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Research Article

Convergence Theorems of Common Fixed Points for Pseudocontractive Mappings

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We consider an implicit iterative process with mixed errors for a finite family of pseudocontractive mappings in the framework of Banach spaces. Our results improve and extend the recent ones announced by many others.

1. Introduction and preliminaries

Let E be a real Banach space and let J denote the normalized duality mapping from E into 2^{E^*} given by

$$J(x) = \{f \in E^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2\}, \quad x \in E, \tag{1.1}$$

where E^* denotes the dual space of E and $\langle \cdot, \cdot \rangle$ denotes the generalized duality pairing. In the sequel, we denote a single-valued normalized duality mapping by j. Throughout this paper, we use $F(T)$ to denote the set of fixed points of the mapping T. $\rightharpoonup$ and $\rightarrow$ denote weak and strong convergence, respectively. Let K be a nonempty subset of E. For a given sequence $\{x_n\} \subset K$, let $\omega_\omega(x_n)$ denote the weak ω-limit set.

Recall that $T : K \rightarrow K$ is nonexpansive if the following inequality holds:

$$\|Tx - Ty\| \leq \|x - y\|, \quad \forall x, y \in K. \tag{1.2}$$

T is said to be strictly pseudocontractive in the terminology of Browder and Petryshyn [1] if for all $x, y \in K$, there exist $\lambda > 0$ and $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \lambda\|x - y - (Tx - Ty)\|^2. \tag{1.3}$$

T is said to be pseudocontractive if for all $x, y \in K$, there exists $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y) \rangle \leq \|x-y\|^2. \tag{1.4}$$

It is well known that [2] (1.4) is equivalent to the following:

$$\|x-y\| \leq \|x-y-s[(I-T)x-(I-T)y]\|, \quad \forall s > 0. \tag{1.5}$$

Recently, concerning the convergence problems of an implicit (or nonimplicit) iterative process to a common fixed point,a finite family of nonexpansive mappings and its extensions in Hilbert spaces or Banach spaces have been considered by several authors (see [1–18]) for more details.

In 2001, Xu and Ori [17] introduced the following implicit iteration process for a finite family of nonexpansive mappings $\{T_1, T_2, \ldots, T_N\}$ with $\{\alpha_n\}$ a real sequence in $(0,1)$ and an initial point $x_0 \in K$:

$$\begin{aligned} x_1 &= \alpha_1 x_0 + (1-\alpha_1) T_1 x_1, \\ x_2 &= \alpha_2 x_1 + (1-\alpha_2) T_2 x_2, \\ &\cdots \\ x_N &= \alpha_N x_{N-1} + (1-\alpha_N) T_N x_N, \\ x_{N+1} &= \alpha_{N+1} x_N + (1-\alpha_{N+1}) T_1 x_{N+1}, \\ &\cdots \end{aligned} \tag{1.6}$$

which can be written in the following compact form:

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n) T_n x_n, \quad \forall n \geq 1, \tag{1.7}$$

where $T_n = T_{n(\mathrm{mod} N)}$ (here the mod N takes values in $\{1, 2, \ldots, N\}$).

Xu and Ori [17] proved weak convergence theorems of this iterative process to a common fixed point of the finite family of nonexpansive mappings in a Hilbert space. Chidume and Shahzad [3] improved Xu and Ori's [17] results to some extent. They obtained a strong convergence theorem for a finite family of nonexpansive mappings if one of the mappings is semicompact. Osilike [8] improved the results of Xu and Ori [17] from nonexpansive mappings to strict pseudocontractions in the framework of Hilbert spaces. Recently, Chen et al. [7] obtained the following results in Banach spaces.

Theorem CSZ. *Let E be a real q-uniformly smooth Banach space which is also uniformly convex and satisfies Opial's condition. Let K be a nonempty closed convex subset of E and $T_i : K \to K$, $i = 1, 2, \ldots, N$ be strictly pseudocontractive mapping in the terminology of Browder-Petryshyn such that $F = \bigcap_{i=1}^{N} F(T_i) \neq \varnothing$, and let $\{\alpha_n\}$ be a real sequence satisfying the conditions:*

$$0 < a \leq \alpha_n \leq b < 1. \tag{1.8}$$

Let $x_0 \in K$ and let $\{x_n\}$ be defined by (1.7), *where $T_n = T_{n \bmod N}$. Then $\{x_n\}$ weakly converges to a common fixed point of the mappings $\{T_i\}_{i=1}^{N}$.*

Very recently, Zhou [18] still considered the iterative Algorithm (1.7) in the framework of Banach spaces. Zhou [18] improved Theorem CSZ from strict pseudocontractions to Lipschitzian pseudocontractions. To be more precise, he proved the following theorem.

Theorem Z. *Let E be a real uniformly convex Banach space with a Fréchet differentiable norm. Let K be a closed convex subset of E, and $\{T_i\}$ be a finite family of Lipschitzian pseudocontractive self-mappings of K such that $F = \bigcap_{i=1}^{r} F(T_i) \neq \varnothing$. Let $\{x_n\}$ be defined by* (1.7). *If $\{\alpha_n\}$ is chosen so that $\alpha_n \in (0,1)$ with $\limsup \alpha_n < 1$, then $\{x_n\}$ converges weakly to a common fixed point of the family $\{T_i\}_{i=1}^{r}$.*

In this paper, motivated and inspired by Chidume and Shahzad [3], Chen et al. [7], Osilike [8], Qin et al. [10], Xu and Ori [17], and Zhou [18], we consider an implicit iteration process with mixed errors for a finite family of pseudocontractive mappings. To be more precise, we consider the following implicit iterative algorithm:

$$x_0 \in K, \qquad x_n = \alpha_n x_{n-1} + \beta_n T_n x_n + \gamma_n u_n, \quad \forall n \geq 1, \tag{1.9}$$

where $\{\alpha_n\}$, $\{\beta_n\}$, and $\{\gamma_n\}$ are three sequences in $[0,1]$ such that $\alpha_n + \beta_n + \gamma_n = 1$ and $\{u_n\}$ is a bounded sequence in K.

We remark that, from the view of computation, the implicit iterative scheme (1.7) is often impractical since, for each step, we must solve a nonlinear operator equation. Therefore, one of the interesting and important problems in the theory of implicit iterative algorithm is to consider the iterative algorithm with errors. That is an efficient iterative algorithm to compute approximately fixed point of nonlinear mappings.

The purpose of this paper is to use a new analysis technique and establish weak and strong convergence theorems of the implicit iteration process (1.9) for a finite family of pseudocontractive mappings in Banach spaces. Our results improve and extend the corresponding ones announced by many others.

Next, we will recall some well-known concepts and results.

(1) A space E is said to satisfy Opial's condition [9] if, for each sequence $\{x_n\}$ in E, the convergence $x_n \to x$ weakly implies that

$$\limsup_{n\to\infty} \|x_n - x\| < \limsup_{n\to\infty} \|x_n - y\|, \quad \forall y \in E \ (y \neq x). \tag{1.10}$$

(2) A mapping $T : K \to K$ is said to be demiclosed at the origin if, for each sequence $\{x_n\}$ in K, the convergences $x_n \to x_0$ weakly and $Tx_n \to 0$ strongly imply that $Tx_0 = 0$.

(3) A mapping $T : K \to K$ is semicompact if any sequence $\{x_n\}$ in K satisfying $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$ has a convergent subsequence.

In order to prove our main results, we also need the following lemmas.

Lemma 1.1 (see [16]). *Let $\{r_n\}$, $\{s_n\}$, and $\{t_n\}$ be three nonnegative sequences satisfying the following condition:*

$$r_{n+1} \leq (1+s_n)r_n + t_n, \quad \forall n \geq 1. \tag{1.11}$$

If $\sum_{n=1}^{\infty} s_n < \infty$ and $\sum_{n=1}^{\infty} t_n < \infty$, then $\lim_{n\to\infty} r_n$ exists.

Lemma 1.2 (see [2]). *Let E be a real uniformly convex Banach space whose norm is Fréchet differentiable. Let K be a closed convex subset of E and $\{T_n\}$ be a family of Lipschitzian self-mappings on K such that $\sum_{n=1}^{\infty}(L_n - 1) < \infty$ and $F = \bigcap_{i=1}^{r} F(T_i)$. For arbitrary $x_1 \in K$, define $x_{n+1} = T_n x_n$, for all $n \geq 1$. Then $\lim_{n\to\infty} \langle x_n, j(p-q)\rangle$ exists for all $p, q \in F$ and, in particular, for all $u, v \in \omega_\omega(x_n)$, and $p, q \in F$, $\langle u - v, j(p-q)\rangle = 0$.*

Lemma 1.3 (see [19]). *Let E be a uniformly convex Banach space, K be a nonempty closed convex subset of E, and $T : K \to K$ be a pseudocontractive mapping. Then $I - T$ is demiclosed at zero.*

Lemma 1.4 (see [15]). *Suppose that E is a uniformly convex Banach space and $0 < p \leq t_n \leq q < 1$, for all $n \in N$. Suppose further that $\{x_n\}$ and $\{y_n\}$ are sequences of E such that*

$$\begin{gathered} \limsup_{n\to\infty} \|x_n\| \leq r, \qquad \limsup_{n\to\infty} \|y_n\| \leq r, \\ \lim_{n\to\infty} \|t_n x_n + (1-t_n) y_n\| = r \end{gathered} \tag{1.12}$$

hold for some $r \geq 0$. Then $\lim_{n\to\infty} \|x_n - y_n\| = 0$.

2. Main results

Lemma 2.1. *Let E be a uniformly convex Banach space and K a nonempty closed convex subset of E. Let T_i be an L_i-Lipschitz pseudocontractive mappings from K into itself with $F = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$. Assume that the control sequences $\{\alpha_n\}$, $\{\beta_n\}$, and $\{\gamma_n\}$ satisfy the following conditions:*

(i) $\alpha_n + \beta_n + \gamma_n = 1$;

(ii) $\sum_{n=1}^{\infty} \gamma_n < \infty$;

(iii) $0 \leq a \leq \alpha_n \leq b < 1$.

Let $\{x_n\}$ be defined by (1.9). *Then*

(1) $\lim_{n\to\infty} \|x_n - p\|$ *exists, for all $p \in F$;*

(2) $\lim_{n\to\infty} \|x_n - T_m x_n\| = 0$, *for all $1 \leq m \leq N$.*

Proof. Since $F = \bigcap_{n=1}^{N} F(T_i) \neq \emptyset$, for any given $p \in F$, we have

$$\begin{aligned} \|x_n - p\|^2 &= \langle \alpha_n x_{n-1} + \beta_n T_n x_n + \gamma_n u_n - p, j(x_n - p)\rangle \\ &= \alpha_n \langle x_{n-1} - p, j(x_n - p)\rangle + \beta_n \langle T_n x_n - p, j(x_n - p)\rangle + \gamma_n \langle u_n - p, j(x_n - p)\rangle \\ &\leq \alpha_n \|x_{n-1} - p\| \|x_n - p\| + \beta_n \|x_n - p\|^2 + \gamma_n \|u_n - p\| \|x_n - p\|. \end{aligned} \tag{2.1}$$

Simplifying the above inequality, we have

$$\|x_n - p\|^2 \le \frac{\alpha_n}{\alpha_n + \gamma_n}\|x_{n-1} - p\|\|x_n - p\| + \frac{\gamma_n}{\alpha_n + \gamma_n}\|u_n - p\|\|x_n - p\|. \tag{2.2}$$

If $\|x_n - p\| = 0$, then the result is apparent. Letting $\|x_n - p\| > 0$, we obtain

$$\begin{aligned}\|x_n - p\| &\le \frac{\alpha_n}{\alpha_n + \gamma_n}\|x_{n-1} - p\| + \frac{\gamma_n}{\alpha_n + \gamma_n}\|u_n - p\| \\ &\le \|x_{n-1} - p\| + \gamma_n M,\end{aligned} \tag{2.3}$$

where M is an appropriate constant such that $M \ge \sup_{n\ge1}\{\|u_n - p\|/a\}$. Noticing the condition (ii) and applying Lemma 1.1 to (2.3), we have $\lim_{n\to\infty}\|x_n - p\|$ exists. Next, we assume that

$$\lim_{n\to\infty}\|x_n - p\| = d. \tag{2.4}$$

On the other hand, from (1.5) and (1.9), we see

$$\begin{aligned}\|x_n - p\| &= \left\|x_n - p + \frac{1-\alpha_n}{2\alpha_n}(x_n - T_n x_n)\right\| \\ &= \left\|x_n - p + \frac{1-\alpha_n}{2\alpha_n}[\alpha_n(x_{n-1} - T_n x_n) + \gamma_n(u_n - T_n x_n)]\right\| \\ &= \left\|x_n - p + \frac{1-\alpha_n}{2}(x_{n-1} - T_n x_n) + \frac{\gamma_n(1-\alpha_n)}{2\alpha_n}(u_n - T_n x_n)\right\| \\ &= \left\|\frac{x_{n-1}}{2} + x_n - \frac{1}{2}[\alpha_n x_{n-1} + (1-\alpha_n)T_n x_n + \gamma_n(u_n - T_n x_n)]\right. \\ &\qquad \left. + \frac{\gamma_n}{2}(u_n - T_n x_n) + \frac{\gamma_n(1-\alpha_n)}{2\alpha_n}(u_n - T_n x_n)\right\| \\ &= \left\|\frac{1}{2}(x_{n-1} - p) + \frac{1}{2}(x_n - p) + \frac{\gamma_n}{2\alpha_n}(u_n - T_n x_n)\right\| \\ &\le \left\|\frac{1}{2}(x_{n-1} - p) + \frac{1}{2}(x_n - p)\right\| + \frac{\gamma_n}{2\alpha_n}\|u_n - T_n x_n\|.\end{aligned} \tag{2.5}$$

Noting that the conditions (ii) and (iii) and (2.4), we obtain

$$\liminf_{n\to\infty}\left\|\frac{1}{2}(x_{n-1} - p) + \frac{1}{2}(x_n - p)\right\| \ge d. \tag{2.6}$$

On the other hand, we have

$$\limsup_{n\to\infty}\left\|\frac{1}{2}(x_{n-1} - p) + \frac{1}{2}(x_n - p)\right\| \le \limsup_{n\to\infty}\left[\frac{1}{2}\|x_{n-1} - p\| + \frac{1}{2}\|x_n - p\|\right] \le d. \tag{2.7}$$

Combining (2.6) with (2.7), we arrive at

$$\lim_{n\to\infty}\left\|\frac{1}{2}(x_{n-1}-p)+\frac{1}{2}(x_n-p)\right\|=d. \tag{2.8}$$

By using Lemma 1.4, we get

$$\lim_{n\to\infty}\|x_{n-1}-x_n\|=0. \tag{2.9}$$

That is,

$$\lim_{n\to\infty}\|x_{n+i}-x_n\|=0,\quad \forall i\in\{1,2,\dots,N\}. \tag{2.10}$$

It follows from (1.9) that

$$\begin{aligned}\|x_{n-1}-T_nx_n\|&=\frac{1}{1-\alpha_n}\|x_n-x_{n-1}-\gamma_n(u_n-T_nx_n)\|\\&\le\frac{1}{1-\alpha_n}\|x_n-x_{n-1}\|+\frac{\gamma_n}{1-\alpha_n}\|u_n-T_nx_n\|.\end{aligned} \tag{2.11}$$

From the conditions (ii) and (iii), we obtain

$$\lim_{n\to\infty}\|x_{n-1}-T_nx_n\|=0. \tag{2.12}$$

On the other hand, we have

$$\|x_n-T_nx_n\|\le\alpha_n\|x_{n-1}-T_nx_n\|+\gamma_n\|u_n-T_nx_n\|. \tag{2.13}$$

From the condition (ii) and (2.12), we see

$$\lim_{n\to\infty}\|x_n-T_nx_n\|=0. \tag{2.14}$$

For each $1\le i\le N$, we have

$$\|x_n-T_{n+i}x_n\|\le(1+L)\|x_n-x_{n+i}\|+\|x_{n+i}-T_{n+i}x_{n+i}\|, \tag{2.15}$$

where $L=\max\{L_i:1\le i\le N\}$. It follows from (2.10) and (2.14) that

$$\lim_{n\to\infty}\|x_n-T_{n+i}x_n\|=0. \tag{2.16}$$

Therefore, for each $1 \leq m \leq N$, there exists some $i \in \{1, 2, \ldots, N\}$ such that $n + i = m \bmod N$. It follows that

$$\|x_n - T_m x_n\| = \|x_n - T_{n+i} x_n\|, \tag{2.17}$$

which combines with (2.16) yields that

$$\lim_{n \to \infty} \|x_n - T_m x_n\| = 0, \quad \forall m \in \{1, 2, \ldots, N\}. \tag{2.18}$$

This completes the proof. □

Next, we give two weak convergence theorems.

Theorem 2.2. *Let E be a uniformly convex Banach space with a Fréchet differentiable norm and K a nonempty closed convex subset of E. Let T_i be an L_i-Lipschitz pseudocontractive mapping from K into itself with $F = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$. If the control sequences $\{\alpha_n\}$, $\{\beta_n\}$, and $\{\gamma_n\}$ satisfy the followings conditions:*

(i) $\alpha_n + \beta_n + \gamma_n = 1$;

(ii) $\sum_{n=1}^{\infty} \gamma_n < \infty$;

(iii) $0 \leq a \leq \alpha_n \leq b < 1$,

then the sequence $\{x_n\}$ defined by (1.9) *converges weakly to a common fixed point of $\{T_1, T_2, \ldots, T_N\}$.*

Proof. From Lemma 1.3, we see that $\omega_\omega(x_n) \subset F$. It follows from Lemma 1.2 that $\omega_\omega(x_n)$ is singleton. Hence, $\{x_n\}$ converges weakly to a common fixed point of $\{T_1, T_2, \ldots, T_N\}$. This completes the proof. □

Remark 2.3. Theorem 2.2 includes Theorem 3.1 of Zhou [18] as a special case. If $\{\gamma_n\} = 0$ for all $n \geq 1$, then Theorem 2.2 reduces to Theorem 3.1 of Zhou [18]. It derives to mention that the method in this paper is also different from Zhou's [18].

Theorem 2.4. *Let E be a uniformly convex Banach space satisfying Opial's condition and K a nonempty closed convex subset of E. Let T_i be an L_i-Lipschitz pseudocontractive mapping from K into itself with $F = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$. If the control sequences $\{\alpha_n\}$, $\{\beta_n\}$, and $\{\gamma_n\}$ satisfy the followings conditions:*

(i) $\alpha_n + \beta_n + \gamma_n = 1$;

(ii) $\sum_{n=1}^{\infty} \gamma_n < \infty$;

(iii) $0 \leq a \leq \alpha_n \leq b < 1$,

then the sequence $\{x_n\}$ defined by (1.9) *converges weakly to a common fixed point of $\{T_1, T_2, \ldots, T_N\}$.*

Proof. Since E is uniformly convex and $\{x_n\}$ is bounded, we see that there exists a subsequence $\{x_{n_i}\} \subset \{x_n\}$ such that $\{x_{n_i}\}$ converges weakly to a point $x^* \in K$. It follows from Lemma 1.3 and arbitrariness of $m \in \{1, 2, \ldots, N\}$ that $x^* \in F$.

On the other hand, since the space E satisfies Opial's condition, we can prove that the sequence $\{x_n\}$ converges weakly to a common fixed point of $\{T_1, T_2, \ldots, T_N\}$ by the standard proof. This completes the proof. □

Remark 2.5. Theorem 2.4 improves Theorem 2.6 of Chen et al. [7] in several respects.

(a) From q-uniformly smooth Banach spaces which both are uniformly convex and satisfy Opial's condition extend to uniformly convex Banach spaces which satisfy the Opial's condition.

(b) From strict pseudocontractions extend to Lipschitzian pseudocontractions.

(c) From view of computation, the iterative Algorithm (1.9) also can be viewed as an improvement of its analogue in [7].

Now, we are in a position to state a strong convergence theorem.

Theorem 2.6. *Let E be a uniformly convex Banach space and K a nonempty closed convex subset of E. Let T_i be an L_i-Lipschitz pseudocontractive mappings from K into itself with $F = \bigcap_{i=1}^{N} F(T_i) \neq \varnothing$. Assume that the control sequences $\{\alpha_n\}$, $\{\beta_n\}$, and $\{\gamma_n\}$ satisfy the followings conditions:*

(i) $\alpha_n + \beta_n + \gamma_n = 1$;

(ii) $\sum_{n=1}^{\infty} \gamma_n < \infty$;

(iii) $0 \leq a \leq \alpha_n \leq b < 1$.

Let the sequence $\{x_n\}$ be defined by (1.9). *If one of the mappings $\{T_1, T_2, \ldots, T_N\}$ is semicompact, then $\{x_n\}$ converges strongly to a common fixed point of $\{T_1, T_2, \ldots, T_N\}$.*

Proof. Without loss of generality, we can assume that T_1 is semicompact. It follows from (2.18) that

$$\lim_{n \to \infty} \|x_n - T_1 x_n\| = 0. \tag{2.19}$$

By the semicompactness of T_1, there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_{n_i} \to x^* \in K$ strongly. From (2.18), we have

$$\lim_{n_i \to \infty} \|x_{n_i} - T_m x_{n_i}\| = \|x^* - T_m x^*\| = 0, \tag{2.20}$$

for all $m = 1, 2, \ldots, N$. This implies that $x^* \in F$. From Lemma 2.1, we know that $\lim_{n \to \infty} \|x_n - p\|$ exists for each $p \in F$. That is, $\lim_{n \to \infty} \|x_n - x^*\|$ exists. From $x_{n_i} \to x^*$, we have

$$\lim_{n \to \infty} \|x_n - x^*\| = 0. \tag{2.21}$$

This completes the proof of Theorem 2.6. □

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