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Stability of common fixed points in uniform spaces

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Abstract

Stability results for a pair of sequences of mappings and their common fixed points in a Hausdorff uniform space using certain new notions of convergence are proved. The results obtained herein extend and unify several known results.

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1 Introduction

The relationship between the convergence of a sequence of self mappings T_n of a metric (resp. topological space) X and their fixed points, known as the stability (or continuity) of fixed points, has been widely studied in fixed point theory in various settings (cf. [1-18]). The origin of this problem seems into a classical result (see Theorem 1.1) of Bonsall [6] (see also Sonnenshein [18]) for contraction mappings. Recall that a self-mapping f of a metric space (X, d) is called a contraction mapping if there exists a constant k, $0 <k < 1$ such that

$$d(f(x), f(y) \leq kd(x, y) \quad \text{for all} \quad x, y \in X.$$

Theorem 1.1. Let (X, d) be a complete metric space and T and $T_n(n = 1, 2,...)$ be contraction mappings of X into itself with the same Lipschitz constant $k < 1$, and with fixed points u and $u_n(n = 1, 2,...)$, respectively. Suppose that $\lim_n T_nx = Tx$ for every $x \in X$. Then, $\lim_n u_n = u$.

Subsequent results by Nadler Jr. [11], and others address mainly the problem of replacing the completeness of the space X by the existence of fixed points (which was ensured otherwise by the completeness of X) and various relaxations on the contraction constant k. In most of these results, pointwise (resp. uniform) convergence plays invariably a vital role. However, if the domain of definition of T_n is different for each $n \in \mathbb{N}$ (naturals), then these notions do not work. An alternative to this problem has recently been presented by Barbet and Nachi [5] (see also [4]) where some new notions of convergence have been introduced and utilized to obtain stability results in a metric space. For a uniform space version of these results, see Mishra and Kalinde [10]. On the other hand, a result of Jungck [19] on common fixed points of commuting continuous mappings has also been found quite useful. We note that the above-mentioned result of Jungck [19] includes the well-known Banach contraction principle. Using the above ideas of Barbet and Nachi [5] and Jungck [19], we obtain stability results for common fixed points in a uniform space whose uniformity is generated by a family of pseudometrics. These results generalize the recent results obtained by Mishra and Kalinde [10] and which in turn include several known results. Locally convex topological vector spaces being completely regular are uniformizable, where the uniformity of the space is induced by a family of seminorms. Therefore, all the results obtained herein for uniform spaces also apply to locally convex spaces (cf. Remark 4.4).

2 Preliminaries

Let $(X,\mathcal{U})$ be a uniform space. A family $P = \{\rho_\alpha : \alpha \in I\}$ of pseudometrics on X, where I is an indexing set is called an associated family for the uniformity $\mathcal{U}$ if the family

$$\mathfrak{B} = \{V(\alpha,\varepsilon) \ : \ \alpha \in I, \varepsilon > 0\},$$

where

$$V(\alpha,\varepsilon) = \{(x,y) \ \in X \times X : \ \rho_\alpha(x,y) < \varepsilon\}$$

is a subbase for the uniformity $\mathcal{U}$. We may assume $\mathfrak{B}$ itself to be a base for $\mathcal{U}$ by adjoining finite intersections of members of $\mathfrak{B}$ if necessary. The corresponding family of pseudometrics is called an augmented associated family for $\mathcal{U}$. An augmented associated family for $\mathcal{U}$ will be denoted by P^*. (cf. Mishra [9] and Thron [20]). In view of Kelley [21], we note that each member $V(\alpha, \varepsilon)$ of $\mathfrak{B}$ is symmetric and ρ_α is uniformly continuous on $X \times X$ for each $\alpha \in I$. Further, the uniformity $\mathcal{U}$ is not necessarily pseudometrizable (resp. metrizable) unless $\mathfrak{B}$ is countable, and in that case, $\mathcal{U}$ may be generated by a single pseudometric (resp. a metric) ρ on X. For an interesting motivation, we refer to Reilly [[22], Example 2] (see also Kelley [[21], Example C, p. 204]). For further details on uniform spaces and a systematic account of fixed point theory there in (including applications), we refer to Kelleyl [21] and Angelov [3] respectively.

Now onwards, unless stated otherwise, $(X,\mathcal{U})$ will denote a uniform space defined by P^* while $\bar{\mathbb{N}} = \mathbb{N} \cup \{\infty\}$.

Definition 2.1. [23] Let $(X,\mathcal{U})$ be a uniform space and let $\{\rho_\alpha : \alpha \in I\} = P^*$. A mapping $T : X \to X$ is called a P^*- contraction if for each $\alpha \in I$, there exists a real $k(\alpha)$, $0 < k(\alpha) < 1$ such that

$$\rho_\alpha\left(T(x),\ T(y)\right) \leq k(\alpha)\rho_\alpha(x,\ y) \quad \text{for all } \ x,\ y \in X.$$

It is well known that $T : X \to X$ is P^*-contraction if and only if it is P- contraction (see Tarafdar [[23], Remark 1]). Hence, now onwards, we shall simply use the term *k-contraction* (resp. *contraction*) to mean either of them. In case the above condition is satisfied for any $k = k(\alpha) > 0$, T will be called *k- Lipschitz* (or simply *Lipschitz*).

The following result due to Tarafdar [[23], Theorem 1.1] (see also Acharya [[24], Theorem 3.1]) presents an exact analog of the well-known Banach contraction principle.

Theorem 2.2. Let $(X,\mathcal{U})$ be a Hausdorff complete uniform space and let $\{\rho_\alpha : \alpha \in I\} = P^*$. Let T be a contraction on X. Then, T has a unique fixed point $a \in X$ such that $T^n x \to a$ in τ_u (the uniform topology) for each $x \in X$.

Definition 2.3. Let $(X,\mathcal{U})$ be a uniform space, $S, T : Y \subseteq X \to X$. Then, the pair (S, T) will be called *J - Lipschitz* (Jungck Lipschitz) if for each $\alpha \in I$, there exists a constant $\mu = \mu(\alpha) > 0$ such that

$$\rho_\alpha(Sx,\ Sy) \le \mu\rho_\alpha(Tx, Ty) \quad \text{for all } x, y \in Y. \tag{2.1}$$

The pair (S, T) is generally called *Jungck contraction* (or simply *J-contraction*) when $0 < \mu < 1$, and the constant μ in this case is a called *Jungck constant* (see, for instance, [13]). Indeed, *J-contractions* and their generalized versions became popular because of the constructive approach of proof adopted by Jungck [19]. Now onwards, a *J-Lpschitz map* (resp. *J-contraction*) with Jungck constant μ will be called a *J-Lipschitz* (resp. *J-contraction*) with constant μ.

The following example illustrates the generality of *J-Lipschitz maps*.

Example 2.4. Let $X = (0, \infty)$ with the usual uniformity induced by $\rho(x, y) = |x - y|$ for all $x, y \in X$. Define $S : X \to X$ by

$$Sx = \frac{1}{x} \quad \text{for all } x \in X.$$

Then,

$$\rho(Sx, Sy) = \frac{1}{xy}\rho(x, y) \quad \text{for all } x, y \in X.$$

Since $\frac{1}{xy} \to \infty$ for small x or $y \in X$, S is not a Lipschitz map. However, if we consider the map $T : X \to X$ defined by

$$Tx = \frac{1}{Lx}, \quad \text{for all } x \in X \quad \text{and some } L > 0,$$

then

$$\rho(Sx, Sy) = L\rho(Tx, Ty)$$

and S is Lipscitz with respect to T or the pair (S, T) is J-Lipschitz.

3 G-convergence and stability

Definition 3.1 [5,10]. Let $(X, \mathcal{U})$ be a uniform space, $\{X_n\}_{n\in\bar{\mathbb{N}}}$ a sequence of nonempty subsets of X and $\{S_n : X_n \to X\}_{n\in\bar{\mathbb{N}}}$ a sequence of mappings. Then $\{S_n\}_{n\in\bar{\mathbb{N}}}$ is said to converge *G-pointwise* to a map S_∞: $X_\infty \to X$, or equivalently $\{S_n\}_{n\in\bar{\mathbb{N}}}$ *satisfies the property* (G), if the following condition holds:

(G) $Gr(S_\infty) \subset \liminf Gr(S_n)$: for every $x \in X_\infty$, there exists a sequence $\{x_n\}$ in $\prod_{n\in\mathbb{N}} X_n$ such that for any $\alpha \in I$,

$$\lim_n \rho_\alpha(x_n, x) = 0 \ \text{ and } \ \lim_n \rho_\alpha(S_n x_n, S_\infty x) = 0,$$

where $Gr(T)$ stands for the graph of T.

In view of Barbet and Nachi [5], we note that:

(i) A G-limit need not be unique.

(ii) The property (G) is more general than pointwise convergence. However, the two notions are equivalent provided the sequence $\{S_n\}_{n\in\mathbb{N}}$ is equicontinuous when the domains of definitions are identical.

The following theorem gives a sufficient condition for the existence of a unique G-limit.

Theorem 3.2. Let $(X, \mathcal{U})$ be a uniform space, $\{X_n\}_{n\in\bar{\mathbb{N}}}$ a family of nonempty subsets of X and $\{S_n : X_n \to X\}_{n\in\bar{\mathbb{N}}}$ a sequence of J-Lipschitz maps relative to a continuous map $T : X \to X$ with Lipschitz constant μ. If $S_\infty : X_\infty \to X$ is a G-limit of the sequence $\{S_n\}$, then S_∞ is unique.

Proof. Let $U \in \mathcal{U}$ be an arbitrary entourage. Then, since $\mathfrak{B}$ is base for $\mathcal{U}$, there exists $V(\alpha, \varepsilon) \in \mathfrak{B}$, $\alpha \in I$, $\varepsilon > 0$ such that $V(\alpha, \varepsilon) \subset U$. Suppose that $S_\infty : X_\infty \to X$ and $S^*_\infty : X_\infty \to X$ are G-limit maps of the sequence $\{S_n\}$. Then, for every $x \in X_\infty$, there exist two sequences $\{x_n\}$ and $\{y_n\}$ in $\prod_{n\in\mathbb{N}} X_n$ such that for any $\alpha \in I$

$$\lim_n \rho_\alpha(x_n, x) = 0 \text{ and } \lim_n \rho_\alpha(S_n x_n, S_\infty x) = 0,$$
$$\lim_n \rho_\alpha(y_n, x) = 0 \text{ and } \lim_n \rho_\alpha(S_n y_n, S^*_\infty x) = 0.$$

Further, since S_n is J-Lipschitz, for any $\alpha \in I$, there exists a constant $\mu = \mu(\alpha) > 0$ such that

$$\rho_\alpha(S_n x_n, S_n y_n) \le \mu \rho_\alpha(T_n x_n, T_n y_n)$$

Therefore, for any $n \in \mathbb{N}$ and $\alpha \in I$,

$$\begin{aligned}\rho_\alpha(S_\infty x, S^*_\infty x) &\le \rho_\alpha(S_\infty x, S_n x_n) + \rho_\alpha(S_n x_n, S_n y_n) + \rho_\alpha(S_n y_n, S^*_\infty x)\\ &\le \rho_\alpha(S_\infty x, S_n x_n) + \mu \rho_\alpha(T x_n, T y_n) + \rho_\alpha(S_n y_n, S^*_\infty x)\\ &\le \rho_\alpha(S_\infty x, S_n x_n) + \mu[\rho_\alpha(T x_n, Tx) + (Tx, T y_n)] + \rho_\alpha(S_n y_n, S^*_\infty x)\end{aligned}$$

Since T is continuous and $x_n \to x$ and $y_n \to x$ as $n \to \infty$, it follows that $Tx_n \to Tx$, $Ty_n \to Tx$. Hence the R.H.S. of the above expression tends to 0 as $n \to \infty$ and so, $\rho_\alpha(S_\infty x, S^*_\infty x) < \varepsilon$ for all $n \ge N(\alpha, \varepsilon)$. Therefore $(S_\infty x, S^*_\infty x) \in V(\alpha, \varepsilon) \subset U$ and since X is Hausdorff, it follows that $S_\infty x = S^*_\infty x$.■

Corollary 3.3. Theorem 3.2 with J-Lipschitz replaced by J-contraction.

Proof. It comes from Theorem 3.2 for $\mu \in (0, 1)$.■

The following result due to Mishra and Kalinde [[10], Proposition 3.1, see also, Remark 3.2)], which in turn includes a result of Barbet and Nachi [[5], Proposition 1], follows as a corollary of Theorem 3.2.

Corollary 3.4. Let $(X, \mathcal{U})$ be a Hausdorff uniform space, $\{X_n\}_{n\in\bar{\mathbb{N}}}$ a family of nonempty subsets of X and $S_n : X_n \to X$ a k- contraction (resp. k-Lipschitz) mapping for each $n \in \bar{\mathbb{N}}$. If $S_\infty : X_\infty \to X$ is a (G) - limit of $\{S_n\}_{n\in\bar{\mathbb{N}}}$ then S_∞ is unique.

Proof. It comes from Theorem 3.2 when T is the identity map and $\mu \in (0, 1)$ (resp. $\mu > 0$).■

Now, we present our first stability result.

Theorem 3.5. Let $(X, \mathcal{U})$ be a uniform space, $\{X_n\}_{n\in\bar{\mathbb{N}}}$ a family of nonempty subsets of X and $\{S_n, T_n : X_n \to X\}_{n\in\bar{\mathbb{N}}}$ two families of maps each satisfying the property (G) and such that for all $n \in \bar{\mathbb{N}}$, the pair (S_n, T_n) is J-contraction with constant μ. If for all $n \in \bar{\mathbb{N}}$, z_n is a common fixed point of S_n and T_n, then, the sequence $\{z_n\}$ converges to z_∞.

Proof. Let $W \in \mathcal{U}$ be arbitrary. Then, there exists $V(\lambda, \varepsilon) \in \mathfrak{B}, \lambda \in I, \varepsilon > 0$ such that $V(\lambda, \varepsilon) \subset W$. Since z_n is a common fixed point of S_n and T_n for each $n \in \bar{\mathbb{N}}$, and the property (G) holds and $z_\infty \in X_\infty$, there exists a sequence $\{y_n\}$ such that $y_n \in X_n$ (for all $n \in \bar{\mathbb{N}}$) such that for any $\lambda \in I$,

$$\lim_n \rho_\lambda(y_n, z_\infty) = 0, \quad \lim_n \rho_\lambda(S_n y_n, S_\infty z_\infty) = 0 \text{ and } \lim_n \rho_\lambda(T_n y_n, T_\infty z_\infty) = 0.$$

Using the fact that the pair (S_n, T_n) is J-contraction, for any $\lambda \in I$, we have

$$\begin{aligned}\rho_\lambda(z_n, z_\infty) &= \rho_\lambda(S_n z_n, S_\infty z_\infty)\\ &\leq \rho_\lambda(S_n z_n, S_n y_n) + \rho_\lambda(S_n y_n, S_\infty z_\infty)\\ &\leq \mu(\lambda)\rho_\lambda(T_n z_n, T_n y_n) + \rho_\lambda(S_n y_n, S_\infty z_\infty)\\ &\leq \mu(\lambda)\rho_\lambda(T_n z_n, T_\infty z_\infty) + \mu(\lambda)\rho_\lambda(T_n y_n, T_\infty z_\infty) + \rho_\lambda(S_n y_n, S_\infty z_\infty).\end{aligned}$$

This gives

$$\rho_\lambda(z_n, z_\infty) \leq \frac{1}{1-\mu(\lambda)}[\mu(\lambda)\rho_\lambda(T_n y_n, T_\infty z_\infty) + \rho_\lambda(S_n y_n, S_\infty z_\infty)].$$

Since $\mu(\lambda) < 1$, it follows that $\rho_\lambda(z_n, z_\infty) \to 0$ as $n \to \infty$. Hence, $\rho_\lambda(z_n, z_\infty) < \varepsilon$ for all $n \geq N(\lambda, \varepsilon)$ and so $(z_n, z_\infty) \in V(\lambda, \varepsilon) \subset W$ and the conclusion follows.■

When for each $n \in \bar{\mathbb{N}}$, T_n is the identity map on X_n in Theorem 3.5, we have the following result due to Mishra and Kalinde [[10], Theorem 3.3], which includes a result of Barbet and Nachi [[5], Theorem 2].

Corollary 3.6. Let $(X, \mathcal{U})$ be a Hausdorff uniform space, $\{X_n\}_{n\in\bar{\mathbb{N}}}$ a family of nonempty subsets of X and $\{S_n : X_n \to X\}_{n\in\bar{\mathbb{N}}}$ a family of mappings satisfying the property (G) and S_n is a k- contraction for each $n \in \bar{\mathbb{N}}$. If x_n is a fixed point of S_n for each $n \in \bar{\mathbb{N}}$, then the sequence $\{x_n\}_{n\in\mathbb{N}}$ converges to x_∞.

Again, when $X_n = X$, for all $n \in \bar{\mathbb{N}}$, we obtain, as a consequence of Theorem 3.5, the following result.

Corollary 3.7. Let $(X, \mathcal{U})$ be a uniform space and S_n, $T_n : X \to X$ be such that the pair (S_n, T_n) is J-contraction with constant μ and with at least one common fixed point z_n for all $n \in \bar{\mathbb{N}}$. If the sequences $\{S_n\}$ and $\{T_n\}$ converge pointwise respectively to S, $T : X \to X$, then the sequence $\{z_n\}$ converges to z_∞.

Notice that Corollary 3.7 includes as a special case a result of Singh [[13], Theorem 1] for metric spaces (metrizable spaces).

We remark that under the conditions of Theorem 3.5 the pair (S_∞, T_∞) of G-limit maps is also a J-contraction. Indeed, we have the following stability result.

Theorem 3.8. Let $(X, \mathcal{U})$ be a uniform space, $\{X_n\}_{n\in\bar{\mathbb{N}}}$ a family of nonempty subsets of X and $\{S_n, T_n : X_n \to X\}_{n\in\overline{\mathbb{N}}}$ two families of maps each satisfying the property (G) and such that for all $n\in \mathbb{N}$, the pair (S_n, T_n) is J-contraction with constant $\{\mu_n\}_{n\in\mathbb{N}}$ a bounded (resp. convergent) sequence. Then, the pair (S_∞, T_∞) is J-contraction with constant $\mu = \sup_{n\in\mathbb{N}}\mu_n$ (resp. $\mu = \lim_n \mu_n$).

Proof. Let $x, y \in X_\infty$. Then, by the property (G), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in $\prod_{n\in\mathbb{N}} X_n$ such that the sequences $\{S_n x_n\}$, $\{S_n y_n\}$, $\{T_n x_n\}$ and $\{T_n y_n\}$ converge respectively to $S_\infty x$, $S_\infty y$, $T_\infty x$, and $T_\infty y$.

Therefore, for any $n\in$ N and each $\alpha \in I$,

$$\begin{aligned}\rho_\alpha(S_\infty x, S_\infty y) &\leq \rho_\alpha(S_\infty x, S_n x_n) + \rho_\alpha(S_n x_n, S_n y_n) + \rho_\alpha(S_n y_n, S_\infty y)\\ &\leq \rho_\alpha(S_\infty x, S_n x_n) + \mu_n\rho_\alpha(T_n x_n, T_n y_n) + \rho_\alpha(S_n y_n, S_\infty y).\end{aligned}$$

Since

$$\limsup_n \mu_n \rho_\alpha(T_n x_n, T_n y_n) \le \mu \rho_\alpha(T_\infty x, T_\infty y),$$

the above inequality yields $\rho_\alpha(S_\infty x, S_\infty y) \le \mu\ \rho_\alpha(T_\infty x, T_\infty y)$ and the conclusion follows.■

Remark 3.9. Theorem 3.8 includes, as a special case, a result of Mishra and Kalinde [[10], Proposition 3.5] for uniform spaces when $X_n = X$ and T_n is an identity mapping for each $n \in \bar{\mathbb{N}}$. Consequently, a result of Barbet and Nachi [[5], Proposition 4] for metric spaces also follows when X is metrizable.

4 H-convergence and stability

Definition 4.1. [5,10] Let $(X, \mathcal{U})$ be a uniform space, $\{X_n\}_{n\in\bar{\mathbb{N}}}$ a family of nonempty subsets of X and $\{S_n : X_n \to X\}_{n\in\bar{\mathbb{N}}}$ a family of mappings. Then,

S_∞ is called an (H) - limit of the sequence $\{S_n\}_{n\in\mathbb{N}}$ in or, equivalently $\{S_n\}_{n\in\bar{\mathbb{N}}}$ satisfies the property (H) if the following condition holds:

(H) For all sequences $\{x_n\}$ in $\prod_{n\in\mathbb{N}} X_n$, there exists a sequence $\{y_n\}$ in X_∞ such that for any $\alpha \in I$,

$$\lim_n \rho_\alpha(x_n, y_n) = 0 \textit{ and } \lim_n \rho_\alpha(S_n x_n, S_n y_n) = 0.$$

In case X is a metrizable uniform space (that is the uniformity $\mathcal{U}$ is generated by a metric d), we get the corresponding definitions due to Barbet and Nachi [5].

In view of [5], we note that:

(a) A G-limit map is not necessarily an H-limit.

(b) If $\{S_n : Y \subseteq X \to X\}_{n\in\mathbb{N}}$ converges uniformly to S_∞ on Y, then S_∞ is an H-limit of $\{S_n\}$.

(c) The converse of (b) holds only when S_∞ is uniformly continuous on Y.

For details and examples, we refer to Barbet and Nachi [5].

Theorem 4.2. Let $(X, \mathcal{U})$ be a uniform space, $\{X_n\}_{n\in\bar{\mathbb{N}}}$ a family of nonempty subsets of X. Let $\{S_n, T_n : X_n \to X\}_{n\in\overline{\mathbb{N}}}$ be two families of maps each satisfying the property (H). Further, let the pair (S_∞, T_∞) be a J-contraction with constant μ_∞. If, for every $n \in \bar{\mathbb{N}}$, z_n is a common fixed point of S_n and T_n, then the sequence $\{z_n\}$ converges to z_∞.

Proof. The property (H) implies that there exists a sequence $\{y_n\}$ in X_∞ such that for any $\alpha \in I$, $\rho_\alpha(z_n, y_n) \to 0$, $\rho_\alpha(S_n z_n, S_\infty y_n) \to 0$ and $\rho_\alpha(T_n z_n, T_\infty y_n) \to 0$ as $n \to \infty$. Then

$$\begin{aligned}
\rho_\alpha(z_n, z_\infty) &= \rho_\alpha(S_n z_n, S_\infty z_\infty) \\
&\le \rho_\alpha(S_n z_n, S_\infty y_n) + \rho_\alpha(S_\infty y_n, S_\infty z_\infty) \\
&\le \rho_\alpha(S_n z_n, S_\infty y_n) + \mu_\infty \rho_\alpha(T_\infty y_n, T_\infty z_\infty) \\
&\le \rho_\alpha(S_n z_n, S_\infty y_n) + \mu_\infty[\rho_\alpha(T_\infty y_n, T_n z_n) + \rho_\alpha(T_n z_n, T_\infty z_\infty)].
\end{aligned}$$

So, we get

$$\rho_\alpha(z_n, z_\infty) \le \frac{1}{(1-\mu_\infty)}[\rho_\alpha(S_n z_n, S_\infty y_n) + \mu_\infty \rho_\alpha(T_\infty y_n, T_n z_n].$$

Since the right hand side of the above inequality tends to 0 as $n \to \infty$, we deduce that $z_n \to z_\infty$ as $n \to \infty$. ■

As a consequence of Theorem 4.2, we have the following result due to Mishra and Kalinde [[10], Theorem 3.13].

Corollary 4.3. Let $(X, \mathcal{U})$ be a Hausdorff uniform space, $\{X_n\}_{n\in\bar{\mathbb{N}}}$ a family of nonempty subsets of X and $\{S_n : X_n \to X\}_{n\in\bar{\mathbb{N}}}$ a family of mappings satisfying the property (H) and such that S_∞ is a k_∞ - contraction. If for any $n \in \bar{\mathbb{N}}$, x_n is a fixed point of T_n, then $\{x_n\}_{n\in\mathbb{N}}$ converges to x_∞.

Proof. It comes from Theorem 4.2 by taking T_n to be the identity mapping for each $n \in \bar{\mathbb{N}}$.■

If X is metrizable, then we get a stability result of Barbet and Nachi [[5], Theorem 11], which in turn includes a result of Nadler [[11], Theorem 1]. Indeed, Nadler's result is a direct consequence of Corollary 4.3 when $X_n = X$ for each $n \in \mathbb{N}$ with X being metrizable.

Remark 4.4. Every locally convex topological vector space X is uniformizable being completely regular (cf. Kelley [21], Shaefer [25]) where the family of pseudometrics $\{\rho_\alpha : \alpha \in I\}$ is induced by a family of seminorms $\{\rho_\alpha : \alpha \in I\}$ so that $\rho_\alpha(x, y) = \rho_\alpha(x - y)$ for all $x, y \in X$. Therefore, all the results proved previously for uniform spaces also apply to locally convex spaces.

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Authors' contributions
A seminar on the basic ideas of G and H-convergence was presented by SNM in 2009. Subsequently, SLS and SS joined him to extend these basic ideas to the setting of J-contractions. SNM finalized the paper in 2010 when SLS was visiting Walter Sisulu University again in 2010. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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