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Convergence theorem for an iterative algorithm of λ-strict pseudocontraction

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Abstract

In this article, we prove strong convergence of sequence generated by the following iteration sequence for a class of Lipschitzian pseudocontractive mapping T:

$$x_{n+1} = \beta_n u + (1 - \beta_n)[\alpha_n T x_n + (1 - \alpha_n)x_n]$$

whenever $\{\alpha_n\}$ and $\{\beta_n\}$ satisfy the appropriate conditions.
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1. Introduction

Let T be a pseudocontractive mapping defined on a real smooth Banach space E. We consider the problem of finding a solution $z \in E$ *of the fixed point equation* $x = Tx$. One classical way to study pseudocontractive mappings is to use a strong pseudocontraction to approximate a pseudocontractive mapping T. More precisely take $t \in (0, 1)$ and $u \in E$ define a strong pseudocontraction T_t by $T_t x = tu + (1 - t)Tx$. In [1, Corollary 2],Deimling proves that T_t has a unique fixed point x_t, i.e.,

$$x_t = tu + (1 - t)Tx_t. \tag{1.1}$$

This implicit iteration was introduced by Browder [2] for a nonexpansive mapping T in Hilbert space. Halpern [3] was the first who introduced the following explicit iteration scheme for a nonexpansive mapping T which was referred to as *Halpern iteration*: for $u, x_0 \in K$, $\alpha_n \in [0, 1]$,

$$x_{n+1} = \alpha_n u + (1 - \alpha_n)Tx_n. \tag{1.2}$$

Convergence of this two schemes have been studied by many researchers with various types of additional conditions. For the studies of a nonexpansive mapping T, see Bruck [4,5], Reich [6,7], Song-Xu [8], Takahashi-Ueda [9], Suzuki [10], and many others. For the studies of a continuous pseudocontractive mapping T, see Morales-Jung [11], Schu [12], Chidume-Zegeye [13], Chidume-Udomene [14], Udomene [15], Chidume-Ofoedu [16], Chen-Song-Zhou [17,18], Song [19-21], Song-Chen [22,23] and others. The following results play a key role in proving strong convergence of Halpern iteration.

Theorem 1.1 [11,22,23]*Let E be a reflexive Banach space which has both the fixed point property for nonexpansive self-mappings and a uniformly Gâteaux differ-entiable norm or be a reflexive and strictly convex Banach space with a uniformly Gâteaux differentiable norm. Assume that K is a nonempty, closed and convex subset of E. Suppose that T is a continuous pseudocontractive mapping from K into E with $F(T) \neq \emptyset$. Then, as $t \to 0$, x_t, defined by (1.1) converges strongly to a fixed point of T.*

Theorem 1.2. [22]*Let K be nonempty, closed and convex subset of a Banach space E with a uniformly Gâteaux differentiable norm and let $T : K \to K$ be a continuous pseudocontractive mapping with a fixed point. Assume that there exists a bounded sequence $\{x_n\}$ such that $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$ and $p = \lim_{t\to 0} z_t$ exists, where $\{z_t\}$ is defined by (1.1). Then,*

$$\limsup_{n\to\infty} \langle u - p, J(x_n - p) \rangle \le 0.$$

Mann [24] introduced the following iteration for T in a Hilbert space:

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n) T x_n, \tag{1.3}$$

where $\{\alpha_n\}$ is a sequence in [0, 1]. Latterly, Reich [25] studied this iteration in a uniformly convex Banach space with a Fréchet differentiable norm, and obtained that if T has a fixed point and $\sum_{n=0}^{\infty} \alpha_n(1 - \alpha_n) = \infty$, then the sequence $\{x_n\}$ converges weakly to a fixed point of T. This Mann's iteration process has extensively been studied over the last 20 years for constructions of fixed points of nonlinear mappings and for solving nonlinear operator equations involving monotone, accretive and pseudocontractive operators (see, e.g., [16,26-34] and others). In an infinite-dimensional Hilbert space, the classical Mann's iteration algorithm (1.3) has, in general, only weak convergence, even for nonexpansive mappings. In order to get strong convergence result, one has to modify the Mann's iteration algorithm. Several attempts have been made and many important results have been reported (see, e.g., [12-16,35-37] and others). Recently, Zhou [37] obtained strong convergence theorem for the following iterative sequence in a 2-uniformly smooth Banach space: for u, $x_0 \in E$ and λ-strict pseudocontraction T,

$$x_{n+1} = \beta_n u + \gamma_n x_n + (1 - \beta_n - \gamma_n)[\alpha_n T x_n + (1 - \alpha_n) x_n], \tag{1.4}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ in (0, 1) satisfy:

(i) $a \le \alpha_n \le \dfrac{\lambda}{K^2}$ for some $a > 0$ and for all $n \ge 0$;

(ii) $\lim_{n\to\infty} \beta_n = 0$ and $\sum_{n=1}^{\infty} \beta_n = \infty$;

(iii) $\lim_{n\to\infty} |\alpha_{n+1} - \alpha_n| = 0$;

(iv) $0 < \liminf_{n\to\infty} \gamma_n \le \limsup_{n\to\infty} \gamma_n < 1$.

Very recently, Zhang and Su [38] extended Zhou's results to q-uniformly smooth Banach space. However, the above results excluded $\gamma_n \equiv 0$ and $\gamma_n \equiv \dfrac{1}{n+1}$.

In this article, we deal with iterative schemes generated by the following iterative sequence (in (1.4), $\gamma_n \equiv 0$) for λ-strict pseudocontraction T:

$$x_{n+1} = \beta_n u + (1 - \beta_n)[\alpha_n T x_n + (1 - \alpha_n) x_n], \tag{1.5}$$

and obtain its strong convergence whenever $\{\alpha_n\}$ and $\{\beta_n\}$ satisfy the following conditions:

(i) $\alpha_n \in \left[a, \dfrac{\lambda}{K^2}\right]$ such that $\sum\limits_{n=1}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty$;

(ii) $\lim\limits_{n \to \infty} \beta_n = 0$, $\sum\limits_{n=1}^{\infty} \beta_n = \infty$ and $\sum\limits_{n=1}^{\infty} |\beta_{n+1} - \beta_n| < \infty$.

Our result not only complements and develops corresponding ones of Zhou [37, Theorem 2.3] (see also Zhang and Su [38, Theorem 4.1], where $\gamma_n \equiv 0$), but also extend main result of Chidume-Chidume [35] and Kim-Xu [36] from nonexpansive mappings to λ-strict pseudocontractions.

2. Preliminaries

Throughout this article, a Banach space E will always be over the real scalar field. We denote its norm by $\| \cdot \|$ and its dual space by E^*. The value of $x^* \in E^*$ at $y \in E$ is denoted by $\langle y, x \rangle$ and the normalized duality mapping from E into 2^{E^*} is denoted by J, that is, $J(x) = \{f \in E^* : \langle x, f \rangle = \|x\|\|f\|, \|x\| = \|f\|\}$. Let $F(T) = \{x \in E : Tx = x\}$ be the set of all fixed point of a mapping T.

Recall that a mapping T with domain $D(T)$ and range $R(T)$ in Banach space E is called *strongly pseudo-contractive* if, for all $x, y \in D(T)$, there exist $k \in (0, 1)$ and $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y) \rangle \leq k\|x - y\|^2 \tag{2.1}$$

or, equivalently,

$$\langle (x - Tx) - (y - Ty), j(x - y) \rangle \geq (1 - k)\|x - y\|^2 \tag{2.2}$$

while T is said to be *pseudo-contractive* if (2.1) or (2.2) holds for $k = 1$. A mapping T is said to be *Lipschitzian* if, for all $x, y \in K$, there exists $L > 0$ such that

$$\|Tx - Ty\| \leq L\|x - y\|.$$

A mapping T is called *non-expansive* if $L = 1$ and, further, T is said to be *contractive* if $L < 1$. An important class of mappings closely related to the class of pseudo-contractive mappings is that of accretive mappings. A mapping A is *accretive* if and only if $(I - A)$ is *pseudo-contractive*. The accretive mappings were independently introduced by Browder [39] and Kato [40] in 1967. The importance of these mappings is well known. A mapping T is called λ-*strictly pseudocontractive*, if for all $x, y \in D(T)$, there exists $\lambda \in (0, 1)$ and $j(x - y) \in J(x - y)$ such that

$$\langle Tx - Ty, j(x - y) \rangle \leq \|x - y\|^2 - \lambda\|x - y - (Tx - Ty)\|^2. \tag{2.3}$$

It is obvious that λ-strictly pseudocontractive mapping is Lipschitzian with $L = \dfrac{\lambda + 1}{\lambda}$.

The class of nonexpansive mappings is a subclass of strictly pseudocontractive mappings in Hilbert space, but the converse implication may be false. We remark that the class of strongly pseudo-contractive mappings is independent from the class of λ-strict pseudo-contractions. This can be seen from the existing examples (see, e.g., [30,37]).

Let $S(E) := \{x \in E; \|x\| = 1\}$ denote the unit sphere of a Banach space E. The space E is said to have (i) *a Gâteaux differentiable norm* (we also say that E is *smooth*), if the limit

$$\lim_{t \to 0} \frac{\|x + ty\| - \|x\|}{t} \tag{2.4}$$

exists for each $x, y \in S(E)$; (ii) *a uniformly Gâteaux differentiable norm*, if for any y in $S(E)$, the limit (2.4) is uniformly attained for $x \in S(E)$; (iii) *a Fréchet differentiable norm*, if for any $x \in S(E)$, the limit (2.4) is attained uniformly for $y \in S(E)$; (iv) *a uniformly Fréchet differentiable norm* (we also say that E is *uniformly smooth*), if the limit (2.4) is attained uniformly for all $(x, y) \in S(E) \times S(E)$; (v) *fixed point property* for non-expansive self-mappings, if each non-expansive self-mapping defined on any bounded, closed convex subset K of E has at least one fixed point. Let $\rho_E : [0, \infty) \to [0, \infty)$ be the modulus of smoothness of E defined by

$$\rho_E(t) = \sup \left\{ \frac{1}{2}(\|x + y\| + \|x - y\|) - 1 : x \in S(E), \|y\| \leq t \right\}.$$

Let $q > 1$. A Banach space E is said to be q-uniformly smooth, if there exists a fixed constant $c > 0$ such that $\rho_E(t) < ct^q$. It is well known that E is uniformly smooth if and only if $\lim_{t \to 0} \frac{\rho_E(t)}{t} = 0$. If E is q-uniformly smooth, then E is uniformly smooth, and hence the norm of E is uniformly Fréchet differentiable, in particular, the norm of E is Fréchet differentiable. Typical example of uniformly smooth Banach spaces is L_p ($p > 1$). More precisely, L_p is min$\{p, 2\}$-uniformly smooth for every $p > 1$.

Lemma 2.1.(Zhou [37]) *Let E be a real 2-uniformly smooth Banach space with the best smooth constant K, C be a nonempty subset of E, and let $T : C \to C$ be a λ-strict pseudocontraction. For any $\alpha \in (0, 1)$, we define $T_\alpha = (1 - \alpha)x + \alpha Tx$. Then, as $\alpha \in \left(0, \frac{\lambda}{K^2}\right]$, $T_\alpha : C \to C$ is nonexpansive such that $F(T_\alpha) = F(T)$.*

Lemma 2.2. (Liu [34] and Xu [41]) *Let $\{a_n\}$ be a sequence of nonnegative real numbers satisfying the property:*

$$a_{n+1} \leq (1 - t_n)a_n + b_n + t_n c_n,$$

where $\{t_n\}$, $\{b_n\}$ and $\{c_n\}$ satisfy the restrictions:

(i) $\sum_{n=0}^{\infty} t_n = \infty$; (ii) $\sum_{n=0}^{\infty} b_n < +\infty$; (iii) $\limsup_{n \to \infty} c_n \leq 0$.

Then, $\{a_n\}$ converges to zero as $n \to \infty$.

3. Main result

Theorem 3.1 *Let E be a real 2-uniformly smooth Banach space with the best smooth constant K and let C be a nonempty, closed and convex subset of E. Suppose that $T : C \to C$ is a λ-strict pseudocontraction with $F(T) \neq \emptyset$. Given $u, x_0 \in C$, a sequence $\{x_n\}$ is generated by*

$$\begin{cases} y_n = \alpha_n T x_n + (1 - \alpha_n)x_n, \\ x_{n+1} = \beta_n u + (1 - \beta_n)y_n, \end{cases} \tag{3.1}$$

where $\{\beta_n\}$ and $\{\alpha_n\}$ in $(0, 1)$ satisfy the following control conditions:

(i) $\alpha_n \in \left[a, \frac{\lambda}{K^2}\right]$ for some constant $a \in \left(0, \frac{\lambda}{K^2}\right)$ such that $\sum_{n=1}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty$;

(ii) $\lim\limits_{n\to\infty} \beta_n = 0$, $\sum\limits_{n=1}^{\infty} \beta_n = \infty$ and $\sum\limits_{n=1}^{\infty} |\beta_{n+1} - \beta_n| < \infty.$

Then, $\{x_n\}$ converges strongly to a fixed point of T.

Proof. The proof will be divided into four steps.

Step 1. The sequence $\{x_n\}$ is bounded. Let $T_{\alpha_n} = \alpha_n T + (1 - \alpha_n)I$. Then, T_{α_n} is nonexpansive for every n by Lemma 2.1 and so, for $p \in F(T)$, we have

$$
\begin{aligned}
||x_{n+1} - p|| &= ||\beta_n(u - p) + (1 - \beta_n)(T_{\alpha_n}x_n - p)|| \\
&\leq \beta_n||u - p|| + (1 - \beta_n)||T_{\alpha_n}x_n - p|| \\
&\leq \beta_n||u - p|| + (1 - \beta_n)||x_n - p|| \\
&\leq \max\{||x_n - p||, ||u - p||\} \\
&\ \ \vdots \\
&\leq \max\{||x_0 - p||, ||u - p||\}.
\end{aligned}
$$

Consequently, both $\{x_n\}$ and $\{y_n\}$ are bounded. This implies the boundedness of $\{Tx_n\}$ from the inequality $||Tx_n - p|| \leq \dfrac{1 + \lambda}{\lambda}||x_n - p||$.

Let $M > 0$ be a constant such that $M \geq \sup_{n\in\mathbb{N}}\{||u||, ||x_n||, ||Tx_n||\}$.

Step 2. Since $y_n = T_{\alpha_n}x_n = \alpha_n Tx_n + (1 - \alpha_n)x_n$, then

$$||y_n|| = ||\alpha_n Tx_n + (1 - \alpha_n)x_n|| \leq \alpha_n||Tx_n|| + (1 - \alpha_n)||x_n|| \leq M.$$

Furthermore, we have

$$
\begin{aligned}
||y_{n+1} - y_n|| &= ||T_{\alpha_{n+1}}x_{n+1} - T_{\alpha_n}x_n|| \\
&\leq ||T_{\alpha_{n+1}}x_{n+1} - T_{\alpha_{n+1}}x_n|| + ||T_{\alpha_{n+1}}x_n - T_{\alpha_n}x_n|| \\
&\leq ||x_{n+1} - x_n|| + ||\alpha_{n+1}Tx_n + (1 - \alpha_{n+1})x_n - \alpha_n Tx_n - (1 - \alpha_n)x_n|| \quad (3.2) \\
&\leq ||x_{n+1} - x_n|| + |\alpha_{n+1} - \alpha_n|||x_n - Tx_n|| \\
&\leq ||x_{n+1} - x_n|| + 2M|\alpha_{n+1} - \alpha_n|.
\end{aligned}
$$

From (3.1), it follows

$$
\begin{aligned}
||x_{n+2} - x_{n+1}|| &= ||\beta_{n+1}u + (1 - \beta_{n+1})y_{n+1} - \beta_n u - (1 - \beta_n)y_n|| \\
&\leq |\beta_{n+1} - \beta_n|(||u|| + ||y_{n+1}||) + (1 - \beta_n)||y_{n+1} - y_n|| \qquad (3.3) \\
&\leq 2M|\beta_{n+1} - \beta_n| + (1 - \beta_n)||y_{n+1} - y_n||.
\end{aligned}
$$

Substituting (3.2) into (3.3) yields

$$||x_{n+2} - x_{n+1}|| \leq (1 - \beta_n)||x_{n+1} - x_n|| + 2M|\alpha_{n+1} - \alpha_n| + 2M|\beta_{n+1} - \beta_n|.$$

From the assumptions on $\{\alpha_n\}$ and $\{\beta_n\}$ and using Lemma 2.3, we conclude that

$$\lim_{n\to\infty} ||x_{n+1} - x_n|| = 0. \tag{3.4}$$

From the definition of x_n and since $\lim_{n\to\infty} \beta_n = 0$, it follows

$$\lim_{n\to\infty} ||x_{n+1} - y_n|| = \lim_{n\to\infty} \beta_n||u - Tx_n|| = 0.$$

Combining (3.4), we have

$$\lim_{n\to\infty} ||x_n - y_n|| = 0. \tag{3.5}$$

Thus, we obtain

$$\lim_{n \to \infty} ||x_n - Tx_n|| = \lim_{n \to \infty} \frac{||x_n - \gamma_n||}{\alpha_n} = 0.$$

Step 3. There exists $z \in F(T)$ such that

$$\limsup_{n \to \infty} \langle u - z, J(x_{n+1} - z) \rangle \leq 0.$$

Since E is 2-uniformly smooth, then E is a reflexive Banach space which has both the fixed point property for non-expansive self-mappings and a uniformly Gâteaux differentiable norm. Then, from Theorem 1.1, as $t \to 0$, x_t, defined by (1.1) converges strongly to a fixed point z of T. The desired conclusion follows from Theorem 1.2.

Step 4. $\lim_{n \to \infty} x_n = z$. In fact,

$$
\begin{aligned}
||x_{n+1} - z||^2 &= \langle \beta_n(u - z) + (1 - \beta_n)(\gamma_n - z), J(x_{n+1} - z) \rangle \\
&\leq \beta_n \langle u - z, J(x_{n+1} - z) \rangle + (1 - \beta_n)||T_{\alpha_n} x_n - z|| ||J(x_{n+1} - z)|| \\
&\leq \beta_n \langle u - z, J(x_{n+1} - z) \rangle + (1 - \beta_n)\frac{||x_n - z||^2 + ||x_{n+1} - z||^2}{2} \\
&\leq (1 - \beta_n)\frac{||x_n - z||^2}{2} + \frac{||x_{n+1} - z||^2}{2} + \beta_n \langle u - z, J(x_{n+1} - z) \rangle,
\end{aligned}
$$

which implies that

$$||x_{n+1} - z||^2 \leq (1 - \beta_n)||x_n - z||^2 + 2\beta_n \langle u - z, J(x_{n+1} - z) \rangle, \tag{3.6}$$

and hence $\lim_{n \to \infty} ||x_n - z|| = 0$ because of Lemma 2.2. This completes the proof.

Remark 1. Theorem 3.1 is applicable to l_p and L_p for all $p \geq 2$, however, we do not know whether it works for L_p for $1 < p < 2$.

Remark 2. In Theorem 3.1, if the condition $\sum_{n=1}^{\infty} |\beta_{n+1} - \beta_n| < \infty$ is replaced by $\lim_{n \to \infty} \frac{\beta_{n+1}}{\beta_n} = 1$, the conclusion still holds.

Remark 3. Theorem 3.1 not only complements and develops corresponding result of Zhou [37, Theorem 3.2] (see also Zhang and Su [38, Theorem 4.1] where $\gamma_n \equiv 0$), but also extend main result of Chidume-Chidume [35] and Kim-Xu [36] from nonexpansive mappings to λ-strict pseudocontractions.

Corollary 3.2 *Let E be a reflexive Banach space which has both the fixed point property for non-expansive self-mappings and a uniformly Gâteaux differentiable norm and let C be a nonempty, closed and convex subset of E. Suppose that $T : C \to C$ is a non-expansive mapping with $F(T) \neq \emptyset$. Given $u, x_0 \in C$, a sequence $\{x_n\}$ is generated by (3.1), where $\{\alpha_n\}$ and $\{\beta_n\}$ in (0,1) satisfy the following control conditions:*

(i) $\alpha_n \in (0, 1)$ such that $\sum_{n=1}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty$;

(ii) $\lim_{n \to \infty} \beta_n = 0$, $\sum_{n=1}^{\infty} \beta_n = \infty$;

(iii) either $\sum_{n=1}^{\infty} |\beta_{n+1} - \beta_n| < \infty$ or $\lim_{n \to \infty} \frac{\beta_{n+1}}{\beta_n} = 1$.

Then, $\{x_n\}$ converges strongly to a fixed point of T.

Proof. Let $T_{\alpha_n} = \alpha_n T + (1 - \alpha_n)I$. Clearly, T_{α_n} is nonexpansive and $F(T) = F(T_{\alpha_n})$ for each n. Therefore, following the same proof technique of Theorem 3.1, the desired result is obtained.

Remark 4. Theorem 3.1 of Chidume-Chidume [35] and Theorem 1 of Kim-Xu [36] can be regarded as a special case of Corollary 3.2, respectively. In fact, if $\alpha_n \equiv \delta \in (0, 1)$ in Corollary 3.2, then Theorem 3.1 of Chidume-Chidume [35] is reached; if in Corollary 3.2, E is a uniformly smooth Banach space and the conditions $\lim_{n\to\infty} a_n = 1$ and $\sum_{n=1}^{\infty} (1 - \alpha_n) = \infty$ are added, then Theorem 1 of Kim-Xu [36] is obtained.

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