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Approximating fixed points for nonself mappings in CAT(0) spaces

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Abstract

Suppose K is a nonempty closed convex subset of a complete CAT(0) space X with the nearest point projection P from X onto K. Let $T : K \to X$ be a nonself mapping, satisfying Condition (E) with $F(T) := \{x \in K : Tx = x\} \neq \emptyset$. Suppose $\{x_n\}$ is generated iteratively by $x_1 \in K$, $x_{n+1} = P((1 - \alpha_n)x_n \oplus \alpha_n TP[(1 - \beta_n)x_n \oplus \beta_n Tx_n]), n \geq 1$, where $\{\alpha_n\}$ and $\{\beta_n\}$ are real sequences in $[\varepsilon, 1 - \varepsilon]$ for some $\varepsilon \in (0, 1)$. Then, $\{x_n\}$ Δ-converges to some point $x^\star$ in $F(T)$. This extends a result of Laowang and Panyanak [Fixed Point Theory Appl. **367274**, 11 (2010)] for nonself mappings satisfying Condition (E).

Keywords: CAT(0) spaces, fixed point, condition (E), nonself mappings

1 Introduction

In 2010, Laowang and Panyanak [1] studied an iterative scheme and proved the following result: let K be a nonempty closed convex subset of a complete CAT(0) space X, (the initials of term "CAT" are in honor of E. Cartan, A.D. Alexanderov and V.A. Toponogov) with the nearest point projection P from X onto K. Let $T : K \to X$ be a nonexpansive nonself mapping with nonempty fixed point set. If $\{x_n\}$ is generated iteratively by

$$x_1 \in K, \quad x_{n+1} = P((1 - \alpha_n)x_n \oplus \alpha_n TP[(1 - \beta_n)x_n \oplus \beta_n Tx_n]), \tag{1.1}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are real sequences in $[\varepsilon, 1 - \varepsilon]$ for some $\varepsilon \in (0, 1)$, then $\{x_n\}$ is Δ-convergent to a fixed point of T. In this article, this result is extended for nonself mappings satisfying Condition (E).

Let K be a nonempty subset of a CAT(0) space X and $T : K \to X$ be a mapping. A point $x \in K$ is called a fixed point of T, if $x = Tx$. We shall denote the fixed point set of T by $F(T)$. Moreover, T is called nonexpansive if for each $x, y \in K$, $d(Tx, Ty) \leq d(x, y)$.

In 2011, Falset et al. [2] introduced Condition (E) as follows:

Definition 1.1. Let K be a bounded closed convex subset of a complete CAT(0) space X. A mapping $T : K \to X$ is called to satisfy Condition (E_μ) on C, if there exists $\mu \geq 1$ such that

$$\mathrm{d}(x, Ty) \leq \mu \mathrm{d}(Tx, x) + \mathrm{d}(x, y)$$

holds, for all $x, y \in K$. It is called, T satisfies Condition (E) on C whenever T satisfies (E_μ) for some $\mu \geq 1$.

Proposition 1.2 [2]. *Every nonexpansive mapping satisfies Condition (E), but the inverse is not true.*

Now, we need some fact about CAT(0) spaces as follows:

Let (X, d) be a metric space. A geodesic path joining $x \in X$ to $y \in X$ (or, more briefly, a geodesic from x to y) is a map c from a closed interval $[0, l] \subset R$ to X such that $c(0) = x$, $c(l) = y$ and $d(c(t), c(t')) = ||t - t'||$ for all $t, t' \in [0, l]$. In particular, c is an isometry and $d(x, y) = l$. The image α of c is called a geodesic (or metric) segment joining x and y. When it is unique, this geodesic is denoted by $[x, y]$. The space (X, d) is said to be a geodesic space, if every two points of X are joined by a geodesic, and X is said to be uniquely geodesic if there is exactly one geodesic joining x to y, for each $x, y \in X$. A subset $Y \subset X$ is said to be convex if Y includes every geodesic segment joining any two of its points. A geodesic triangle $\Delta(x_1, x_2, x_3)$ in a geodesic metric space (X, d) consists of three points in X (the vertices of Δ) and a geodesic segment between each pair of vertices (the edges of Δ). A comparison triangle for geodesic triangle $\Delta(x_1, x_2, x_3)$ in (X, d) is a triangle $\bar{\Delta}(x_1, x_2, x_3) := \Delta(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ in the Euclidean plane E^2 such that $d_{E^2}(\bar{x}_i, \bar{x}_j) = d(x_i, x_j)$ for $i, j \in \{1, 2, 3\}$. A geodesic metric space is said to be a CAT(0) space [3], if all geodesic triangles of appropriate size satisfy the following comparison axiom. Let Δ be a geodesic triangle in X and $\bar{\Delta}$ be a comparison triangle for Δ. Then Δ is said to satisfy the CAT(0) inequality if for all $x, y \in \Delta$ and all comparison points $\bar{x}, \bar{y} \in \bar{\Delta}$,

$$d(x, y) \leq d_{E^2}(\bar{x}, \bar{y}). \tag{1.2}$$

If x, y_1, y_2 are points in a CAT(0) space and y_0 is the midpoint of the segment $[y_1, y_2]$, then the CAT(0) inequality implies

$$d(x, y_0)^2 \leq \frac{1}{2}d(x, y_1)^2 + \frac{1}{2}d(x, y_2)^2 - \frac{1}{4}d(y_1, y_2)^2. \tag{CN}$$

In fact, a geodesic space is a CAT(0) space if and only if it satisfies the (CN) inequality (Courbure negative)[[3], p. 163].

Lemma 1.3. *Let (X, d) be a CAT(0) space.*

1. [[3], Proposition 2.4] *Let K be a convex subset of X which is complete in the induced metric. Then for every $x \in X$, there exists a unique point $P(x) \in K$ such that $d(x, P(x)) = \inf\{d(x, y): y \in K\}$. Moreover, the map $x \to P(x)$ is a nonexpansive retract from X onto K.*
2. [[4], Lemma 2.1] *For $x, y \in X$ and $t \in [0, 1]$, there exists a unique point $z \in [x, y]$ such that*

$$d(x, z) = t d(x, y), \qquad d(y, z) = (1 - t) d(x, y)$$

one uses the notation $(1 - t)x \oplus ty$ for the unique point z.

3. [[4], Lemma 2.4] *For $x, y, z \in X$ and $t \in [0, 1]$, one has*

$$d((1 - t)x \oplus ty, z) \leq (1 - t)d(x, z) + t d(y, z).$$

[[4], Lemma 2.5] *For $x, y, z \in X$ and $t \in [0, 1]$, one has*

$$d((1 - t)x \oplus ty, z)^2 \leq (1 - t)d(x, z)^2 + t d(y, z)^2 - t(1 - t)d(x, y)^2.$$

Let $\{x_n\}$ be a bounded sequence in a CAT(0) space X. For $x \in X$, we set

$$r(x, \{x_n\}) = \limsup_{n \to \infty} \mathrm{d}(x, x_n).$$

The asymptotic radius

$$r(\{x_n\}) = \inf\{r(x, \{x_n\}) : x \in X\},$$

and the asymptotic center $A(\{x_n\})$ of $\{x_n\}$ is the set

$$A(\{x_n\}) = \{x \in X : r(x, \{x_n\}) = r(\{x_n\})\}.$$

It is known [[5], Proposition 7], in a CAT(0) space X, $A(\{x_n\})$ consists of exactly one point.

Definition 1.4. [[6], Definition 3.1] A sequence $\{x_n\}$ in a CAT(0) space X is said Δ-converges to $x \in X$, if x is the unique asymptotic center of $\{u_n\}$ for every subsequence $\{u_n\}$ of $\{x_n\}$. In this case, one can write Δ - $\lim_n x_n = x$ and call x the Δ - lim of $\{x_n\}$.

Lemma 1.5. *Let* (X, d) *be a CAT(0) space.*

1. [[6], p. 3690] *Every bounded sequence in X has a* Δ*-convergent subsequence.*
2. [[7], Proposition 2.1] *If K is a closed convex subset of X and if* $\{x_n\}$ *is a bounded sequence in K, then the asymptotic center of* $\{x_n\}$ *is in K.*
3. [[4], Lemma 2.8] *If* $\{x_n\}$ *is a bounded sequence in X with* $A(\{x_n\}) = \{x\}$ *and* $\{u_n\}$ *is a subsequence of* $\{x_n\}$ *with* $A(\{u_n\}) = \{u\}$ *and the sequence* $\{\mathrm{d}(x_n, u)\}$ *converges, then* $x = u$.

2 Main results

The following lemma was proved by Dhompongsa and Panyanak in the case of nonexpansive [[4], Lemma 2.10].

Lemma 2.1. *Let K be a nonempty closed convex subset of a complete CAT(0) space X, and* $T : K \to X$ *be a nonself mapping, satisfying Condition* (E). *Suppose* $\{x_n\}$ *is a bounded sequence in K such that* $\lim_n \mathrm{d}(x_n, Tx_n) = 0$ *and* $\{\mathrm{d}(x_n, v)\}$ *converges for all* $v \in F(T)$. *Then*

$$\omega_w(x_n) \subset F(T),$$

where $\omega_w(x_n) := \bigcup A(\{u_n\})$ *and the union is taken over all subsequences* $\{u_n\}$ *of* $\{x_n\}$. *Moreover,* $\omega_w(x_n)$ *consists of exactly one point.*

Proof. Let $u \in \omega_w(x_n)$, then there exists a subsequence $\{u_n\}$ of $\{x_n\}$ such that $A(\{u_n\}) = \{u\}$. By part (1) and (2) of Lemma 1.5, there exists a subsequence $\{v_n\}$ of $\{u_n\}$ such that Δ - $lim_n v_n = v \in K$. We show $v \in F(T)$. In order to prove this, by Condition (E), one can write

$$\mathrm{d}(x_n, Tv) \leq \mu \mathrm{d}(Tx_n, x_n) + \mathrm{d}(x_n, v)$$

for some $\mu \geq 1$. Therefore

$$\begin{aligned} \limsup_n \mathrm{d}(x_n, Tv) &\leq \limsup_n (\mu \mathrm{d}(Tx_n, x_n) + \mathrm{d}(x_n, v)) \\ &= \limsup_n \mathrm{d}(x_n, v). \end{aligned}$$

The uniqueness of asymptotic center, implies $v \in K$ and $T(v) = v$. By part (3) Lemma 1.5, $u = v$. Therefore $\omega_w(x_n) \subset F(T)$. Let $\{u_n\}$ be a subsequence of $\{x_n\}$ with $A(\{u_n\}) = \{u\}$ and $A(\{x_n\}) = \{x\}$. Since $u \in \omega_w(x_n) \subset F(T)$, $\{d(x_n, v)\}$ converges. By part (3) Lemma 1.5, $x = u$. This shows that $\omega_w(x_n)$ consists of exactly one point. □

Theorem 2.2. *Let K be a nonempty closed convex subset of a complete CAT(0) space X, and* $T : K \to X$ *be a nonself mapping, satisfying Condition* (*E*) *with* $x^\star \in F(T) = \{x \in K : Tx = x\}$. *Let* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *be sequences in* $[\varepsilon, 1 - \varepsilon]$ *for some* $\varepsilon \in (0, 1)$. *Starting from arbitrary* $x_1 \in K$, *define the sequence* $\{x_n\}$ *by* $x_{n+1} = P((1 - \alpha_n)x_n \oplus \alpha_n TP[(1 - \beta_n) x_n \oplus \beta_n Tx_n])$, $n \geq 1$. *Then* $\lim_{n\to\infty} d(x_n, x^\star)$ *exists.*

Proof. By part (1) of Lemma 1.3, the nearest point projection P from X onto K is nonexpansive. Then,

$$\begin{aligned} d(x_{n+1}, x^\star) &= d(P((1-\alpha_n)x_n \oplus \alpha_n TP[(1-\beta_n)x_n \oplus \beta_n Tx_n]), Px^\star) \\ &\leq d((1-\alpha_n)x_n \oplus \alpha_n TP[(1-\beta_n)x_n \oplus \beta_n Tx_n], x^\star) \\ &= (1-\alpha_n)d(x_n, x^\star) + \alpha_n d(TP[(1-\beta_n)x_n \oplus \beta_n Tx_n], x^\star). \end{aligned}$$

But by Condition (*E*), for some $\mu \geq 1$, we have

$$\begin{aligned} &(1-\alpha_n)d(x_n, x^\star) + \alpha_n d(TP[(1-\beta_n)x_n \oplus \beta_n Tx_n], x^\star) \\ &\leq (1-\alpha_n)d(x_n, x^\star) + \alpha_n(\mu d(Tx^\star, x^\star) + d(P[(1-\beta_n)x_n \oplus \beta_n Tx_n], x^\star)) \\ &\leq (1-\alpha_n)d(x_n, x^\star) + \alpha_n[(1-\beta_n)d(x_n, x^\star) + \beta_n d(x_n, x^\star)] \\ &= d(x_n, x^\star). \end{aligned}$$

Consequently, $d(x_{n+1}, x^\star) \leq d(x_n, x^\star)$. Then $d(x_n, x^\star) \leq d(x_1, x^\star)$ for all $n \geq 1$. This implies $\{d(x_n, x^\star)\}_{n=1}^{\infty}$ is bounded and decreasing. Hence, $\lim_{n\to\infty} d(x_n, x^\star)$ exists. □

Theorem 2.3. *Let K be a nonempty closed convex subset of a complete CAT(0) space X, and* $T : K \to X$ *be a nonself mapping, satisfying Condition* (*E*) *with* $F(T) \neq \emptyset$. *Let* $\{\alpha_n\}$ *and* $\{\beta_n\}$ *be sequences in* $[\varepsilon, 1 - \varepsilon]$ *for some* $\varepsilon \in (0, 1)$. *Starting from arbitrary* $x_1 \in K$, *define the sequence* $\{x_n\}$ *by* $x_{n+1} = P((1 - \alpha_n)x_n \oplus \alpha_n TP\,[(1 - \beta_n)x_n \oplus \beta_n Tx_n])$, $n \geq 1$. *Then* $\lim_{n\to\infty} d(x_n, Tx_n) = 0$.

Proof. Let $x^\star \in F(T)$. By Theorem 2.2, $\lim_{n\to\infty} d(x_n, x^\star)$ exists. Set

$$\lim_{n\to\infty} d(x_n, x^\star) = r.$$

If $r = 0$, by the Condition (*E*), for some $\mu \geq 1$,

$$\begin{aligned} d(x_n, Tx_n) &\leq d(x^\star, x_n) + d(x^\star, Tx_n) \\ &\leq d(x^\star, x_n) + \mu d(x^\star, Tx^\star) + d(x^\star, x_n). \end{aligned}$$

Therefore $\lim_{n\to\infty} d(x_n, Tx_n) = 0$.

If $r > 0$, set $y_n = P\,[(1 - \beta_n)x_n \oplus \beta_n Tx_n]$. By part (4) of Lemma 1.3,

$$\begin{aligned} d(y_n, x^\star)^2 &= d(P[(1-\beta_n)x_n \oplus \beta_n Tx_n], Px^\star)^2 \\ &\leq d([(1-\beta_n)x_n \oplus \beta_n Tx_n], x^\star)^2 \\ &\leq (1-\beta_n)d(x_n, x^\star)^2 + \beta_n d(Tx_n, x^\star)^2 - \beta_n(1-\beta_n)d(x_n, Tx_n)^2 \\ &\leq (1-\beta_n)d(x_n, x^\star)^2 + \beta_n d(Tx_n, x^\star)^2. \end{aligned} \tag{2.3}$$

Using Condition (E), for some $\mu \geq 1$,

$$\begin{aligned}(1-\beta_n)\mathrm{d}(x_n, x^\star)^2 + \beta_n \mathrm{d}(Tx_n, x^\star)^2 \\ &\leq (1-\beta_n)\mathrm{d}(x_n, x^\star)^2 + \beta_n(\mu \mathrm{d}(Tx^\star, x^\star) + \mathrm{d}(x_n, x^\star))^2 \\ &= \mathrm{d}(x_n, x^\star)^2.\end{aligned} \tag{2.4}$$

Therefore by inequities (2.3) and (2.4), one can get

$$\mathrm{d}(y_n, x^\star) \leq \mathrm{d}(x_n, x^\star). \tag{2.5}$$

Part (4) of Lemma 1.3, shows

$$\begin{aligned}\mathrm{d}(x_{n+1}, x^\star)^2 &= \mathrm{d}(P[(1-\alpha_n)x_n \oplus \alpha_n Ty_n], Px^\star)^2 \\ &\leq \mathrm{d}((1-\alpha_n)x_n \oplus \alpha_n Ty_n, x^\star)^2 \\ &\leq (1-\alpha_n)\mathrm{d}(x_n, x^\star)^2 + \alpha_n \mathrm{d}(Ty_n, x^\star)^2 - \alpha_n(1-\alpha_n)\mathrm{d}(x_n, Ty_n)^2 \\ &\leq (1-\alpha_n)\mathrm{d}(x_n, x^\star)^2 + \alpha_n(\mu \mathrm{d}(Tx^\star, x^\star) + \mathrm{d}(y_n, x^\star))^2 \\ &\quad - \alpha_n(1-\alpha_n)\mathrm{d}(x_n, Ty_n)^2 \\ &= (1-\alpha_n)\mathrm{d}(x_n, x^\star)^2 + \alpha_n \mathrm{d}(y_n, x^\star)^2 - \alpha_n(1-\alpha_n)\mathrm{d}(x_n, Ty_n)^2 \\ &\leq (1-\alpha_n)\mathrm{d}(x_n, x^\star)^2 + \alpha_n \mathrm{d}(x_n, x^\star)^2 - \alpha_n(1-\alpha_n)\mathrm{d}(x_n, Ty_n)^2 \\ &= \mathrm{d}(x_n, x^\star)^2 - \alpha_n(1-\alpha_n)\mathrm{d}(x_n, Ty_n)^2.\end{aligned}$$

Therefore

$$\mathrm{d}(x_{n+1}, x^\star)^2 \leq \mathrm{d}(x_n, x^\star)^2 - W(\alpha_n)\mathrm{d}(x_n, Ty_n)^2,$$

where $W(\alpha) = \alpha(1 - \alpha)$. Since $\alpha \in [\varepsilon, 1 - \varepsilon]$, $W(\alpha_n) \geq \varepsilon^2$.
Therefore

$$\varepsilon^2 \sum_{n=1}^{\infty} \mathrm{d}(x_n, Ty_n)^2 \leq \mathrm{d}(x_1, x^\star)^2 < \infty.$$

This implies $\lim_{n\to\infty} \mathrm{d}(x_n, Ty_n) = 0$.
By Condition (E), for some $\mu \geq 1$, we have

$$\begin{aligned}\mathrm{d}(x_n, x^\star) &\leq \mathrm{d}(x_n, Ty_n) + \mathrm{d}(Ty_n, x^\star) \\ &\leq \mathrm{d}(x_n, Ty_n) + \mu \mathrm{d}(Tx^\star, x^\star) + \mathrm{d}(y_n, x^\star) \\ &= \mathrm{d}(x_n, Ty_n) + \mathrm{d}(y_n, x^\star).\end{aligned}$$

Hence

$$r \leq \liminf_{n\to\infty} \mathrm{d}(y_n, x^\star).$$

On the other hand, from (2.5),

$$\limsup_{n\to\infty} \mathrm{d}(y_n, x^\star) \leq r.$$

This implies

$$\lim_{n\to\infty} \mathrm{d}(y_n, x^\star) = r.$$

Thus (2.5) shows

$$\lim_{n\to\infty} \mathrm{d}((1-\beta_n)x_n \oplus \beta_n Tx_n], x^\star) = r.$$

Since T satisfies Condition (E), we have

$$\begin{aligned}\mathrm{d}(Tx_n, x^\star) &\leq \mu \mathrm{d}(Tx^\star, x^\star) + \mathrm{d}(x_n, x^\star)\\ &= \mathrm{d}(x_n, x^\star)\end{aligned}$$

Thus

$$\limsup_{n\to\infty} \mathrm{d}(Tx_n, x^\star) \leq r.$$

Now, by [[1], Lemma 2.9], $\lim_{n\to\infty}$ d(x_n, Tx_n) = 0. □

Theorem 2.4. *Let K be a nonempty closed convex subset of a complete CAT(0) space X, and $T : K \to X$ be a nonself mapping, satisfying Condition (E) with $F(T) \neq \emptyset$. Assume $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in [ε, 1 - ε] for some $\varepsilon \in$ (0, 1). Starting from arbitrary $x_1 \in K$, define the sequence $\{x_n\}$ by $x_{n+1} = P((1 - \alpha_n)x_n \oplus \alpha_n TP[(1 - \beta_n)x_n \oplus \beta_n Tx_n])$, $n \geq 1$. Then $\{x_n\}$ is Δ-convergent to some point $x^\star$ in $F(T)$.*

Proof. By Theorem 2.3, $\lim_{n\to\infty}$ d(x_n, Tx_n) = 0. The proof of Theorem 2.2 shows {d (x_n, v)} is bounded and decreasing for each $v \in F(T)$, and so it is convergent. By Lemma 2.1, $\omega_w(x_n)$ consists exactly one point which is a fixed point of T. Consequently, the sequence $\{x_n\}$ is Δ-convergent to some point $x^\star$ in $F(T)$. □

The following definition is recalled from [8].

Definition 2.5. A mapping $T : K \to X$ is said to satisfy Condition I, if there exists a nondecreasing function f: [0, ∞) → [0, ∞) with $f(0) = 0$ and $f(r) > 0$ for all $r > 0$ such that

$$\mathrm{d}(x, Tx) \geq f(\mathrm{d}(x, F(T))),$$

where $x \in K$.

With respect to the above definition, we have the following theorem [[1], Theorem 3.4].

Theorem 2.6. *Let K be a nonempty closed convex subset of a complete CAT(0) space X, and $T : K \to X$ be a nonself mapping, satisfying condition (E) with $F(T) \neq \emptyset$. Assume $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in [ε, 1 - ε] for some $\varepsilon \in$ (0, 1). Starting from arbitrary $x_1 \in K$, define the sequence $\{x_n\}$ by $x_{n+1} = P((1 - \alpha_n)x_n \oplus \alpha_n TP\ [(1 - \beta_n)x_n \oplus \beta_n Tx_n])$, $n \geq 1$. If T satisfies condition I, then $\{x_n\}$ converges strongly to a fixed point of T.*

We state another strong convergence theorem [[1], Theorem 3.5] as follows:

Theorem 2.7. *Let K be a nonempty compact convex subset of a complete CAT(0) space X, and $T : K \to X$ be a nonself mapping, satisfying condition (E) with $F(T) \neq \emptyset$. Assume $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in [ε, 1 - ε] for some $\varepsilon \in$ (0, 1). Starting from arbitrary $x_1 \in K$, define the sequence $\{x_n\}$ by $x_{n+1} = P((1 - \alpha_n)x_n \oplus \alpha_n TP[(1 - \beta_n)x_n \oplus \beta_n Tx_n])$,$n \geq 1$. Then, $\{x_n\}$ converges strongly to a fixed point of T.*

Another result in [1] is to obtain the Δ-convergence of a defined sequence, to a common fixed point of two nonexpansive self-mappings. According to the present setting, we can state the following result.

Theorem 2.8. *Let K be a nonempty closed convex subset of a complete CAT(0) space X, and S, $T : K \to X$ be two nonself mappings, satisfying Condition (E) with $F(S) \cap F(T)$*

$\neq \emptyset$. Assume $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $[\varepsilon, 1 - \varepsilon]$ for some $\varepsilon \in (0, 1)$. Starting from arbitrary $x_1 \in K$, define the sequence $\{x_n\}$ by $x_{n+1} = (1 - \alpha_n)x_n \oplus \alpha_n S[(1 - \beta_n)x_n \oplus \beta_n T x_n]$, $n \geq 1$. Then $\{x_n\}$ is Δ-convergent to a common fixed point of S and T.

Authors' contributions
The authors have contributed in obtaining the new results presented in this article. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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