d\big(d(y_{2t+1}, y_{2t})\big)\big)\bigg\}\bigg)$$
$$- \phi\big(d(y_{2t+1}, y_{2t}), 0, d(y_{2t+2}, y_{2t})\big)$$
$$= \psi\big(\max\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1})\}\big) - \phi\big(d(y_{2t+1}, y_{2t}), 0, d(y_{2t+2}, y_{2t})\big)$$
$$\leq \psi\big(\max\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1})\}\big).$$

If

$$\max\big\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1})\big\} = d(y_{2t+2}, y_{2t+1}),$$

then

$$\phi\big(d(y_{2t+1}, y_{2t}), 0, d(y_{2t+2}, y_{2t})\big) = 0,$$

and hence $d(y_{2t+1}, y_{2t}) = d(y_{2t+2}, y_{2t}) = 0$. Thus $y_{2t+1} = y_{2t}$, a contradiction. So,

$$\max\big\{d(y_{2t+1}, y_{2t}), d(y_{2t+2}, y_{2t+1})\big\} = d(y_{2t+1}, y_{2t}). \tag{3.4}$$

Therefore

$$\psi\big(d(y_{2t+2}, y_{2t+1})\big) \leq \psi\big(d(y_{2t+1}, y_{2t})\big) - \phi\big(d(y_{2t+1}, y_{2t}), 0, d(y_{2t+2}, y_{2t})\big). \tag{3.5}$$

From (3.2) and (3.4), we have

$$d(y_n, y_{n+1}) \leq d(y_{n-1}, y_n). \tag{3.6}$$

Therefore $\{d(y_{n+1}, y_n) : n \in \mathbb{N}\}$ is a nonincreasing sequence. Thus there exists $r \geq 0$ such that

$$\lim_{n \to +\infty} d(y_n, y_{n+1}) = r.$$

On taking $\limsup$ in (3.3) and (3.5), we have

$$\psi(r) \leq \psi(r) - \liminf_{t \to +\infty} \phi\big(d(y_{2t-1}, y_{2t}), d(y_{2t-1}, y_{2t+1}), 0\big)$$

and

$$\psi(r) \leq \psi(r) - \liminf_{t \to +\infty} \phi\big(d(y_{2t+1}, y_{2t}), 0, d(y_{2t}, y_{2t+2})\big).$$

Thus

$$\liminf_{t \to +\infty} d(y_{2t-1}, y_{2t}) = \liminf_{t \to +\infty} d(y_{2t-1}, y_{2t+1}) = \liminf_{t \to +\infty} d(y_{2t}, y_{2t+2}) = \liminf_{t \to +\infty} d(y_{2t}, y_{2t+1}) = 0.$$

Therefore, $r = 0$ and hence

$$\lim_{n \to +\infty} d(y_n, y_{n+1}) = 0. \tag{3.7}$$

Now, we show that $\{y_n\}$ is a Cauchy sequence in the metric space (X, d). It is sufficient to show that $\{y_{2n}\}$ is a Cauchy sequence in (X, d). Suppose to the contrary; that is, $\{y_{2n}\}$ is not a Cauchy sequence in (X, d). Then there exists $\epsilon > 0$ for which we can find two subsequences $\{y_{2m(i)}\}$ and $\{y_{2n(i)}\}$ of $\{y_{2n}\}$ such that $n(i)$ is the smallest index for which

$$n(i) > m(i) > i, \quad d(y_{2m(i)}, y_{2n(i)}) \geq \epsilon \tag{3.8}$$

and

$$d(y_{2m(i)}, y_{2n(i)-2}) < \epsilon. \tag{3.9}$$

From (3.8), (3.9) and the triangular inequality, we get that

$$\epsilon \leq d(y_{2m(i)}, y_{2n(i)})$$
$$\leq d(y_{2m(i)}, y_{2n(i)-2}) + d(y_{2n(i)-2}, y_{2n(i)-1}) + d(y_{2n(i)-1}, y_{2n(i)})$$
$$< \epsilon + d(y_{2n(i)-2}, y_{2n(i)-1}) + d(y_{2n(i)-1}, y_{2n(i)}).$$

On letting $i \to +\infty$ in the above inequalities and using (3.7), we have

$$\lim_{i \to +\infty} d(y_{2m(i)}, y_{2n(i)}) = \epsilon. \tag{3.10}$$

Again, from (3.8) and the triangular inequality, we get that

$$\epsilon \le d(y_{2m(i)}, y_{2n(i)})$$
$$\le d(y_{2n(i)}, y_{2n(i)-1}) + d(y_{2n(i)-1}, y_{2m(i)})$$
$$\le d(y_{2n(i)}, y_{2n(i)-1}) + d(y_{2n(i)-1}, y_{2m(i)+1}) + d(y_{2m(i)+1}, y_{2m(i)})$$
$$\le d(y_{2n(i)}, y_{2n(i)-1}) + d(y_{2n(i)-1}, y_{2m(i)}) + 2d(y_{2m(i)+1}, y_{2m(i)})$$
$$\le 2d(y_{2n(i)}, y_{2n(i)-1}) + d(y_{2n(i)}, y_{2m(i)}) + 2d(y_{2m(i)+1}, y_{2m(i)}).$$

Letting $i \to +\infty$ in the above inequalities and using (3.7) and (3.10), we get that

$$\lim_{i \to +\infty} d(y_{2m(i)}, y_{2n(i)}) = \lim_{i \to +\infty} d(y_{2m(i)+1}, y_{2n(i)-1})$$
$$= \lim_{i \to +\infty} d(y_{2m(i)+1}, y_{2n(i)})$$
$$= \lim_{i \to +\infty} d(y_{2m(i)}, y_{2n(i)-1})$$
$$= \epsilon.$$

Since $x_{2n(i)}$ and $x_{2m(i)+1}$ are comparable, we have

$$\psi\big(d(y_{2n(i)}, y_{2m(i)+1})\big)$$
$$= \psi\big(d(fx_{2n(i)}, gx_{2m(i)+1})\big)$$
$$\le \psi\bigg(\max\bigg\{d(Sx_{2n(i)}, Tx_{2m(i)+1}), d(fx_{2n(i)}, Sx_{2n(i)}), d(gx_{2m(i)+1}, Tx_{2m(i)+1}),$$
$$\frac{1}{2}\big(d(Sx_{2n(i)}, gx_{2m(i)+1}) + d(fx_{2n(i)}, Tx_{2m(i)+1})\big)\bigg\}\bigg)$$
$$- \phi\big(d(Sx_{2n(i)}, Tx_{2m(i)+1}), d(Sx_{2n(i)}, gx_{2m(i)+1}), d(fx_{2n(i)}, Tx_{2m(i)+1})\big)$$
$$+ L\min\big\{d(Sx_{2n(i)}, Tx_{2m(i)+1}), d(Sx_{2n(i)}, gx_{2m(i)+1}), d(fx_{2n(i)}, Tx_{2m(i)+1})\big\}$$
$$= \psi\bigg(\max\bigg\{d(y_{2n(i)-1}, y_{2m(i)}), d(y_{2n(i)}, y_{2n(i)-1}), d(y_{2m(i)+1}, y_{2m(i)}),$$
$$\frac{1}{2}\big(d(y_{2n(i)}, y_{2m(i)}) + d(y_{2n(i)-1}, y_{2m(i)+1})\big)\bigg\}\bigg)$$
$$- \phi\big(d(y_{2n(i)-1}, y_{2m(i)}), d(y_{2n(i)-1}, y_{2m(i)+1}), d(y_{2n(i)}, y_{2m(i)})\big)$$
$$+ L\min\big\{d(y_{2n(i)-1}, y_{2m(i)}), d(y_{2n(i)-1}, y_{2m(i)+1}), d(y_{2n(i)}, y_{2m(i)})\big\}.$$

Letting $i \to +\infty$ and using the continuity of ψ, we get that

$$\psi(\epsilon) \le \psi(\epsilon) - \phi(\epsilon, \epsilon, \epsilon) + L\epsilon.$$

By condition (3), we get $\psi(\epsilon) = 0$ and hence $\epsilon = 0$, a contradiction. Thus $\{y_n\}$ is a Cauchy sequence in (X, d).

Since (X, d) is complete, there is $y \in X$ such that $y_n \to y$ in the metric space (X, d). Thus

$$\lim_{n \to +\infty} d(y_n, y) = 0. \tag{3.11}$$

Now we show that y is the fixed point of g and T. Assume that TX is closed, since $\{y_{2n} = Tx_{2n+1}\}$ is a sequence in TX converging to y, we have $y \in TX$. So, there exists $u \in X$ such that $y = Tu$. Therefore,

$$\lim_{n \to +\infty} fx_{2n} = \lim_{n \to +\infty} gx_{2n+1} = \lim_{n \to +\infty} Tx_{2n+1} = \lim_{n \to +\infty} Sx_{2n+2} = y = Tu.$$

Now, we show that $gu = y$. Since $x_{2n} \preceq fx_{2n}$ and $y_{2n} = fx_{2n} \to y$, we have $x_{2n} \preceq y$. Since the mapping f is dominating and weak annihilator of T, we obtain $x_{2n} \preceq y = Tu \preceq fTu \preceq u$. Thus

$$\psi\big(d(y_{2n}, gu)\big)$$
$$= \psi\big(d(fx_{2n}, gu)\big)$$
$$\leq \psi\left(\max\left\{d(Sx_{2n}, Tu), d(fx_{2n}, Sx_{2n}), d(gu, Tu), \frac{1}{2}\big(d(Sx_{2n}, gu) + d(fx_{2n}, Tu)\big)\right\}\right)$$
$$\quad - \phi\big(d(Sx_{2n}, Tu), d(Sx_{2n}, gu), d(fx_{2n}, Tu)\big)$$
$$\quad + L\min\big\{d(Sx_{2n}, Tu), d(Sx_{2n}, gu), d(fx_{2n}, Tu)\big\}$$
$$= \psi\left(\max\left\{d(y_{2n-1}, y), d(y_{2n}, y_{2n-1}), d(gu, y), \frac{1}{2}\big(d(y_{2n-1}, gu) + d(y_{2n}, y)\big)\right\}\right)$$
$$\quad - \phi\big(d(y_{2n-1}, y), d(y_{2n-1}, gu), d(y_{2n}, y)\big) + L\min\big\{d(y_{2n-1}, y), d(y_{2n-1}, gu), d(y_{2n}, y)\big\}.$$

Letting $n \to +\infty$ in the above inequalities and using (3.7), we get that

$$\psi\big(d(y, gu)\big) \leq \psi\big(d(gu, y)\big) - \phi\big(0, d(y, gu), 0\big).$$

Therefore $\phi(0, d(y, gu), 0) = 0$ and hence $d(gu, y) = 0$. Thus $gu = y$. Since g and T are weakly compatible and $gu = gTu = Tgu = Ty$, we have

$$gy = gTu = Tgu = Ty.$$

Again, since x_{2n} and y are comparable, we have

$$\psi\big(d(y_{2n}, gy)\big)$$
$$= \psi\big(d(fx_{2n}, gy)\big)$$
$$\leq \psi\left(\max\left\{d(Sx_{2n}, Ty), d(fx_{2n}, Sx_{2n}), d(gy, Ty), \frac{1}{2}\big(d(Sx_{2n}, gy) + d(fx_{2n}, Ty)\big)\right\}\right)$$
$$\quad - \phi\big(d(Sx_{2n}, Ty), d(Sx_{2n}, gy), d(fx_{2n}, Ty)\big)$$
$$\quad + L\min\big\{d(Sx_{2n}, Ty), d(Sx_{2n}, gy), d(fx_{2n}, Ty)\big\}$$

$$= \psi \left(\max \left\{ d(y_{2n-1}, gy), d(y_{2n}, y_{2n-1}), d(gy, gy), \frac{1}{2} \big(d(y_{2n-1}, gy) + d(y_{2n}, gy) \big) \right\} \right)$$
$$- \phi \big(d(y_{2n-1}, gy), d(y_{2n-1}, gy), d(y_{2n}, gy) \big) + \min \{ d(y_{2n-1}, gy), d(y_{2n-1}, gy), d(y_{2n}, gy) \}.$$

On letting $n \to +\infty$ in the above inequalities and using (3.7), we have

$$\psi \big(d(y, gy) \big) \le \psi \big(d(y, gy) \big) - \phi \big(d(y, gy), d(y, gy), d(y, gy) \big) + L d(y, gy).$$

Using condition (3), we get $\psi(d(y, gy)) = 0$. Thus $d(y, gy) = 0$ and hence $gy = y$.

Finally, we have to show that y is also a fixed point of f and T. Since $gX \subseteq SX$, there exists $v \in X$ such that $y = gy = Sv$. Since the mapping g is dominating and weak annihilator of S, we have $y \preceq gy = Sv \preceq gSv \preceq v$. Thus y and v are comparable, and hence

$$\psi \big(d(fv, Sv) \big)$$
$$= \psi \big(d(fv, gy) \big)$$
$$\le \psi \left(\max \left\{ d(Sv, Ty), d(fv, Sv), d(gy, Ty), \frac{1}{2} \big(d(Sv, gy) + d(fv, Ty) \big) \right\} \right)$$
$$\quad - \phi \big(d(Sv, Ty), d(Sv, gy), d(fv, Ty) \big) + L \min \{ d(Sv, Ty), d(Sv, gy), d(fv, Ty) \}$$
$$= \psi \left(\max \left\{ d(Sv, Sv), d(fv, Sv), d(Sv, Sv), \frac{1}{2} \big(d(Sv, Sv) + d(fv, Sv) \big) \right\} \right)$$
$$\quad - \phi \big(d(Sv, Sv), d(Sv, Sv), d(fv, Sv) \big) + L \min \{ d(Sv, Sv), d(Sv, Sv), d(fv, Sv) \}$$
$$= \psi \big(d(fv, Sv) \big) - \phi \big(0, 0, d(fv, Sv) \big) + L d(fv, Sv).$$

Using condition (3), we get $\psi(d(fv, Sv)) = 0$. Thus $d(fv, Sv) = 0$ and hence $fv = Sv = gy = y$. Since f and S are weakly compatible, we have $fy = fSv = Sfv = Sy$. Since y and y are comparable, we have

$$\psi \big(d(fy, y) \big)$$
$$= \psi \big(d(fy, gy) \big)$$
$$\le \psi \left(\max \left\{ d(Sy, Ty), d(fy, Sy), d(gy, Ty), \frac{1}{2} \big(d(Sy, gy) + d(fy, Ty) \big) \right\} \right)$$
$$\quad - \phi \big(d(Sy, Ty), d(Sy, gy), d(fy, Ty) \big) + L \min \{ d(Sy, Ty), d(Sy, gy), d(fy, Ty) \}$$
$$= \psi \left(\max \left\{ d(fy, y), d(fy, fy), d(y, y), \frac{1}{2} \big(d(fy, y) + d(fy, y) \big) \right\} \right)$$
$$\quad - \phi \big(d(fy, y), d(fy, y), d(fy, y) \big) + L \min \{ d(fy, y), d(fy, y), d(fy, y) \}$$
$$= \psi \big(d(fy, y) \big) - \phi \big(d(fy, y), d(fy, y), d(fy, y) \big) + L d(fy, y).$$

Using condition (3), we get $\psi(d(fy, y)) = 0$. Thus $d(fy, y) = 0$ and hence $fy = y$. So, y is a common fixed point of f, g, T and S. In case SX, fX or gX is closed, the proof of the existence of a common fixed point is similar to the arguments above. □

Corollary 3.1 *Let $(X, d, \preceq)$ be a complete ordered metric space, and let $f, g, T, S : X \to X$ be four mappings. Assume that there exist $\psi \in \Psi$ and $\phi \in \Phi$ such that*

$$\psi\big(d(fx, gy)\big) \le \psi\left(\max\left\{d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2}\big(d(Sx, gy) + d(fx, Ty)\big)\right\}\right)$$
$$- \phi\big(\max\big\{d(Sx, Ty), d(Sx, gy), d(fx, Ty)\big\}\big)$$

holds for any two comparable elements $x, y \in X$. Suppose that f, g, S and T satisfy the following conditions:

  (1) *$fX \subseteq TX$*;
  (2) *$gX \subseteq SX$*;
  (3) *f is dominating and weak annihilator of T*;
  (4) *g is dominating and weak annihilator of S*;
  (5) *$\{f, S\}$ and $\{g, T\}$ are weakly compatible*;
  (6) *one of fX, gX, SX and TX is a closed subspace of X*.

If X satisfies the property (π), then f, g, S and T have a common fixed point.

Corollary 3.2 *Let $(X, d, \preceq)$ be a complete ordered metric space, and let $f, g, T, S : X \to X$ be four mappings. Assume that there exist $\phi_1, \psi \in \Psi$ such that*

$$\psi\big(d(fx, gy)\big) \le \psi\left(\max\left\{d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2}\big(d(Sx, gy) + d(fx, Ty)\big)\right\}\right)$$
$$- \phi_1\big(\max\big\{d(Sx, Ty), d(Sx, gy), d(fx, Ty)\big\}\big)$$

holds for any two comparable elements $x, y \in X$. Suppose that f, g, S and T satisfy the following conditions:

  (1) *$fX \subseteq TX$*;
  (2) *$gX \subseteq SX$*;
  (3) *f is dominating and weak annihilator of T*;
  (4) *g is dominating and weak annihilator of S*;
  (5) *$\{f, S\}$ and $\{g, T\}$ are weakly compatible*;
  (6) *one of fX, gX, SX and TX is a closed subspace of X*.

If X satisfies the property (π), then f, g, S and T have a common fixed point.

Proof Follows from Corollary 3.1 by defining $\phi : [0, +\infty) \times [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ via $\phi(s, t, u) = \phi_1(\max\{s, t, u\})$ and noting that $\phi \in \Phi$.   □

Corollary 3.3 *Let $(X, d, \preceq)$ be a complete ordered metric space, and let $f, g, T, S : X \to X$ be four mappings. Assume that there exist $\phi_1, \psi \in \Psi$ such that*

$$\psi\big(d(fx, gy)\big) \le \psi\left(\max\left\{d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2}\big(d(Sx, gy) + d(fx, Ty)\big)\right\}\right)$$
$$- \phi_1\left(\max\left\{d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2}\big(d(Sx, gy) + d(fx, Ty)\big)\right\}\right)$$

holds for any two comparable elements $x, y \in X$. Suppose that f, g, S and T satisfy the following conditions:

(1) $fX \subseteq TX$;

(2) $gX \subseteq SX$;

(3) f *is dominating and weak annihilator of* T;

(4) g *is dominating and weak annihilator of* S;

(5) $\{f, S\}$ *and* $\{g, T\}$ *are weakly compatible*;

(6) *one of* fX, gX, SX *and* TX *is a closed subspace of* X.

If X *satisfies the property* (π), *then* f, g, S *and* T *have a common fixed point*.

Proof By noting that

$$\phi_1 \big(\max \big\{ d(Sx, Ty), d(Sx, gy), d(fx, Ty) \big\} \big)$$

$$\leq \phi_1 \bigg(\max \bigg\{ d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2} \big(d(Sx, gy) + d(fx, Ty) \big) \bigg\} \bigg),$$

the proof follows from Corollary 3.2. □

Jachymski [40] proved that some conditions for generalized contractions in (ordered) metric spaces are equivalent. By the aid of Lemma 1 [40], we have the following result.

Theorem 3.2 *Let* f, g, S *and* T *be self-mappings on a partial ordered metric space* (X, d), *and set*

$$M(x, y) = \max \bigg\{ d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2} \big(d(Sx, gy) + d(fx, Ty) \big) \bigg\}.$$

Then the following are equivalent:

(i) *There exist* $\alpha \in [0, 1)$ *and* $\psi \in \Psi$ *such that*

$$\psi \big(d(fx, gy) \big) \leq \alpha \psi \big(M(x, y) \big)$$

for all comparable elements $x, y \in X$.

(ii) *There exist* $\psi, \phi \in \Psi$ *such that*

$$\psi \big(d(fx, gy) \big) \leq \psi \big(M(x, y) \big) - \phi \big(M(x, y) \big)$$

for all comparable elements $x, y \in X$.

(iii) *There exists a continuous nondecreasing function* $\phi : [0, +\infty) \to [0, +\infty)$ *such that* $\phi(t) < t$ *for all* $t > 0$ *and for any* $x, y \in X$,

$$d(fx, gy) \leq \phi \big(M(x, y) \big).$$

Proof Set $D = \{(M(x, y), d(fx, gy)) : x, y$ are two comparable elements in $X\}$. Then the proof follows from Lemma 1 of [40]. □

By the aid of Theorem 3.2 and Corollary 3.3, we have the following results.

Theorem 3.3 *Let* $(X, d, \preceq)$ *be a complete ordered metric space, and let* $f, g, T, S : X \to X$ *be four mappings. Assume that there exist* $\psi \in \Psi$ *and* $k \in [0, 1)$ *such that*

$$\psi\left(d(fx, gy)\right) \le k\psi\left(\max\left\{d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2}\big(d(Sx, gy) + d(fx, Ty)\big)\right\}\right)$$

holds, for any two comparable elements $x, y \in X$. *Suppose that* f, g, S *and* T *satisfy the following conditions:*

   (1) $fX \subseteq TX$;

   (2) $gX \subseteq SX$;

   (3) *f is dominating and weak annihilator of* T;

   (4) *g is dominating and weak annihilator of* S;

   (5) $\{f, S\}$ *and* $\{g, T\}$ *are weakly compatible*;

   (6) *one of* fX, gX, SX *and* TX *is a closed subspace of* X.

If X *satisfies the property* (π), *then* f, g, S *and* T *have a common fixed point.*

Theorem 3.4 *Let* $(X, d, \preceq)$ *be a complete ordered metric space, and let* $f, g, T, S : X \to X$ *be four mappings. Assume that there exists a continuous and nondecreasing function* $\phi : [0, +\infty) \to [0, +\infty)$ *with* $\phi(t) < t$ *for all* $t > 0$ *such that*

$$d(fx, gy) \le \phi\left(\max\left\{d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2}\big(d(Sx, gy) + d(fx, Ty)\big)\right\}\right)$$

holds for any two comparable elements $x, y \in X$. *Suppose that* f, g, S *and* T *satisfy the following conditions:*

   (1) $fX \subseteq TX$;

   (2) $gX \subseteq SX$;

   (3) *f is dominating and weak annihilator of* T;

   (4) *g is dominating and weak annihilator of* S;

   (5) $\{f, S\}$ *and* $\{g, T\}$ *are weakly compatible*;

   (6) *one of* fX, gX, SX *and* TX *is a closed subspace of* X.

If X *satisfies the property* (π), *then* f, g, S *and* T *have a common fixed point.*

As a direct result of our theorems, we have the following result.

Corollary 3.4 *Let* $(X, d, \preceq)$ *be a complete ordered metric space, and let* $f, g, T, S : X \to X$ *be four mappings. Assume that there exists* $k \in [0, 1)$ *such that*

$$d(fx, gy) \le k\max\left\{d(Sx, Ty), d(fx, Sx), d(gy, Ty), \frac{1}{2}\big(d(Sx, gy) + d(fx, Ty)\big)\right\}$$

holds for any two comparable elements $x, y \in X$. *Suppose that* f, g, S *and* T *satisfy the following conditions:*

   (1) $fX \subseteq TX$;

   (2) $gX \subseteq SX$;

   (3) *f is dominating and weak annihilator of* T;

   (4) *g is dominating and weak annihilator of* S;

   (5) $\{f, S\}$ *and* $\{g, T\}$ *are weakly compatible*;

(6) *one of fX, gX, SX and TX is a closed subspace of X.*
If X satisfies the property (π), then f, g, S and T have a common fixed point.

Remark 3.1 By using the method of Haghi *et al.* [33], its an easy matter to show that Theorem 2.1 is a consequence result of Corollary 3.4.

To support our results, we introduce the following example.

Example 3.1 On $X = \{0, 1, 2, 3, \ldots\}$, define

$$d : X \times X \to X, \qquad d(x, y) = \begin{cases} 0 & \text{if } x = y; \\ \max\{x, y\} & \text{if } x \neq y. \end{cases}$$

We introduce a relation on X by $x \preceq y$ if and only if $y \leq x$. Also, define $f, g, S, T : X \to X$ by the formulas

$$fx = gx = \begin{cases} 0 & \text{if } x \in \{0, 1, 2\}; \\ 3 & \text{if } x \in \{3, 4\}; \\ x - 2 & \text{if } x \geq 5 \end{cases}$$

and

$$Tx = Sx = \begin{cases} 0 & \text{if } x = 0; \\ x + 2 & \text{if } x \geq 1. \end{cases}$$

Consider $\psi : [0, +\infty) \to [0, +\infty)$, $\psi(t) = t^2$ and $\phi : [0, +\infty) \times [0, +\infty) \times [0, +\infty) \to [0, +\infty)$, $\phi(t, s, u) = \max\{t, s, u\}$. Then, for every two comparable elements $x, y \in X$, we have

$$\psi\left(d(fx, fy)\right) \leq \psi\left(\max\left\{d(Tx, Ty), d(fx, Tx), d(fy, Ty), \frac{1}{2}\left(d(Tx, fy) + d(fx, Ty)\right)\right\}\right)$$
$$- \phi\left(d(Tx, Ty), d(Tx, fy), d(fx, Ty)\right). \tag{3.12}$$

Proof Let

$$M(x, y) = \max\left\{d(Tx, Ty), d(fx, Tx), d(fy, Ty), \frac{1}{2}\left(d(Tx, fy) + d(fx, Ty)\right)\right\}$$

and

$$N(x, y) = \phi\left(d(Tx, Ty), d(Tx, fy), d(fx, Ty)\right) = \max\left\{d(Tx, Ty), d(Tx, fy), d(fx, Ty)\right\}.$$

Given $x, y \in X$, without loss of generality, we assume that $x \leq y$.

Now, we divide the proof into the following cases:

- CASE I: $x = y$. Here, we have $\psi(d(fx, fy)) = 0$ and get (3.12).
- CASE II: $x < y$ and $x, y \in \{0, 1, 2\}$. Here, $\psi(d(fx, fy)) = 0$, hence (3.12) holds.
- CASE III: $x \in \{0, 1, 2\}$ and $y = 3$. Here, $fx = 0$, $fy = 3$, $Tx \in \{0, 3, 4\}$ and $Ty = 5$. Thus, $d(fx, fy) = 3$, $M(x, y) = 5$ and $N(x, y) = 5$. Since $9 \leq 25 - 5$, we obtain (3.12).

- CASE IV: $x \in \{0, 1, 2\}$ and $y = 4$. Here, $fx = 0$, $fy = 3$, $Tx \in \{0, 3, 4\}$ and $Ty = 6$. Thus, $d(fx, fy) = 3$, $M(x, y) = 6$ and $N(x, y) = 6$. Since $9 \leq 36 - 6$, we deduce (3.12).
- CASE V: $x \in \{0, 1, 2\}$ and $y \geq 5$. Here, $fx = 0$, $fy = y - 2$, $Tx \in \{0, 3, 4\}$ and $Ty = y + 2$. Thus, $d(fx, fy) = y - 2$, $M(x, y) = y + 2$ and $N(x, y) = y + 2$. Since $(y - 2)^2 \leq (y + 2)^2 - (y + 2)$, we have (3.12).
- CASE VI: $x = 3$ and $y = 4$. Here, $fx = 0$, $fy = 3$, $Tx = 5$ and $Ty = 6$. Thus, $d(fx, fy) = 3$, $M(x, y) = 6$ and $N(x, y) = 6$. Since $9 \leq 36 - 6$, we get (3.12).
- CASE VII: $x = 3$ and $y \geq 5$. Here, $fx = 3$, $fy = y - 2$, $Tx = 5$ and $Ty = y + 2$. Thus, $d(fx, fy) = y - 2$, $M(x, y) = y + 2$ and $N(x, y) = y + 2$. Since $(y - 2)^2 \leq (y + 2)^2 - (y + 2)$, we have (3.12) and

$$\psi\left(p(fx, gy)\right)$$
$$\leq e^{-1} \psi\left(\max\left\{p(Sx, Ty), p(fx, Sx), p(gy, Ty), \frac{1}{2}\left(p(Sx, gy) + p(fx, Ty)\right)\right\}\right). \quad (3.13)$$

- CASE VIII: $x = 4$ and $y \geq 5$. Here, $fx = 3$, $fy = y - 2$, $Tx = 6$ and $Ty = y + 2$. Thus, $d(fx, fy) = y - 2$, $M(x, y) = y + 2$ and $N(x, y) = y + 2$. Since $(y - 2)^2 \leq (y + 2)^2 - (y + 2)$, we have (3.12) and (3.13).
- CASE IX: $y > x \geq 5$. Here, $fx = x - 2$, $fy = y - 2$, $Tx = x + 2$ and $Ty = y + 2$. Thus, $d(fx, fy) = y - 2$, $M(x, y) = y + 2$ and $N(x, y) = y + 2$. Since $(y - 2)^2 \leq (y + 2)^2 - (y + 2)$, we have (3.12) and (3.13). □

Remark 3.2 Note that Example 3.1 satisfies all the hypotheses of Corollary 3.1. But Example 3.1 does not satisfy the hypotheses of Theorem 2.1.

4 Conclusions

In this paper, we proved some common fixed point theorems for four mappings f, g, S and T satisfying a nonlinear contraction in ordered metric spaces, where the mappings f and g are dominating and weak annihilators of the mappings T and S, respectively. We utilized the techniques of Haghi *et al.* [33] to derive our main result, which is a generalization of the result of Shobkolaei *et al.* [38]. Also, we introduced an example to support the usability of our results.

Author details
[1]Department of Mathematics, Hashemite University, Zarqa, Jordan. [2]Faculty of Applied Sciences, University Politehnica of Bucharest, 313 Splaiul Independenţei, Bucharest, 060042, Romania.

References
1. Ran, ACM, Reurings, MCB: A fixed point theorem in partially ordered sets and some applications to matrix equations. Proc. Am. Math. Soc. **132**(5), 1435-1443 (2004)
2. Nieto, JJ, Lopez, RR: Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations. Order **22**, 223-239 (2005)
3. Nieto, JJ, Lopez, RR: Existence and uniqueness of fixed point in partially ordered sets and applications to ordinary differential equations. Acta Math. Sin. **23**(12), 2205-2212 (2007)

4. O'Regan, D, Petruşel, A: Fixed point theorems for generalized contractions in ordered metric spaces. J. Math. Anal. Appl. **341**(2), 241-1252 (2008)

5. Agarwal, RP, El-Gebeily, MA, O'Regan, D: Generalized contractions in partially ordered metric spaces. Appl. Anal. **87**, 1-8 (2008)

6. Aghajani, A, Radenović, S, Roshan, JR: Common fixed point results for four mappings satisfying almost generalized (S, T)-contractive condition in partially ordered metric spaces. Appl. Math. Comput. **218**, 5665-5670 (2012)

7. Aydi, H, Shatanawi, W, Postolache, M, Mustafa, Z, Tahat, N: Theorems for Boyd-Wong type contractions in ordered metric spaces. Abstr. Appl. Anal. **2012**, Article ID 359054 (2012)

8. Aydi, H, Karapınar, E, Postolache, M: Tripled coincidence point theorems for weak φ-contractions in partially ordered metric spaces. Fixed Point Theory Appl. **2012**, 44 (2012)

9. Cho, YJ, Rhoades, BE, Saadati, R, Samet, B, Shatanawi, W: Nonlinear coupled fixed point theorems in ordered generalized metric spaces with integral type. Fixed Point Theory Appl. **2012**, 8 (2012)

10. Ćirić, L, et al.: Common fixed points of almost generalized contractive mappings in ordered metric spaces. Appl. Math. Comput. **217**, 5784-5789 (2011)

11. Gholizadeh, L, Saadati, R, Shatanawi, W, Vaezpour, SM: Contractive mapping in generalized, ordered metric spaces with application in integral equations. Math. Probl. Eng. **2011**, Article ID 380784 (2011)

12. Luong, NV, Thuan, N: Fixed point theorem for generalized weak contractions satisfying rational expressions in ordered metric spaces. Fixed Point Theory Appl. **2011**, 46 (2011)

13. Radenović, S, Kadelburg, Z: Generalized weak contractions in partially ordered metric spaces. Comput. Math. Appl. **60**, 1776-1783 (2010)

14. Nashine, HK, Samet, B: Fixed point results for mappings satisfying (ϕ, ψ)-weakly contractive condition in partially ordered metric spaces. Nonlinear Anal. **74**, 2201-2209 (2011)

15. Chandok, S, Postolache, M: Fixed point theorem for weakly Chatterjea-type cyclic contractions. Fixed Point Theory Appl. **2013**, 28 (2013)

16. Shatanawi, W, Postolache, M: Common fixed point results of mappings for nonlinear contractions of cyclic form in ordered metric spaces. Fixed Point Theory Appl. **2013**, 60 (2013)

17. Shatanawi, W, Al-Rawashdeh, A: Common fixed points of almost generalized (ψ, ϕ)-contractive mappings in ordered metric spaces. Fixed Point Theory Appl. **2012**, 80 (2012)

18. Shatanawi, W, Mustafa, Z, Tahat, N: Some coincidence point theorems for nonlinear contraction in ordered metric spaces. Fixed Point Theory Appl. **2011**, 68 (2011)

19. Shatanawi, W, Samet, B: On (ψ, ϕ)-weakly contractive condition in partially ordered metric spaces. Comput. Math. Appl. **62**, 3204-3214 (2011)

20. Berinde, V: Iterative Approximation of Fixed Points. Springer, Berlin (2007)

21. Berinde, V: General constructive fixed point theorems for Ćirić-type almost contractions in metric spaces. Carpath. J. Math. **24**(2), 10-19 (2008)

22. Berinde, V: Approximating common fixed points of noncommuting almost contractions in metric spaces. Fixed Point Theory **11**(2), 179-188 (2010)

23. Berinde, V: Common fixed point of noncommuting almost contractions in cone metric spaces. Math. Commun. **15**, 229-241 (2010)

24. Berinde, V: Common fixed points of noncommuting discontinuous weakly contractive mappings in cone metric spaces. Taiwan. J. Math. **14**, 1763-1776 (2010)

25. Babu, GVR, Sandhya, ML, Kameswari, MVR: A note on a fixed point theorem of Berinde on weak contractions. Carpath. J. Math. **24**, 8-12 (2008)

26. Berinde, V, Păcurar, M: Fixed points and continuity of almost contractions. Fixed Point Theory **9**(1), 23-34 (2008)

27. Ćirić, L, et al.: Common fixed points of almost generalized contractive mappings in ordered metric spaces. Appl. Math. Comput. **217**, 5784-5789 (2011)

28. Haghi, RH, Postolache, M, Rezapour, S: On T-stability of the Picard iteration for generalized φ-contraction mappings. Abstr. Appl. Anal. **2012**, Article ID 658971 (2012)

29. Olatinwo, MO, Postolache, M: Stability results for Jungck-type iterative processes in convex metric spaces. Appl. Math. Comput. **218**(12), 6727-6732 (2012)

30. Samet, B, Vetro, C: Berinde mappings in orbitally complete metric spaces. Chaos Solitons Fractals (2011). doi:10.1016/j.chaos.2011.08.009

31. Shatanawi, W: Some fixed point results for a generalized ψ-weak contraction mappings in orbitally metric spaces. Chaos Solitons Fractals **45**, 520-526 (2012)

32. Matthews, SG: Partial metric topology. Ann. N.Y. Acad. Sci. **728**, 183-197 (1994)

33. Haghi, RH, Rezapour, S, Shahzad, N: Be careful on partial metric fixed point results. Topol. Appl. **160**, 450-454 (2013)

34. Jungck, G: Common fixed points for noncontinuous nonself maps on nonmetric spaces. Far East J. Math. Sci. **4**, 199-215 (1996)

35. Abbas, M, Talat, N, Radenović, S: Common fixed points of four maps in partially ordered metric spaces. Appl. Math. Lett. **24**, 1520-1526 (2011)

36. Shatanawi, W, Postolache, M: Coincidence and fixed point results for generalized weak contractions in the sense of Berinde on partial metric spaces. Fixed Point Theory Appl. **2013**, 54 (2013)

37. Shatanawi, W, Pitea, A: Some coupled fixed point theorems in quasi-partial metric spaces. Fixed Point Theory Appl. **2013**, 153 (2013)

38. Shobkolaei, N, Sedghi, S, Roshan, JR, Altun, I: Common fixed point of mappings satisfying almost generalized (S, T)-contractive condition in partially ordered partial metric spaces. Appl. Math. Comput. **219**, 443-452 (2012)

39. Khan, MS, Swaleh, M, Sessa, S: Fixed point theorems by altering distances between the points. Bull. Aust. Math. Soc. **30**, 1-9 (1984)

40. Jachymski, J: Equivalent conditions for generalized contractions on (ordered) metric spaces. Nonlinear Anal. **74**, 768-774 (2011)