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Research Article

Approximating Fixed Points of Non-Lipschitzian Mappings by Metric Projections

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We define and study a new iterative algorithm inspired by Matsushita and Takahashi (2008). We establish a strong convergence theorem of the proposed algorithm for asymptotically nonexpansive in the intermediate sense mappings in uniformly convex and smooth Banach spaces by using metric projections. This theorem generalizes and refines Matsushita and Takahashi's strong convergence theorem which was established for nonexpansive mappings.

1. Introduction and Preliminaries

Let E be a real Banach space, and let C be a nonempty subset of E. A mapping $T : C \to C$ is said to be *asymptotically nonexpansive* if there exists a sequence $\{k_n\}$ in $[1, \infty)$ with $\lim_{n\to\infty} k_n = 1$ such that

$$\|T^n x - T^n y\| \le k_n \|x - y\| \tag{1.1}$$

for all $x, y \in C$ and each $n \ge 1$. If $k_n \equiv 1$, then T is known as *nonexpansive* mapping. T is said to be *asymptotically nonexpansive in the intermediate sense* [1] provided T is uniformly continuous and

$$\limsup_{n\to\infty} \sup_{x,y\in C} \left(\|T^n x - T^n y\| - \|x - y\|\right) \le 0. \tag{1.2}$$

T is said to be *Lipschitzian* if there exists a constant $L > 0$ such that

$$\|Tx - Ty\| \leq L\|x - y\| \tag{1.3}$$

for all $x, y \in C$.

It follows from the above definitions that every asymptotically nonexpansive mapping is asymptotically nonexpansive in the intermediate sense and Lipschitzian mapping but the converse does not hold such as in the following example.

Example 1.1. Let $E = \mathbb{R}$, $C = [0,1]$ and $k \in (0,1)$. We define $T(0) = 0$ and for each $x \in (0,1]$

$$T(x) = \begin{cases} k(1-2nx), & \text{if } x \in \left(\dfrac{1}{2n+1}, \dfrac{1}{2n}\right], \\ k(2nx-1), & \text{if } x \in \left(\dfrac{1}{2n}, \dfrac{1}{2n-1}\right], \end{cases} \quad (n \in \mathbb{N}). \tag{1.4}$$

We see that T is continuous on the compact interval $[0,1]$ and so it is uniformly continuous. Consider the function $f : [1,\infty) \to [0,1]$ defined as

$$f(x) = \begin{cases} 2n - x, & \text{if } \ x \in [2n-1, 2n), \\ x - 2n, & \text{if } \ x \in [2n, 2n+1), \end{cases} \quad (n \in \mathbb{N}). \tag{1.5}$$

Then, $T(x) = kxf(1/x)$ for all $x \in (0,1]$ and $|T^n(x)| \leq k^n \to 0$ uniformly. On the other hand, compactness of $[0,1]$ gives that for each $n \in \mathbb{N}$ there exist $x_n, y_n \in [0,1]$ such that

$$\sup_{x,y\in[0,1]} \left(\left|T^n x - T^n y\right| - \left|x - y\right|\right) = \left|T^n x_n - T^n y_n\right| - \left|x_n - y_n\right|. \tag{1.6}$$

Therefore,

$$\begin{aligned} \limsup_{n\to\infty} \sup_{x,y\in[0,1]} \left(\left|T^n x - T^n y\right| - \left|x - y\right|\right) &= \limsup_{n\to\infty} \left(\left|T^n x_n - T^n y_n\right| - \left|x_n - y_n\right|\right) \\ &\leq \limsup_{n\to\infty} \left(\left|T^n x_n\right| + \left|T^n y_n\right|\right) \\ &\leq \limsup_{n\to\infty} 2\, k^n = 0. \end{aligned} \tag{1.7}$$

Thus, T is asymptotically nonexpansive in the intermediate sense.

It is easy to see that T is differentiable on $(1/(n+1), 1/n)$ and $|T'(x)| = 2nk$ for all $n \in \mathbb{N}$. Let there exist $L > 0$ such that

$$\left|Tx - Ty\right| \leq L\left|x - y\right|, \tag{1.8}$$

for all $x, y \in [0,1]$. Now, choose $n \in \mathbb{N}$ such that $n > L/2k$. Then, for each $x, y \in (1/(n+1), 1/n)$ with $x \neq y$, it follows from (1.8) that

$$2nk = \left|T'(x)\right| = \left|\lim_{y \to x} \frac{Tx - Ty}{x - y}\right| \leq L. \tag{1.9}$$

This contradiction shows that T is not Lipschitzian mapping and so it is not asymptotically nonexpansive mapping. Another example of an asymptotically nonexpansive in the intermediate sense mapping which is not asymptotically nonexpansive can be found in [2].

It is known [3] that if E is a uniformly convex Banach space and T is asymptotically nonexpansive in the intermediate sense self-mapping of a bounded closed convex subset C of E, then $F(T) \neq \emptyset$, where $F(T)$ denotes the set of all fixed points of T. Let E^* be the dual of E. We denote the value of $x^* \in E^*$ at $x \in E$ by $\langle x, x^* \rangle$. When $\{x_n\}$ is a sequence in E, we denote strong convergence of $\{x_n\}$ to $x \in E$ by $x_n \to x$ and weak convergence by $x_n \rightharpoonup x$. A Banach space E is said to be *strictly convex* if $\|(x+y)/2\| < 1$ for all $x, y \in E$ with $\|x\| = \|y\| = 1$ and $x \neq y$. A Banach space E is also said to be *uniformly convex* if $\lim_{n\to\infty}\|x_n - y_n\| = 0$ for any two sequences $\{x_n\}$ and $\{y_n\}$ in E such that $\|x_n\| = \|y_n\| = 1$ and $\lim_{n\to\infty}\|x_n + y_n\| = 2$. A Banach space E is said to have Kadec-Klee property if for every sequence $\{x_n\}$ in E, $x_n \rightharpoonup x$ and $\|x_n\| \to \|x\|$ imply that $x_n \to x$. Every uniformly convex Banach space has the Kadec-Klee property [4]. Let $U = \{x \in E : \|x\| = 1\}$ be the unit sphere of E. Then the Banach space E is said to be *smooth* if

$$\lim_{t \to 0} \frac{\|x + ty\| - \|x\|}{t} \tag{1.10}$$

exists for each $x, y \in U$. The normalized duality mapping J from E to 2^{E^*} is defined by

$$J(x) = \left\{x^* \in E^* : \langle x, x^* \rangle = \|x\|^2 = \|x^*\|^2\right\} \tag{1.11}$$

for all $x \in E$. It is known that a Banach space E is smooth if and only if the normalized duality mapping J is single-valued. Some properties of duality mapping have been given in [4–6]. Let C be a closed convex subset of a reflexive, strictly convex and smooth Banach space E. Then for any $x \in E$ there exists a unique point $x_0 \in C$ such that

$$\|x_0 - x\| = \min_{y \in C}\|y - x\|. \tag{1.12}$$

The mapping $P_C : E \to C$ defined by $P_C x = x_0$ is called the *metric projection* from E onto C. Let $x \in E$ and $u \in C$. Then, it is known that $u = P_C x$ if and only if

$$\langle u - y, J(x - u) \rangle \geq 0 \tag{1.13}$$

for all $y \in C$ (see [4, 6, 7]).

Fixed points of nonlinear mappings play an important role in solving systems of equations and inequalities that often arise in applied sciences. Approximating fixed points of asymptotically nonexpansive and nonexpansive mappings with implicit and explicit iterative schemes has been studied by many authors (see, e.g., [8–14]).

On the other hand, using the metric projection, Nakajo and Takahashi [15] introduced an iterative algorithm in the framework of Hilbert spaces and gave strong convergence theorem for nonexpansive mappings. Xu [16] extended Nakajo and Takahashi's theorem to Banach spaces by using the generalized projection. Recently, Matsushita and Takahashi [17] introduced an iterative algorithm for nonexpansive mappings in Banach spaces as follows.

Let C be a nonempty convex bounded subset of a uniformly convex and smooth Banach space E, and let T be a nonexpansive self-mapping of C. For a given $x_1 = x \in C$, compute the sequence $\{x_n\}$ by the iterative algorithm

$$\begin{aligned} C_n &= \overline{\text{co}}\{z \in C : \|z - Tz\| \leq t_n\|x_n - Tx_n\|\}, \\ D_n &= \{z \in C : \langle x_n - z, J(x - x_n)\rangle \geq 0\}, \\ x_{n+1} &= P_{C_n \cap D_n} x, \quad n \geq 1, \end{aligned} \tag{1.14}$$

where $\overline{\text{co}}\, D$ denotes the convex closure of the set D and $\{t_n\}$ is a sequence in $(0, 1)$ with $\lim_{n\to\infty} t_n = 0$. They proved that $\{x_n\}$ generated by (1.14) converges strongly to a fixed point of T.

Inspired and motivated by these facts, we introduce a new iterative algorithm to find fixed points of asymptotically nonexpansive in the intermediate sense mappings in a uniformly convex and smooth Banach space. Let $x_1 = x \in C, C_0 = D_0 = C$, and compute the sequence $\{x_n\}$ by the iterative algorithm

$$\begin{aligned} C_n &= \overline{\text{co}}\{z \in C_{n-1} : \|z - T^n z\| \leq t_n\|x_n - T^n x_n\|\}, \\ D_n &= \{z \in D_{n-1} : \langle x_n - z, J(x - x_n)\rangle \geq 0\}, \\ x_{n+1} &= P_{C_n \cap D_n} x, \quad n \geq 1, \end{aligned} \tag{1.15}$$

where $\{t_n\}$ is a sequence in $(0, 1)$ with $\lim_{n\to\infty} t_n = 0$ and $P_{C_n \cap D_n}$ is the metric projection from E onto $C_n \cap D_n$.

In the sequel, the following lemmas are needed to prove our main convergence theorem.

Lemma 1.2 (see [18, Lemma 1.5]). *Let C be a nonempty bounded closed convex subset of a uniformly convex Banach space E and $T : C \to C$ be a mapping which is asymptotically nonexpansive in the intermediate sense. For each $\epsilon > 0$, there exist integers $K_\epsilon > 0$ and $\delta_\epsilon > 0$ such that if $n \geq 2$ is any integer, $j \geq K_\epsilon$, $z_1, \ldots, z_n \in C$ and if $\|z_i - z_k\| - \|T^j z_i - T^j z_k\| \leq \delta_\epsilon$ for $1 \leq i, k \leq n$, then*

$$\left\| \sum_{i=1}^{n} \lambda_i T^j x_i - T^j\left(\sum_{i=1}^{n} \lambda_i x_i\right) \right\| \leq \epsilon \tag{1.16}$$

for any numbers $\lambda_1, \ldots, \lambda_n \geq 0$ with $\lambda_1 + \cdots + \lambda_n = 1$.

Lemma 1.3 (see [18, Lemma 1.6]). *Let E be a real uniformly convex Banach space, let C be a nonempty closed convex subset of E, and let $T : C \to C$ be a mapping which is asymptotically nonexpansive in the intermediate sense. If $\{x_n\}$ is a sequence in C converging weakly to x and if*

$$\lim_{j\to\infty}\left(\limsup_{n\to\infty}\|x_n - T^j x_n\|\right) = 0 \tag{1.17}$$

then $(I-T)$ is demiclosed at zero; that is, for each sequence $\{x_n\}$ in C, if $x_n \rightharpoonup x$ for some $x \in C$ and $(I-T)x_n \to 0$, then $(I-T)x = 0$.

2. Main Results

In this section, we study the iterative algorithm (1.15) to find fixed points of asymptotically nonexpansive in the intermediate sense mappings in a uniformly convex and smooth Banach space. We first prove that the sequence $\{x_n\}$ generated by (1.15) is well-defined. Then, we prove that $\{x_n\}$ converges strongly to $P_{F(T)}x$, where $P_{F(T)}$ is the metric projection from E onto $F(T)$.

Lemma 2.1. *Let C be a nonempty closed convex subset of a reflexive, strictly convex, and smooth Banach space E, and let $T : C \to C$ be a mapping which is asymptotically nonexpansive in the intermediate sense. If $F(T) \neq \emptyset$, then the sequence $\{x_n\}$ generated by* (1.15) *is well-defined.*

Proof. It is easy to check that $C_n \cap D_n$ is closed and convex and $F(T) \subseteq C_n$ for each $n \geq 1$. Moreover $D_1 = C$ and so $F(T) \subseteq C_1 \cap D_1$. Suppose $F(T) \subseteq C_k \cap D_k$. Since $x_{k+1} = P_{C_k \cap D_k}x$, it follows from (1.13) that

$$\left\langle x_{k+1} - y, J(x - x_{k+1})\right\rangle \geq 0 \tag{2.1}$$

for all $y \in C_k \cap D_k$ and so for all $y \in F(T)$, that is $F(T) \subseteq D_{k+1}$. Thus, $F(T) \subseteq C_{k+1} \cap D_{k+1}$. By mathematical induction, we obtain that $F(T) \subseteq C_n \cap D_n$ for all $n \geq 1$. Therefore, $\{x_n\}$ is well-defined. □

In order to prove our main result, the following lemma is needed.

Lemma 2.2. *Let C be a nonempty bounded closed convex subset of a uniformly convex and smooth Banach space E, and let $T : C \to C$ be a mapping which is asymptotically nonexpansive in the intermediate sense. If $\{x_n\}$ is the sequence generated by* (1.15), *then*

$$\lim_{n\to\infty}\|x_{n+k} - T^n x_{n+k}\| = 0 \tag{2.2}$$

for all integers $k \geq 1$.

Proof. Let $k \geq 1$ be fixed, and let $n \geq 1$ be arbitrary. We take $m = n + k$ for simplicity. Since $x_m = P_{C_{m-1}\cap D_{m-1}}x$, we have $x_m \in C_{m-1} \subseteq C_{m-2} \subseteq \cdots \subseteq C_n$. Since $t_n > 0$, there exist elements $z_1, \ldots, z_N$ in C and numbers $\lambda_1, \ldots, \lambda_N \geq 0$ with $\lambda_1 + \cdots + \lambda_N = 1$ such that

$$\left\| x_m - \sum_{i=1}^{N} \lambda_i z_i \right\| < t_n, \tag{2.3}$$

$$\|z_i - T^n z_i\| \leq t_n \|x_n - T^n x_n\|, \tag{2.4}$$

for all $i = 1, 2, \ldots, N$. We put $u = P_{F(T)}x$, $M = \sup_n \|x_n - u\|$, and $G_n = \sup_{x,y\in C}(\|T^n x - T^n y\| - \|x - y\|)$. The inequality (2.4) implies that

$$\begin{aligned} \|z_i - T^n z_i\| &\leq t_n \|x_n - T^n x_n\| \leq t_n (\|x_n - u\| + \|T^n u - T^n x_n\|) \\ &\leq t_n (2\|x_n - u\| + G_n) \leq 2Mt_n + G_n t_n \\ &\leq 2Mt_n + G_n \end{aligned} \tag{2.5}$$

for all $i = 1, 2, \ldots, N$. Now, let $\epsilon > 0$, and choose an integer $K_{\epsilon/3} > 0$ and $\delta_{\epsilon/3} > 0$ with $\delta_{\epsilon/3} < (\epsilon/3)$ as in Lemma 1.2. Since $\lim_{n\to\infty} t_n = 0$ and $\limsup_{n\to\infty} G_n \leq 0$, we may choose an integer $K_0 \geq K_{\epsilon/3}$ such that for all $n \geq K_0$

$$t_n, G_n < \min\left\{ \frac{\delta_{\epsilon/3}}{4}, \frac{\delta_{\epsilon/3}}{8M}, \frac{\epsilon}{6(1+M)} \right\}. \tag{2.6}$$

This together with (2.5) implies that

$$\|z_i - T^n z_i\| \leq 2Mt_n + G_n < 2M\left(\frac{\delta_{\epsilon/3}}{8M}\right) + \frac{\delta_{\epsilon/3}}{4} = \frac{\delta_{\epsilon/3}}{2} \tag{2.7}$$

for all $n \geq K_0$ and all $i = 1, 2, \ldots, N$. Thus,

$$\|z_i - z_j\| - \|T^n z_i - T^n z_j\| \leq \|z_i - T^n z_i\| + \|z_j - T^n z_j\| < \delta_{\epsilon/3} \tag{2.8}$$

and so by Lemma 1.2 we have

$$\left\| \sum_{i=1}^{N} \lambda_i T^n z_i - T^n \left(\sum_{i=1}^{N} \lambda_i z_i \right) \right\| \leq \frac{\epsilon}{3}, \tag{2.9}$$

where $n \geq K_0$. It follows from (2.3)–(2.9) that

$$\begin{aligned}
\|x_m - T^n x_m\| &\leq \left\| x_m - \sum_{i=1}^{N} \lambda_i z_i \right\| + \left\| \sum_{i=1}^{N} \lambda_i z_i - \sum_{i=1}^{N} \lambda_i T^n z_i \right\| \\
&\quad + \left\| \sum_{i=1}^{N} \lambda_i T^n z_i - T^n \left(\sum_{i=1}^{N} \lambda_i z_i \right) \right\| + \left\| T^n \left(\sum_{i=1}^{N} \lambda_i z_i \right) - T^n x_m \right\| \\
&\leq t_n + \sum_{i=1}^{N} \lambda_i \|z_i - T^n z_i\| + \left\| \sum_{i=1}^{N} \lambda_i T^n z_i - T^n \left(\sum_{i=1}^{N} \lambda_i z_i \right) \right\| \\
&\quad + \left\| x_m - \sum_{i=1}^{N} \lambda_i z_i \right\| + G_n \\
&\leq 2t_n + 2Mt_n + 2G_n + \left\| \sum_{i=1}^{N} \lambda_i T^n z_i - T^n \left(\sum_{i=1}^{N} \lambda_i z_i \right) \right\| \\
&< 2(1+M)\left(\frac{\epsilon}{6(1+M)} \right) + 2\left(\frac{\epsilon}{6} \right) + \frac{\epsilon}{3} = \epsilon
\end{aligned} \tag{2.10}$$

for all $n \geq K_0$; that is,

$$\lim_{n \to \infty} \|x_{n+k} - T^n x_{n+k}\| = \lim_{n \to \infty} \|x_m - T^n x_m\| = 0. \tag{2.11}$$

This completes the proof. □

Now, we state and prove the strong convergence theorem of the iterative algorithm (1.15).

Theorem 2.3. *Let C be a nonempty bounded closed convex subset of a uniformly convex and smooth Banach space E, let $T : C \to C$ be a mapping which is asymptotically nonexpansive in the intermediate sense and let $\{x_n\}$ be the sequence generated by* (1.15). *Then $\{x_n\}$ converges strongly to the element $P_{F(T)}x$ of $F(T)$.*

Proof. Put $u = P_{F(T)}x$. Since $F(T) \subseteq C_n \cap D_n$ and $x_{n+1} = P_{C_n \cap D_n} x$, we have

$$\|x - x_{n+1}\| \leq \|x - u\| \tag{2.12}$$

for all $n \geq 1$. On the other hand, we observe that

$$\|x_{n+2} - Tx_{n+2}\| \leq \left\| x_{n+2} - T^{n+1} x_{n+2} \right\| + \left\| T^{n+1} x_{n+2} - Tx_{n+2} \right\| \tag{2.13}$$

and so by uniform continuity of T and Lemma 2.2 we have

$$\lim_{n \to \infty} \|x_n - Tx_n\| = 0. \tag{2.14}$$

Since $\{x_n\}$ is bounded, there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $x_{n_i} \rightharpoonup v$. It follows from (2.14) and Lemma 1.3 (demiclosedness of $(I-T)$) that $v \in F(T)$. From the weakly lower semicontinuity of norm and (2.12), we obtain

$$\|x-u\| \leq \|x-v\| \leq \liminf_{i\to\infty}\|x-x_{n_i}\| \leq \limsup_{i\to\infty}\|x-x_{n_i}\| \leq \|x-u\|. \tag{2.15}$$

This together with the uniqueness of $P_{F(T)}x$ implies that $u=v$, and hence $x_{n_i} \rightharpoonup u$. This gives that $x_n \rightharpoonup u$. By using the same argument as in proof of (2.15), we have

$$\lim_{n\to\infty}\|x-x_n\| = \|x-u\|. \tag{2.16}$$

Since E is uniformly convex, by Kadec-Klee property, we obtain that $x-x_n \to x-u$. It follows that $x_n \to u$. This completes the proof. □

Since every nonexpansive mapping is asymptotically nonexpansive and every asymptotically nonexpansive mapping is asymptotically nonexpansive in the intermediate sense, we have the following result which generalizes and refines the strong convergence theorem of Matsushita and Takahashi [17, Theorem 3.1].

Corollary 2.4. *Let C be a nonempty bounded closed convex subset of a uniformly convex and smooth Banach space E, let T be a nonexpansive self-mapping of C, and let $\{x_n\}$ be the sequence generated by* (1.15)*. Then $\{x_n\}$ converges strongly to the element $P_{F(T)}x$ of $F(T)$.*

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