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Well-posedness for parametric strong vector quasi-equilibrium problems with applications

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Abstract

In this article, we generalize the concept of well-posedness to the parametric strong vector quasi-equilibrium problem. Under some suitable conditions, we establish some characterizations of well-posedness for parametric strong vector quasi-equilibrium problems. The corresponding concept of well-posedness in the generalized sense is also investigated for the parametric strong vector quasi-equilibrium problem. As applications, we investigate the well-posedness for strong vector quasi-variational inequality problems and strong vector quasi-optimization problems.

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1 Introduction

Equilibrium problem was first introduced by Blum and Oettli [1], which includes optimization problems, fixed point problems, variational inequality problems, and complementarity problems as special cases. During the recent years, equilibrium problem has been extensively studied and generalized (e.g., [2,3]).

It is well known that well-posedness is very important for both optimization theory and numerical methods of optimization problems, which guarantees that, for every approximating solution sequence, there is a subsequence which converges to a solution. Well-posedness of unconstrained and constrained scalar optimization problems was first introduced and studied by Tykhonov [4] and Levitin and Polyak [5], respectively. Since then, various concepts of well-posedness have been introduced and extensively studied for minimization problems and vector optimization problems. For details, we refer readers to [6-13] and the references therein.

In recent years, the concept of well-posedness has been generalized to several related problems: variational inequality problems [14-18], saddle point problems [19], Nash equilibrium problems [16,20-23], inclusion problems [24,25], and fixed point problems [24-26].

Recently Fang et al. [27] generalized the concepts of well-posedness to equilibrium problems and to optimization problems with equilibrium constraints and gave some metric characterizations of well-posedness for equilibrium and optimization problems with equilibrium constraints. Subsequently, the well-posedness has been extensively studied by many authors for various equilibrium problems, such as scalar equilibrium

problems [28], vector equilibrium problems [29-31], parametric vector equilibrium problems [32], vector quasi-equilibrium problems [33], generalized vector quasi-equilibrium problems [34], and symmetric quasi-equilibrium problems [35]. It is worth mentioning that the most of results of well-posedness are for scalar equilibrium problem or for weak vector equilibrium problem which depends on $\text{int}C \neq \emptyset$. However, in many cases, the cone C has an empty interior. For example, in the classical Banach spaces l^p and $L^p(\Omega)$, where $1 < p < \infty$, the standard ordering cone has an empty interior. In this case, we cannot study the well-posedness for the weak vector equilibrium problem. However, we can study the strong vector equilibrium problem since the strong vector equilibrium problem does not need $\text{int}C \neq \emptyset$. On the other hand, to the best of our knowledge, the result of well-posedness for the strong vector equilibrium problem is very few.

On the other hand, the strong vector equilibrium problem is a class of important equilibrium problems since a solution for the strong vector equilibrium problem is an ideal solution, which is better than other solutions such as efficient solution, weak efficient solution, Henig efficient solution and supper efficient solution (e.g., [36]). Thus, it is important to investigate the well-posedness for strong vector equilibrium problems.

Motivated and inspired by the studies mentioned above, in this article, we introduce and study the well-posedness for parametric strong vector quasi-equilibrium problems. Under suitable conditions, we obtain some necessary and sufficient conditions for the well-posedness of parametric strong vector quasi-equilibrium problems. As applications, we investigate the well-posedness for strong vector quasi-variational inequality problems and strong vector quasi-optimization problems.

2 Preliminaries

Throughout this article, unless otherwise specified, we use the following notations. Let X be a nonempty closed subset of a metric space (E, d) ; Y be a Hausdorff topological vector space; and Λ and P be two nonempty closed subsets of two metric spaces, respectively. Let $K : \Lambda \times X \rightarrow 2^X$ and $C : \Lambda \times X \rightarrow 2^Y$ be set-valued mappings such that, for each $(\lambda, x) \in \Lambda \times X$, $C(\lambda, x)$ is a nonempty closed convex cone in Y . Let $e : X \rightarrow Y$ be a continuous mapping such that, for each $x \in X$ and $\lambda \in \Lambda$, $e(x) \in C(\lambda, x)$. Let $f : P \times X \times X \rightarrow Y$ be a vector-valued mapping. For any fixed $(\lambda, p) \in \Lambda \times P$, the parametric strong vector quasi-equilibrium problem (for short, $(\text{PSVQEP})_{\lambda,p}$) is to find $\bar{x} \in X$ such that $\bar{x} \in K(\lambda, \bar{x})$ and

$$f(p, \bar{x}, \gamma) \in C(\lambda, \bar{x}), \quad \forall \gamma \in K(\lambda, \bar{x}).$$

Denote by (PSVQEP) the family $\{(\text{PSVQEP})_{\lambda,p} : (\lambda, p) \in \Lambda \times P\}$. For each $(\lambda, p) \in \Lambda \times P$, let $S(\lambda, p)$ be the solution set for $(\text{PSVQEP})_{\lambda,p}$, i.e.,

$$S(\lambda, p) = \{x \in X : x \in K(\lambda, x) \text{ and } f(p, x, \gamma) \in C(\lambda, x), \quad \forall \gamma \in K(\lambda, x)\}.$$

Remark 2.1. If E is a real Banach space, $Y = R$, $f : P \times X \times X \rightarrow R$, $K(\lambda, x) = X$ and $C(\lambda, x) = (0, +\infty)$ for all $(\lambda, x) \in \Lambda \times X$, then $(\text{PSVQEP})_{\lambda,p}$ reduces to the following parametric equilibrium problem (for short, $(\text{PEP})_p$): find $\bar{x} \in X$ such that

$$f(p, \bar{x}, \gamma) \geq 0, \quad \forall \gamma \in X,$$

which has been studied in [27].

Definition 2.1. Let $(\lambda, p) \in \Lambda \times P$ and $\{(\lambda_n, p_n)\} \subseteq \Lambda \times P$ with $(\lambda_n, p_n) \rightarrow (\lambda, p)$. A sequence $\{x_n\} \subseteq X$ is said to be approximating for $(PSVQEP)_{\lambda,p}$ corresponding to $\{(\lambda_n, p_n)\}$ if there exists a sequence $\{\varepsilon_n\} \subseteq R^+$ with $\varepsilon_n \rightarrow 0$ such that, for each $n \in N$,

$$x_n \in K(\lambda_n, p_n)$$

and

$$f(p_n, x_n, \gamma) + \varepsilon_n e(x_n) \in C(\lambda_n, p_n), \quad \forall \gamma \in K(\lambda_n, p_n).$$

Definition 2.2. $(PSVQEP)$ is said to be well-posed if, for every $(\lambda, p) \in \Lambda \times P$,

- (i) $(PSVQEP)_{\lambda,p}$ has a unique solution $x_{\lambda,p}$, i.e., $S(\lambda, p) = \{x_{\lambda,p}\}$;
- (ii) for any sequence $\{(\lambda_n, p_n)\} \subseteq \Lambda \times P$ with $(\lambda_n, p_n) \rightarrow (\lambda, p)$, every approximating sequence $\{x_n\}$ for $(PSVQEP)_{\lambda,p}$ corresponding to $\{(\lambda_n, p_n)\}$ converges to $x_{\lambda,p}$.

Definition 2.3. $(PSVQEP)$ is said to be well-posed in the generalized sense if, for every $(\lambda, p) \in \Lambda \times P$,

- (i) $S(\lambda, p) \neq \emptyset$;
- (ii) for any sequence $\{(\lambda_n, p_n)\} \subseteq \Lambda \times P$ with $(\lambda_n, p_n) \rightarrow (\lambda, p)$, every approximating sequence $\{x_n\}$ for $(PSVQEP)_{\lambda,p}$ corresponding to $\{(\lambda_n, p_n)\}$ has a subsequence which converges to some point of $S(\lambda, p)$.

Remark 2.2. When $(PSVQEP)_{\lambda,p}$ reduces to $(PEP)_p$, Definition 2.1 coincides with 3.1 of [27], and Definitions 2.2 and 2.3 coincide with 3.2 of [27].

Definition 2.4. Let Λ, P , and X be topological spaces and Y be a topological vector space. Let $C : \Lambda \times X \rightarrow 2^Y$ be a set-valued mapping such that, for each $(\lambda, x) \in \Lambda \times X$, $C(\lambda, x)$ is a nonempty closed convex cone in Y . Let $f : P \times X \times X \rightarrow Y$ be a vector-valued mapping. For any fixed $(\lambda, x) \in \Lambda \times X$, f is said to be upper (resp. lower) $C(\lambda, x)$ continuous if, for any $(p, y) \in P \times X$ and any neighborhood V of 0 in Y , there exist neighborhoods U_p, U_x and U_y of p, x , and y , respectively, such that

$$f(p', x', y') \in f(p, x, y) + V + C(\lambda, x), \quad \forall (p', x', y') \in U_p \times U_x \times U_y,$$

$$(\text{resp. } f(p', x', y') \in f(p, x, y) + V - C(\lambda, x), \quad \forall (p', x', y') \in U_p \times U_x \times U_y).$$

Remark 2.3. If $f : P \times X \times X \rightarrow Y$ is continuous, then for any fixed $(\lambda, x) \in \Lambda \times X$, f is both upper $C(\lambda, x)$ continuous and lower $C(\lambda, x)$ continuous.

Definition 2.5 [37]. Let X and Y be two topological spaces. A set-valued mapping $T : X \rightarrow 2^Y$ is said to be

- (i) upper semicontinuous (for short, u.s.c.) at $x_0 \in X$ if, for each open set V in Y with $T(x_0) \subseteq V$, there exists an open neighborhood $U(x_0)$ of x_0 such that $T(x) \subseteq V$ for all $x \in U(x_0)$;
- (ii) lower semicontinuous (for short, l.s.c.) at $x_0 \in X$ if, for each open set V in Y with $T(x_0) \cap V \neq \emptyset$, there exists an open neighborhood $U(x_0)$ of x_0 such that $T(x) \cap V \neq \emptyset$ for all $x \in U(x_0)$;
- (iii) u.s.c. (resp. l.s.c.) on X if it is u.s.c. (resp. l.s.c.) at every point $x \in X$;

- (iv) continuous on X if it is both u.s.c. and l.s.c. on X ;
- (v) closed if the graph $\text{Gr}(T) = \{(x, y) \in X \times Y : y \in T(x)\}$ is a closed subset of $X \times Y$.

Lemma 2.1 [37]. Let X and Y be two topological spaces, and $F : X \rightarrow 2^Y$ a set-valued mapping.

- (i) If F is u.s.c. and close-valued, then F is closed;
- (ii) If $F(x)$ is a compact set, then F is u.s.c. at $x \in X$, if and only if for any net $\{x_\alpha\} \subseteq X$ with $x_\alpha \rightarrow x$ and any net $\{y_\alpha\} \subseteq Y$ with $y_\alpha \in F(x_\alpha)$ for all α , there exist $y \in F(x)$ and a subnet $\{y_\beta\}$ of $\{y_\alpha\}$ such that $y_\beta \rightarrow y$;
- (iii) F is l.s.c. at $x \in X$, if and only if for any $y \in F(x)$ and for any net $\{x_\alpha\}$ with $x_\alpha \rightarrow x$, there exists a net $\{y_\alpha\}$ with $y_\alpha \in F(x_\alpha)$ for all α such that $y_\alpha \rightarrow y$.

We also need the concepts of noncompactness measure and Hausdorff metric.

Definition 2.6 [38]. Let E be completed. The Kuratowski measure of noncompactness of a set $A \subseteq E$ is defined by

$$\mu(A) = \inf\{\varepsilon > 0 : A \subset \cup_{i=1}^n A_i, \text{diam}A_i < \varepsilon, i = 1, 2, \dots, n\},$$

where $\text{diam}A$ denotes the diameter of A defined by $\text{diam}A = \sup\{d(x_1, x_2) : x_1, x_2 \in A\}$.

Definition 2.7 [38]. Let A and B be nonempty subsets of E . The Hausdorff metric $H(\cdot, \cdot)$ between A and B is defined by

$$H(A, B) = \max\{h(A, B), h(B, A)\},$$

where $h(A, B) = \sup_{a \in A} d(a, B)$ with $d(a, B) = \inf_{b \in B} d(a, b)$.

3 Well-posedness for (PSVQEP)

In this section, we shall establish some characterizations of well-posedness for (PSVQEP).

3.1 Continuity characterization of well-posedness for (PSVQEP)

For each $(\lambda, p) \in \Lambda \times P$ and $\varepsilon \geq 0$, the ε -solution set for $(\text{PSVQEP})_{\lambda, p}$ is defined by

$$\Pi(\lambda, p, \varepsilon) := \{x \in X : x \in K(\lambda, x) \text{ and } f(p, x, y) + \varepsilon e(x) \in C(\lambda, x), \quad \forall y \in K(\lambda, x)\}.$$

We call Π an ε -solution mapping.

Clearly, for each $(\lambda, p) \in \Lambda \times P$ and $\varepsilon \geq 0$, $S(\lambda, p) \subseteq \Pi(\lambda, p, \varepsilon)$, and $S(\lambda, p) = \Pi(\lambda, p, 0)$.

Lemma 3.1. For any fixed $(\lambda, x) \in \Lambda \times X$, if f is lower $C(\lambda, x)$ continuous, then for every $\varepsilon \geq 0$, $f(p, x, y) + \varepsilon e(x)$ is lower $C(\lambda, x)$ continuous.

Proof. Let $(\lambda, x) \in \Lambda \times X$ be such that f is lower $C(\lambda, x)$ continuous. For any given $\varepsilon \geq 0$, define a vector-valued mapping $F : P \times X \times X \rightarrow Y$ as follows:

$$F(p, x, y) = f(p, x, y) + \varepsilon e(x), \quad \forall (p, x, y) \in P \times X \times X.$$

For any neighborhood V of 0 in Y , there exists a balanced neighborhood V_1 of 0 in Y such that

$$V_1 + V_1 \subseteq V. \tag{3.1}$$

For any $(p, y) \in P \times X$, since e is continuous and f is lower $C(\lambda, x)$ continuous, there exist neighborhoods U_p, U_x and U_y of p, x , and y , respectively, such that

$$f(p', x', \gamma') \in f(p, x, \gamma) + V_1 - C(\lambda, x), \quad \forall (p', x', \gamma') \in U_p \times U_x \times U_y. \tag{3.2}$$

and

$$\varepsilon e(x') \in \varepsilon e(x) + V_1, \quad \forall x' \in U_x. \tag{3.3}$$

Let $U = U_p \times U_x \times U_y$, then by (3.2), (3.3), and (3.1), we have, for each $(p', x', \gamma') \in U$,

$$\begin{aligned} F(p', x', \gamma') &= f(p', x', \gamma') + \varepsilon e(x') \\ &\in f(p, x, \gamma) + V_1 - C(\lambda, x) + \varepsilon e(x) + V_1 \\ &\subseteq f(p, x, \gamma) + \varepsilon e(x) + V - C(\lambda, x) \\ &= F(p, x, \gamma) + V - C(\lambda, x). \end{aligned}$$

that is,

$$F(p', x', \gamma') \in F(p, x, \gamma) + V - C(\lambda, x), \quad \forall (p', x', \gamma') \in U.$$

Thus, F is lower $C(\lambda, x)$ continuous. This completes the proof.

Lemma 3.2. Assume that

- (i) for each $\lambda \in \Lambda$, $K(\lambda, \cdot)$ is continuous and close-valued on X ;
- (ii) for each λ , $C(\lambda, \cdot)$ is u.s.c. on X ;
- (iii) for each $(\lambda, x) \in \Lambda \times X$, f is lower $C(\lambda, x)$ continuous.

Then for each $(\lambda, p, \varepsilon) \in \Lambda \times P \times R^+$, $S(\lambda, p)$ and $\Pi(\lambda, p, \varepsilon)$ are closed subsets of X .

Proof. (1) Let $(\lambda, p) \in \Lambda \times P$ be any given sequence. Let $\{x_n\} \subseteq S(\lambda, p)$ be any sequence such that $x_n \rightarrow x \in X$. Then, for each n , we have

$$x_n \in K(\lambda, x_n)$$

and

$$f(p, x_n, \gamma) \in C(\lambda, x_n), \quad \forall \gamma \in K(\lambda, x_n). \tag{3.4}$$

Since $K(\lambda, \cdot)$ is u.s.c. and close-valued, $K(\lambda, \cdot)$ is closed and so $x \in K(\lambda, x)$. For each $y \in K(\lambda, x)$, since $K(\lambda, \cdot)$ is l.s.c. at x , there exists a sequence $\{y_n\}$ with $y_n \in K(\lambda, x_n)$ such that $y_n \rightarrow y$. Then, by (3.4), we have

$$f(p, x_n, \gamma_n) \in C(\lambda, x_n), \quad \forall n \in N. \tag{3.5}$$

For any neighborhood V of 0 in Y , there exists a balanced neighborhood V_0 of 0 in Y such that

$$V_0 + V_0 \subseteq V. \tag{3.6}$$

For V_0 , since $C(\lambda, \cdot)$ is u.s.c. at x and f is lower $C(\lambda, x)$ continuous, there exist neighborhoods U_x and U_y of x and y , respectively, such that

$$C(\lambda, x') \subseteq C(\lambda, x) + V_0, \quad \forall x' \in U_x. \tag{3.7}$$

$$f(p, x', y') \in f(p, x, y) + V_0 - C(\lambda, x), \quad \forall (x', y') \in U_x \times U_y. \tag{3.8}$$

Since $x_n \rightarrow x$ and $y_n \rightarrow y$, there exists $n_0 \in \mathbb{N}$ such that, for all $n \geq n_0$, $x_n \in U_x$ and $y_n \in U_y$. Then, by (3.7) and (3.8), for all $n \geq n_0$,

$$C(\lambda, x_n) \subseteq C(\lambda, x) + V_0, \tag{3.9}$$

$$f(p, x_n, y_n) \in f(p, x, y) + V_0 - C(\lambda, x). \tag{3.10}$$

Noting that V_0 is balanced, we have $-V_0 = V_0$. Then, by (3.10), (3.5), (3.9), and (3.6), we have, for each $n \geq n_0$,

$$\begin{aligned} f(p, x, y) &\in f(p, x_n, y_n) - V_0 + C(\lambda, x) \\ &= f(p, x_n, y_n) + V_0 + C(\lambda, x) \\ &\subseteq C(\lambda, x_n) + V_0 + C(\lambda, x) \\ &\subseteq C(\lambda, x) + V_0 + V_0 + C(\lambda, x) \\ &\subseteq C(\lambda, x) + V. \end{aligned}$$

By the arbitrary of V , we get

$$f(p, x, y) \in C(\lambda, x).$$

Thus, $x \in S(\lambda, p)$, and this implies that $S(\lambda, p)$ is closed.

(2) For each $\varepsilon > 0$, let

$$F(p, x, y) = f(p, x, y) + \varepsilon e(x), \quad \forall (p, x, y) \in P \times X \times X.$$

Then, by Lemma 3.1, for each $(\lambda, x) \in \Lambda \times X$, F is lower $C(\lambda, x)$ continuous. It follows from (1) that, for each $(\lambda, p) \in \Lambda \times P$, the set

$$\{x \in X : x \in K(\lambda, x) \text{ and } F(p, x, y) \in C(\lambda, x), \forall y \in K(\lambda, x)\}$$

is closed, i.e., $\Pi(\lambda, p, \varepsilon)$ is closed. This completes the proof.

The following theorem gives a characterization of the well-posedness in the generalized sense for (PSVQEP).

Theorem 3.1. (PSVQEP) is well-posed in the generalized sense, if and only if for every $(\lambda, p) \in \Lambda \times P$, $S(\lambda, p)$ is a nonempty compact subset of X and Π is u.s.c. at $(\lambda, p, 0)$.

Proof. For any $(\lambda, p) \in \Lambda \times P$, suppose that $S(\lambda, p)$ is a nonempty compact subset of X and Π is u.s.c. at $(\lambda, p, 0)$. Let $\{(\lambda_n, p_n)\} \subseteq \Lambda \times P$ be an arbitrary sequence with $(\lambda_n, p_n) \rightarrow (\lambda, p)$ and $\{x_n\}$ be an approximating sequence of $(\text{PSVQEP})_{\lambda, p}$ corresponding to $\{(\lambda_n, p_n)\}$. Then, there exists a sequence $\{\varepsilon_n\} \subseteq \mathbb{R}^+$ with $\varepsilon_n \rightarrow 0$ such that, for each $n \in \mathbb{N}$,

$$x_n \in K(\lambda_n, x_n)$$

and

$$f(p_n, x_n, y) + \varepsilon_n e(x_n) \in C(\lambda_n, x_n), \quad \forall y \in K(\lambda_n, x_n).$$

Thus, $x_n \in \Pi(\lambda_n, p_n, \varepsilon_n)$. Note that $\Pi(\lambda, p, 0) = S(\lambda, p)$ is compact. Since Π is u.s.c. at $(\lambda, p, 0)$, there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ which converges to some point $x_0 \in \Pi(\lambda, p, 0) = S(\lambda, p)$. Therefore, (PSVQEP) is well-posed in the generalized sense.

Conversely, suppose that (PSVQEP) is well-posed in the generalized sense. Then, for each $(\lambda, p) \in \Lambda \times P$, $S(\lambda, p)$ is nonempty and compact, and so $\Pi(\lambda, p, 0)$ is nonempty and compact. Let $\{(\lambda_n, p_n, \varepsilon_n)\} \subseteq \Lambda \times P \times R^+$ be an arbitrary sequence with $(\lambda_n, p_n, \varepsilon_n) \rightarrow (\lambda, p, 0)$ and $x_n \in \Pi(\lambda_n, p_n, \varepsilon_n)$. Then, for each $n \in N$,

$$x_n \in K(\lambda_n, x_n)$$

and

$$f(p_n, x_n, \gamma) + \varepsilon_n e(x_n) \in C(\lambda_n, x_n), \quad \forall \gamma \in K(\lambda_n, x_n).$$

Thus, $\{x_n\}$ is an approximating sequence of $(\text{PSVQEP})_{\lambda, p}$ corresponding to $\{(\lambda_n, p_n)\}$. By the well-posedness in the generalized sense of (PSVQEP), $\{x_n\}$ has a subsequence $\{x_{n_i}\}$ which converges to some point of $S(\lambda, p) = \Pi(\lambda, p, 0)$. Thus, Π is u.s.c. at $(\lambda, p, 0)$. This completes the proof.

For the well-posedness of (PSVQEP), we have the following result.

Theorem 3.2. (PSVQEP) is well-posed, if and only if for every $(\lambda, p) \in \Lambda \times P$, $S(\lambda, p)$ has a unique point and Π is u.s.c. at $(\lambda, p, 0)$.

Proof. The necessity is a direct conclusion of Definition 2.2 and Theorem 3.1.

For the sufficiency, suppose that for any $(\lambda, p) \in \Lambda \times P$, $S(\lambda, p)$ has a unique point $x_{\lambda, p}$ and Π is u.s.c. at $(\lambda, p, 0)$. Let $\{(\lambda_n, p_n)\} \subseteq \Lambda \times P$ be an arbitrary sequence with $(\lambda_n, p_n) \rightarrow (\lambda, p)$ and $\{x_n\}$ be an approximating sequence of $(\text{PSVQEP})_{\lambda, p}$ corresponding to $\{(\lambda_n, p_n)\}$. Then, there exists a sequence $\{\varepsilon_n\} \subseteq R^+$ with $\varepsilon_n \rightarrow 0$ such that, for each $n \in N$,

$$x_n \in K(\lambda_n, x_n)$$

and

$$f(p_n, x_n, \gamma) + \varepsilon_n e(x_n) \in C(\lambda_n, x_n), \quad \forall \gamma \in K(\lambda_n, x_n).$$

Thus, $x_n \in \Pi(\lambda_n, p_n, \varepsilon_n)$. Note that $\Pi(\lambda, p, 0) = S(\lambda, p) = \{x_{\lambda, p}\}$ is a singleton set and so is compact. Since Π is u.s.c. at $(\lambda, p, 0)$, for any neighborhood V of $x_{\lambda, p}$, there exists a neighborhood U of $(\lambda, p, 0)$ such that

$$\Pi(\lambda', p', \varepsilon') \subseteq V, \quad \forall (\lambda', p', \varepsilon') \in U.$$

Since $(\lambda_n, p_n, \varepsilon_n) \rightarrow (\lambda, p, 0)$, there exists $n_0 \in N$ such that

$$(\lambda_n, p_n, \varepsilon_n) \in U, \quad \forall n \geq n_0.$$

It follows that

$$x_n \in \Pi(\lambda_n, p_n, \varepsilon_n) \subseteq V, \quad \forall n \geq n_0.$$

Hence $x_n \rightarrow x_{\lambda, p}$, and this implies that (PSVQEP) is well-posed. This completes the proof.

Now we give a sufficient condition for the upper semicontinuity of the ε -solution mapping.

Theorem 3.3. Let X be a nonempty compact subset of E . Assume that

- (i) K is continuous and close-valued on $\Lambda \times X$;
- (ii) C is u.s.c. on $X \times X$;
- (iii) for any fixed $(\lambda, x) \in \Lambda \times X$, f is lower $C(\lambda, x)$ continuous;
- (iv) for each $\lambda \in \Lambda$ and $p \in P$, $S(\lambda, p) \neq \emptyset$.

Then, Π is u.s.c. on $\Lambda \times P \times R^+$.

Proof. Let $(\lambda_0, p_0, \varepsilon_0) \in \Lambda \times P \times R^+$ be any fixed point. Then, by (iv) and Lemma 3.2, $\Pi(\lambda_0, p_0, \varepsilon_0)$ is nonempty and closed. Furthermore, $\Pi(\lambda_0, p_0, \varepsilon_0)$ is nonempty and compact since $\Pi(\lambda_0, p_0, \varepsilon_0) \subseteq X$ and X is compact. Let $(\lambda_n, p_n, \varepsilon_n) \rightarrow (\lambda_0, p_0, \varepsilon_0)$ and $x_n \in \Pi(\lambda_n, p_n, \varepsilon_n)$. Observe that $\{x_n\} \subseteq X$ and X is compact. We may assume that $x_n \rightarrow x_0$ for some $x_0 \in X$. Then, by similar arguments as Lemma 3.2, we can show that $x_0 \in \Pi(\lambda_0, p_0, \varepsilon_0)$. This implies that Π is u.s.c. at $(\lambda_0, p_0, \varepsilon_0)$. Then, by the arbitrary of $(\lambda_0, p_0, \varepsilon_0)$, we know that Π is u.s.c. on $\Lambda \times P \times R^+$. This completes the proof.

Next, we give an example to illustrate Theorem 3.3.

Example 3.1. Let $E = R$, $X = [0, 2]$, $\Lambda = P = [0, 1] \subseteq R$, $Y = R^2$. Let

$$\begin{aligned}
 e(x) &= (1, 0), \quad \forall x \in X, \\
 C(\lambda, x) &= \left\{ (\xi, \eta) \in R^2 : \eta \geq 0, \xi + \eta \left[\frac{2}{\pi} \arctan(\lambda x) \right] \geq 0 \right\}, \quad \forall (\lambda, x) \in \Lambda \times X, \\
 K(\lambda, x) &= [\lambda x, x], \quad \forall (\lambda, x) \in \Lambda \times X, \\
 f(p, x, y) &= (p - x, x - y), \quad \forall (p, x, y) \in P \times X \times X.
 \end{aligned}$$

Then, it is easy to see that all the conditions of Theorem 3.3 are satisfied and so Π is u.s.c. on $\Lambda \times P \times R^+$. Indeed, by simple computation, we have $\Pi(\lambda, p, \varepsilon) = [0, p + \varepsilon] \cap [0, 2]$ for all $(\lambda, p, \varepsilon) \in \Lambda \times P \times R^+$. Thus, Π is u.s.c. on $\Lambda \times P \times R^+$.

3.2 Metric characterization of well-posedness for (PSVQEP)

In order to give metric characterization of well-posedness for (PSVQEP), we introduce the following notation.

Let $(\lambda, p) \in \Lambda \times P$ be given sequence. The approximating solution set of (PSVQEP) $_{\lambda, p}$ is defined by, for each $\delta \geq 0$ and $\varepsilon \geq 0$,

$$\begin{aligned}
 \Omega_{\lambda, p}(\delta, \varepsilon) &= \bigcup_{\substack{\lambda' \in B(\lambda, \delta) \cap \Lambda \\ p' \in B(p, \delta) \cap P}} \Pi(\lambda', p', \varepsilon) \\
 &= \bigcup_{\substack{\lambda' \in B(\lambda, \delta) \cap \Lambda \\ p' \in B(p, \delta) \cap P}} \{x \in X : x \in K(\lambda', x) \text{ and } f(p', x, y) + \varepsilon e(x) \in C(\lambda', x), \quad \forall y \in K(\lambda', x)\},
 \end{aligned}$$

where $B(a, r)$ denotes the closed ball centered at a with radius r .

Clearly, we have, for every $(\lambda, p) \in \Lambda \times P$,

- (i) $\Omega_{\lambda, p}(0, 0) = \Pi(\lambda, p, 0) = S(\lambda, p)$;
- (ii) $S(\lambda, p) \subseteq \Pi(\lambda, p, \varepsilon) \subseteq \Omega_{\lambda, p}(\delta, \varepsilon)$, $\forall \delta, \varepsilon > 0$;
- (iii) if $0 \leq \delta_1 \leq \delta_2$ and $0 \leq \varepsilon_1 \leq \varepsilon_2$, then $\Omega_{\lambda, p}(\delta_1, \varepsilon_1) \subseteq \Omega_{\lambda, p}(\delta_2, \varepsilon_2)$.

Lemma 3.3. Assume that

- (i) for each $x \in X$, $K(\cdot, x)$ is continuous and close-valued;
- (ii) for each $x \in X$, $C(\cdot, x)$ is u.s.c. and close-valued;
- (iii) for each $x \in X$, $f(\cdot, x, \cdot)$ is continuous.

Then, for every $(\lambda, p) \in \Lambda \times P$, $S(\lambda, p) = \bigcap_{\delta>0, \varepsilon>0} \Omega_{\lambda,p}(\delta, \varepsilon)$.

Proof. Let $(\lambda, p) \in \Lambda \times P$ be any given sequence. Clearly, $S(\lambda, p) \subseteq \bigcap_{\delta>0, \varepsilon>0} \Omega_{\lambda,p}(\delta, \varepsilon)$.

Thus, we only need to show that $\bigcap_{\delta>0, \varepsilon>0} \Omega_{\lambda,p}(\delta, \varepsilon) \subseteq S(\lambda, p)$. Indeed, if $x \in \bigcap_{\delta>0, \varepsilon>0} \Omega_{\lambda,p}(\delta, \varepsilon)$, then, for each $\delta > 0$, and $\varepsilon > 0$, $x \in \Omega_{\lambda,p}(\delta, \varepsilon)$. Thus, for each $n \in \mathbb{N}$, $x \in \Omega_{\lambda,p}(\frac{1}{n}, \frac{1}{n})$ and, so there exists $(\lambda_n, p_n) \in \Lambda \times P$ with $\lambda_n \in B(\lambda, \frac{1}{n})$ and $p_n \in B(p, \frac{1}{n})$ such that

$$x \in K(\lambda_n, x) \tag{3.11}$$

and

$$f(p_n, x, y) + \frac{1}{n}e(x) \in C(\lambda_n, x), \quad \forall y \in K(\lambda_n, x). \tag{3.12}$$

Clearly, $\lambda_n \rightarrow \lambda$ and $p_n \rightarrow p$ as $n \rightarrow \infty$. Since $K(\cdot, x)$ is u.s.c. and closed-valued, $K(\cdot, x)$ is closed and so $x \in K(\lambda, x)$. For each $y \in K(\lambda, x)$, since $K(\cdot, x)$ is l.s.c., there exists $y_n \in K(\lambda_n, x)$ such that $y_n \rightarrow y$. Then, by (3.12), we have

$$f(p_n, x, y_n) + \frac{1}{n}e(x) \in C(\lambda_n, x), \quad \forall n \in \mathbb{N}.$$

Since $C(\cdot, x)$ is u.s.c. and close-valued, $C(\cdot, x)$ is closed. Then, by the continuity of $f(\cdot, x, \cdot)$, we get

$$f(p, x, y) \in C(\lambda, x).$$

By the arbitrary of y , we know that $x \in S(\lambda, p)$ and this implies that $\bigcap_{\delta>0, \varepsilon>0} \Omega_{\lambda,p}(\delta, \varepsilon) \subseteq S(\lambda, p)$. Therefore, $S(\lambda, p) = \bigcap_{\delta>0, \varepsilon>0} \Omega_{\lambda,p}(\delta, \varepsilon)$. This completes the proof.

Lemma 3.4. Assume that

- (i) Λ and P are finite dimensional;
- (ii) K is continuous and close-valued;
- (iii) C is u.s.c. and close-valued;
- (iii) f is continuous.

Then, for each $\delta > 0$ and $\varepsilon > 0$, $\Omega_{\lambda,p}(\delta, \varepsilon)$ is closed.

Proof. Let $\delta \geq 0$ and $\varepsilon \geq 0$ be given. Let $\{x_n\} \subseteq \Omega_{\lambda,p}(\delta, \varepsilon)$ be any sequence such that $x_n \rightarrow x$. Then, for each $n \in \mathbb{N}$, there exist $(\lambda_n, p_n) \in \Lambda \times P$ with $\lambda_n \in B(\lambda, \delta)$ and $p_n \in B(p, \delta)$ such that

$$x_n \in K(\lambda_n, x_n) \tag{3.13}$$

and

$$f(p_n, x_n, \gamma) + \varepsilon e(x_n) \in C(\lambda_n, x_n), \quad \forall \gamma \in K(\lambda_n, x_n). \quad (3.14)$$

Noting that Λ and P are finite dimensional, without loss of generality, we may assume that $\lambda_n \rightarrow \lambda_0$ and $p_n \rightarrow p_0$ for some $\lambda_0 \in \Lambda$ and $p_0 \in P$. It follows that

$$d(\lambda_0, \lambda) = \lim_{n \rightarrow \infty} d(\lambda_n, \lambda) \leq \delta \text{ and } d(p_0, p) = \lim_{n \rightarrow \infty} d(p_n, p) \leq \delta.$$

Hence, $\lambda_0 \in B(\lambda, \delta)$ and $p_0 \in B(p, \delta)$. Since K is u.s.c. and close-valued, K is closed and so $x \in K(\lambda, x)$. For each $y \in K(\lambda, x)$, since K is l.s.c., there exists $y_n \in K(\lambda_n, x_n)$ such that $y_n \rightarrow y$. Then, by (3.14), we have

$$f(p_n, x_n, \gamma_n) + \varepsilon e(x_n) \in C(\lambda_n, x_n), \quad \forall n \in N.$$

Since C is u.s.c. and close-valued, C is closed. Then, by the continuity of f and e , we get

$$f(p_0, x, \gamma) + \varepsilon e(x) \in C(\lambda_0, x).$$

Thus, $x \in \Omega_{\lambda,p}(\delta, \varepsilon)$, and so $\Omega_{\lambda,p}(\delta, \varepsilon)$ is closed. This completes the proof.

The following theorem shows that the well-posedness in the generalized sense for (PSVQEP) can be characterized by considering the noncompactness of approximating solution set.

Theorem 3.4. *Let Λ and P be finite dimensional. Assume that*

- (i) *K is continuous and close-valued;*
- (ii) *C is continuous and close-valued;*
- (iii) *f is continuous.*

Then, (PQSVEP) is well-posed in the generalized sense, if and only if for every $(\lambda, p) \in \Lambda \times P$

$$\Omega_{\lambda,p}(\delta, \varepsilon) \neq \emptyset, \quad \forall \delta, \varepsilon > 0, \text{ and } \mu(\Omega_{\lambda,p}(\delta, \varepsilon)) \rightarrow 0 \text{ as } (\delta, \varepsilon) \rightarrow (0, 0). \quad (3.15)$$

Proof. Suppose that (PSVQEP) is well-posed in the generalized sense. Then, for each $(\lambda, p) \in \Lambda \times P$, $S(\lambda, p)$ is nonempty and compact. Clearly, $S(\lambda, p) \subseteq \Omega_{\lambda,p}(\delta, \varepsilon)$ for all $\delta, \varepsilon > 0$. It follows that $\Omega_{\lambda,p}(\delta, \varepsilon) \neq \emptyset$ for all $\delta, \varepsilon > 0$. Now we shall show that

$$\mu(\Omega_{\lambda,p}(\delta, \varepsilon)) \rightarrow 0 \text{ as } (\delta, \varepsilon) \rightarrow (0, 0).$$

Observe that for each $\delta, \varepsilon > 0$,

$$H(\Omega_{\lambda,p}(\delta, \varepsilon), S(\lambda, p)) = \max\{h(\Omega_{\lambda,p}(\delta, \varepsilon), S(\lambda, p)), h(S(\lambda, p), \Omega_{\lambda,p}(\delta, \varepsilon))\} = h(\Omega_{\lambda,p}(\delta, \varepsilon), S(\lambda, p)).$$

Taking into account the compactness of $S(\lambda, p)$, we get

$$\mu(\Omega_{\lambda,p}(\delta, \varepsilon)) \leq 2H(\Omega_{\lambda,p}(\delta, \varepsilon), S(\lambda, p)) + \mu(S(\lambda, p)) = 2h(\Omega_{\lambda,p}(\delta, \varepsilon), S(\lambda, p)).$$

To prove (3.15), it is sufficient to show that

$$h(\Omega_{\lambda,p}(\delta, \varepsilon), S(\lambda, p)) \rightarrow 0 \text{ as } (\delta, \varepsilon) \rightarrow (0, 0).$$

If $h(\Omega_{\lambda,p}(\delta, \varepsilon), S(\lambda, p)) \not\rightarrow 0$ as $(\delta, \varepsilon) \rightarrow (0, 0)$, then there exist $l > 0$ and $\delta_n > 0, \varepsilon_n > 0$ with $\delta_n \rightarrow 0, \varepsilon_n \rightarrow 0$, and $x_n \in \Omega_{\lambda,p}(\delta_n, \varepsilon_n)$ such that

$$x_n \notin S(\lambda, p) + B(0, l), \quad \forall n \in N. \tag{3.16}$$

As $x_n \in \Omega_{\lambda,p}(\delta_n, \varepsilon_n), \{x_n\}$ is an approximating sequence for $(\text{PSVQEP})_{\lambda,p}$. By the well-posedness in the generalized sense of (PSVQEP) , there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ converging to some point of $S(\lambda, p)$. This contradicts (3.16) and so

$$h(\Omega_{\lambda,p}(\delta, \varepsilon), S(\lambda, p)) \rightarrow 0 \text{ as } (\delta, \varepsilon) \rightarrow (0, 0).$$

Conversely, suppose that (3.15) holds. For each $(\lambda, p) \in \Lambda \times P$, by Lemma 3.4, $\Omega_{\lambda,p}(\delta, \varepsilon)$ is closed for all $\delta, \varepsilon > 0$. By Lemma 3.3, we have $S(\lambda, p) = \bigcap_{\delta > 0, \varepsilon > 0} \Omega_{\lambda,p}(\delta, \varepsilon)$. Since

$$\mu(\Omega_{\lambda,p}(\delta, \varepsilon)) \rightarrow 0,$$

the theorem on p.412 in [38] can be applied and one concludes that $S(\lambda, p)$ is nonempty, compact, and

$$h(\Omega_{\lambda,p}(\delta, \varepsilon), S(\lambda, p)) = H(\Omega_{\lambda,p}(\delta, \varepsilon), S(\lambda, p)) \rightarrow 0 \text{ as } (\delta, \varepsilon) \rightarrow (0, 0). \tag{3.17}$$

Let $(\lambda_n, p_n) \rightarrow (\lambda, p) \in \Lambda \times P$ and $\{x_n\} \subseteq X$ be an approximating sequence for $(\text{PSVQEP})_{\lambda,p}$ corresponding to $\{(\lambda_n, p_n)\}$. Then there exist $\varepsilon_n > 0$ with $\varepsilon_n \rightarrow 0$ such that, for each $n \in N$,

$$x_n \in K(\lambda_n, x_n)$$

and

$$f(p_n, x_n, \gamma) + \varepsilon_n e(x_n) \in C(\lambda_n, x_n), \quad \forall \gamma \in K(\lambda_n, x_n).$$

For each $n \in N$, let $\delta_n = \max\{d(\lambda_n, \lambda), d(p_n, p)\}$. Then $\delta_n \rightarrow 0$ and $x_n \in \Omega_{\lambda,p}(\delta_n, \varepsilon_n)$. It follows from (3.17) that

$$d(x_n, S(\lambda, p)) \leq h(\Omega_{\lambda,p}(\delta_n, \varepsilon_n), S(\lambda, p)) \rightarrow 0.$$

Since $S(\lambda, p)$ is compact, there exists $\bar{x}_n \in S(\lambda, p)$ such that

$$d(x_n, \bar{x}_n) = d(x_n, S(\lambda, p)) \rightarrow 0.$$

Again from the compactness of $S(\lambda, p)$, $\{\bar{x}_n\}$ has a subsequence $\{\bar{x}_{n_k}\}$ converging to $\bar{x} \in S(\lambda, p)$. Hence, the corresponding subsequence $\{x_{n_k}\}$ of $\{x_n\}$ converges to \bar{x} . Therefore, (PSVQEP) is well-posed in the generalized sense. This completes the proof.

Example 3.2. Let $E, X, \Lambda, P, Y, e, K, C$, and f be the same as in Example 3.1. Then, it is easy to see that the conditions (i)-(iii) of Theorem 3.4 are satisfied. Moreover, by simple computation, we have, for each $(\lambda, p) \in \Lambda \times P$ and each $\delta, \varepsilon \geq 0$ and each $\lambda' \in [\lambda - \delta, \lambda + \delta] \cap \Lambda$ and each $p' \in [p - \delta, p + \delta] \cap P$,

$$\Pi(\lambda', p', \varepsilon) = [0, p' + \varepsilon] \cap [0, 2].$$

It follows that

$$\Omega_{\lambda,p}(\delta, \varepsilon) = \bigcup_{p' \in [p - \delta, p + \delta] \cap P} ([0, p' + \varepsilon] \cap [0, 2]) = [0, p + \delta + \varepsilon] \cap [0, 1 + \varepsilon] \cap [0, 2].$$

Thus, $\Omega_{\lambda,p}(\delta, \varepsilon) \neq \emptyset$ for all $\delta, \varepsilon > 0$ and $\mu(\Omega_{\lambda,p}(\delta, \varepsilon)) \rightarrow 0$ as $(\delta, \varepsilon) \rightarrow (0, 0)$. By Theorem 3.4, (PSVQEP) is well-posed in the generalized sense.

Indeed, by simple computation, we have $S(\lambda, p) = [0, p]$ for all $(\lambda, p) \in \Lambda \times P$. Suppose that $\{(\lambda_n, p_n)\} \subseteq \Lambda \times P$ with $(\lambda_n, p_n) \rightarrow (\lambda, p)$ and $\{x_n\}$ is an approximating sequence for (PSVQEP) $_{\lambda,p}$ corresponding to $\{(\lambda_n, p_n)\}$. Then there exists $\{\varepsilon_n\} \subseteq R^+$ with $\varepsilon_n \rightarrow 0$ such that $x_n \in \Pi(\lambda_n, p_n, \varepsilon_n)$, i.e., $x_n \in [0, p_n + \varepsilon_n] \cap [0, 2]$. Since $p_n \rightarrow p \in P = [0, 1]$ and $\varepsilon_n \rightarrow 0$, there exists $n_0 \in N$ such that, for each $n \geq n_0$, $p_n + \varepsilon_n < 2$, and so $x_n \in [0, p_n + \varepsilon_n]$. Since $\{x_n\}$ is bounded, it has a subsequence which converges to some point of $[0, p] = S(\lambda, p)$. Therefore, (PSVQEP) is well-posed in the generalized sense.

4 Applications

Since vector quasi-equilibrium problems contain vector quasi-variational inequality problems, vector quasi-optimization problems, and vector quasi-saddle point problems as special cases, we can derive from the result in Section 3, some consequences for such special cases. In this section, we discuss only some applications of our results to strong vector quasi-variational inequality problems and strong vector quasi-optimization problems.

Let X and Y be two real Banach spaces and C be a closed convex cone in Y . Let $L(X, Y)$ be the set of all the continuous linear operators from X into Y . Let $e \in C$ be a fixed point.

4.1 Strong vector quasi-variational inequality problems

Let $F : X \rightarrow 2^X$ be a set-valued mapping and $G : X \rightarrow L(X, Y)$ a vector-valued mapping. We consider the following strong vector quasi-variational inequality problem (for short, SVQVIP): find $x \in X$ such that

$$x \in F(x) \quad \text{and} \quad \langle Gx, y - x \rangle \in C, \quad \forall y \in F(x).$$

Denote by S the set of solutions for (SVQVIP). For each $\varepsilon \geq 0$, we denote by $\Pi(\varepsilon)$ the ε -solution set of (SVQVIP), i.e.,

$$\Pi(\varepsilon) = \{x \in X : x \in F(x) \text{ and } \langle Gx, y - x \rangle + \varepsilon e \in C, \quad \forall y \in F(x)\}.$$

Let $f(x, y) = \langle Gx, y - x \rangle$. We can obtain the following results for (SVQVIP).

Theorem 4.1. (SVQVIP) is well-posed in the generalized sense, if and only if S is a nonempty compact subset of X and Π is u.s.c. at 0.

Theorem 4.2. (SVQVIP) is well-posed, if and only if S has a unique point and Π is u.s.c. at 0.

4.2 Strong vector quasi-optimization problems

Let $F : X \rightarrow 2^X$ be a set-valued mapping and $\phi : X \rightarrow Y$ be a vector-valued mapping. We consider the following strong vector quasi-optimization problem (for short, SVQOP):

$$\min \phi(x) \quad \text{subject to } x \in F(x).$$

A point $x_0 \in X$ is called a strongly efficient solution of (SVQOP), if

$$x_0 \in F(x_0) \text{ and } \phi(y) - \phi(x_0) \in C, \quad \forall y \in F(x_0).$$

Denote by S the set of all strongly efficient solutions for (SVQOP). For each $\varepsilon \geq 0$, we denote by $\Pi(\varepsilon)$ the ε -strongly efficient solution set of (SVQOP) as follows:

$$\Pi(\varepsilon) = \{x \in X : x \in F(x) \text{ and } \varphi(y) - \varphi(x) + \varepsilon e \in C, \quad \forall y \in F(x)\}.$$

Let $f(x, y) = \phi(y) - \phi(x)$. We can obtain the following results for (SVQOP).

Theorem 4.3. (SVQOP) is well-posed in the generalized sense, if and only if S is a nonempty compact subset of X and Π is u.s.c. at 0.

Theorem 4.4. (SVQOP) is well-posed, if and only if S has a unique point and Π is u.s.c. at 0.

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Authors' contributions

Q-YL carried out the study of well-posedness for parametric strong vector quasi-equilibrium problems, strong vector quasi-variational inequality problems and strong vector quasi-optimization problems, and participated in the sequence alignment and drafted the manuscript. S-HW conceived of the study, and participated in its design and coordination. All authors read and approved the final manuscript.

Competing interests

The authors declare that they have no competing interests.

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