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Comment on 'Fixed point theorems for contraction mappings in modular metric spaces, Fixed Point Theory and Applications, doi:10.1186/1687-1812-2011-93, 20 pages'

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Abstract
In this paper, we provide an example to show that some results obtained in [Mongkolkeha *et al.* in Fixed Point Theory Appl. 2011, doi:10.1186/1687-1812-2011-93] are not valid.

MSC: 47H09; 47H10

Keywords: contraction mappings; modular metric spaces; metric space

We begin with the definition of a modular metric space.

Definition 1 [1] Let X be a nonempty set. A function $\omega : (0,\infty) \times X \times X \to [0,\infty]$ is said to be *metric modular* on X if for all $x, y, z \in X$, the following conditions hold:

(i) $\omega_\lambda(x,y) = 0$ for all $\lambda > 0$ iff $x = y$;
(ii) $\omega_\lambda(x,y) = \omega_\lambda(y,x)$ for all $\lambda > 0$;
(iii) $\omega_{\lambda+\mu}(x,y) \le \omega_\lambda(x,z) + \omega_\mu(z,y)$ for all $\lambda, \mu > 0$.

Given $x_\star \in X$, the set $X_\omega(x_\star) = \{x \in X : \lim_{\lambda\to\infty} \omega_\lambda(x, x_\star) = 0\}$ is called a *modular metric space* generated by $x_\star$ and induced by ω. If its generator $x_\star$ does not play any role in the situation, we will write X_ω instead of $X_\omega(x_\star)$.

We need the following theorems in the proof of the main result of this paper.

Theorem 2 [1, Theorem 2.6] *If ω is metric (pseudo) modular on X, then the modular set X_ω is a (pseudo) metric space with (pseudo) metric given by*

$$d^\circ_\omega(x,y) = \inf\{\lambda > 0 : \omega_\lambda(x,y) \le \lambda\}, \quad x, y \in X_\omega.$$

Theorem 3 [1, Theorem 2.13] *Let ω be (pseudo) modular on a set X. Given a sequence $\{x_n\} \subset X_\omega$ and $x \in X_\omega$, we have $d^\circ_\omega(x_n, x) \to 0$ as $n \to \infty$ if and only if $\omega_\lambda(x_n, x) \to 0$ as $n \to \infty$ for all $\lambda > 0$. A similar assertion holds for Cauchy sequences.*

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Let ω be modular on a set X. A mapping $T : X_\omega \to X_\omega$ is said to be contraction [2, Definition 3.1] if there exists $k \in [0,1)$ such that

$$\omega_\lambda(Tx, Ty) \le k\omega_\lambda(x,y) \tag{1}$$

for all $\lambda > 0$ and $x, y \in X_\omega$.

Recently, Mongkolkeha *et al.* [2] proved the following theorems.

Theorem 4 [2, Theorem 3.2] *Let ω be metric modular on X and X_ω be a modular metric space induced by ω. If X_ω is a complete modular metric space and $T : X_\omega \to X_\omega$ is a contraction mapping, then T has a unique fixed point in X_ω. Moreover, for any $x \in X_\omega$, iterative sequence $\{T^n(x)\}$ converges to the fixed point.*

Theorem 5 [2, Theorem 3.4] *Let ω be metric modular on X and X_ω be a modular metric space induced by ω. If X_ω is a complete modular metric space and $T : X_\omega \to X_\omega$ is a mapping, which T^N is a contraction mapping for some positive integer N. Then, T has a unique fixed point in X_ω.*

We show that Theorems 4 and 5 are not correct. To this end, we give the following example.

Example 6 Let $X = \mathbb{R}$ and define modular ω by $\omega_\lambda(x,y) = \infty$ if $\lambda \le |x-y|$, and $\omega_\lambda(x,y) = 0$ if $\lambda > |x-y|$. It is easy to verify that (see also [1, Example 2.7]) $X_\omega = \mathbb{R}$ and $d^\circ_\omega(x,y) = |x-y|$. It follows from Theorem 3 that $\mathbb{R}$ is a complete modular metric space. Now, define $T : \mathbb{R} \to \mathbb{R}$ by $Tx = x + 1$. We show that T is a contraction while it has no fixed point. Let $k \in [0,1)$ (for example, $k = 1/2$) and $x, y \in \mathbb{R}$. If $\lambda \le |x-y|$, then $\omega_\lambda(x,y) = \infty$ and (1) holds. If $|x-y| < \lambda$, then $|Tx - Ty| = |x-y| < \lambda$. Therefore, $\omega_\lambda(Tx,Ty) = \omega_\lambda(x,y) = 0$. Hence T is a contraction. On the other hand, by definition of T, it is easy to see that T has no fixed point. So, Theorems 4 and 5 are not correct.

Remark 7 In [2, Example 3.7], the authors mentioned that 'Thus, T is not a contraction mapping and then the Banach contraction mapping cannot be applied to this example.' It is true that T is not contraction with the Euclidean metric, but one can easily verify that

$$d^\circ_\omega(Tx, Ty) \le \frac{\sqrt{3}}{2} d^\circ_\omega(x,y).$$

Thus, the Banach contraction guarantees the existence of a fixed point. Note that

$$d^\circ_\omega\big((a_1,0),(a_2,0)\big) = \sqrt{\frac{4|a_1 - a_2|}{3}}, \qquad d^\circ_\omega\big((0,b_1),(0,b_2)\big) = \sqrt{|b_1 - b_2|}$$

and

$$d^\circ_\omega\big((a,0),(0,b)\big) = \sqrt{\frac{4a}{3} + b}.$$

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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