Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH **Open Access**

Tripled fixed point and tripled coincidence point theorems in intuitionistic fuzzy normed spaces

Mujahid Abbas[1], Basit Ali[2], Wutiphol Sintunavarat[3*] and Poom Kumam[3*]

*Correspondence: poom_teun@hotmail.com; poom.kum@kmutt.ac.th
[3]Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok, 10140, Thailand
Full list of author information is available at the end of the article

Abstract
The aim of this paper is to prove the existence of tripled fixed point and tripled coincidence point theorems in intuitionistic fuzzy normed spaces (IFNS). Our results generalize and extend recent coupled fixed point theorems in IFNS.
MSC: 47H09; 47H10; 54H25

Keywords: IFNS; *t*-norm and *t*-conorm; tripled fixed point; tripled coincidence point

1 Introduction and preliminaries

The evolution of fuzzy mathematics commenced with an introduction of the notion of fuzzy sets by Zadeh [1] in 1965 as a new way to represent vagueness in every day life. The idea of intuitionistic fuzzy sets (IFS) was introduced by Atanassov [2]. Saadati and Park [3, 4] introduced intuitionistic fuzzy normed spaces (IFNS). For the detailed survey on fixed point results in fuzzy metric spaces, fuzzy normed spaces and IFNS, we refer the reader to [5–8]. Recently coupled fixed point theorems have been proved in IFNS; for details of these we refer to Gordji [9] and Sintunavarat *et al.* [10]. More recently, tripled fixed point theorems have been introduced in partially ordered metric spaces by Berinde [11]. In this paper, we have proved tripled fixed point and tripled coincidence point theorems in IFNS. Now we give some definitions, examples and lemmas for our main results.

For the sake of completeness, we recall some definitions and known results in a fuzzy metric space.

Definition 1.1 ([1]) Let X be any set. A *fuzzy set* A in X is a function with domain X and values in $[0,1]$.

Definition 1.2 ([12]) A binary operation $*: [0,1] \times [0,1] \to [0,1]$ is called a *continuous t-norm* if

(1) $*$ is associative and commutative;
(2) $*$ is continuous;
(3) $a * 1 = a$ for all $a \in [0,1]$;
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$.

Example 1.3 Three typical examples of continuous t-norms are $a * b = \min\{a,b\}$ (minimum t-norm), $a * b = ab$ (product t-norm), and $a * b = \max\{a + b - 1, 0\}$ (Lukasiewicz t-norm).

Definition 1.4 ([12]) A binary operation $\diamond : [0,1] \times [0,1] \longrightarrow [0,1]$ is called a *continuous t-conorm* if

(1) $\diamond$ is associative and commutative;
(2) $\diamond$ is continuous;
(3) $a \diamond 0 = a$ for all $a \in [0,1]$;
(4) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$.

Example 1.5 Two typical examples of continuous t-conorms are $a \diamond b = \min\{a+b, 1\}$ and $a \diamond b = \max\{a, b\}$.

Using the continuous t-norm and continuous t-conorm, Saadati and Park [3] introduced the concept of intuitionistic fuzzy normed spaces.

Definition 1.6 ([3]) The 5-tuple $(X, \mu, \upsilon, *, \diamond)$ is called an *intuitionistic fuzzy normed space (for short, IFNS)* if X is a vector space, $*$ and $\diamond$ are continuous t-norm and continuous t-conorm respectively and μ, υ are fuzzy sets on $X \times (0, \infty)$ satisfying the following conditions: for all $x, y \in X$ and $s, t > 0$,

(IF_1) $\mu(x,t) + \upsilon(x,t) \leq 1$;
(IF_2) $\mu(x,t) > 0$;
(IF_3) $\mu(x,t) = 1$ if and only if $x = 0$;
(IF_4) $\mu(\alpha x, t) = \mu(x, \frac{t}{|\alpha|})$ for all $\alpha \neq 0$;
(IF_5) $\mu(x,t) * \mu(y,s) \leq \mu(x+y, t+s)$;
(IF_6) $\mu(x, \cdot) : (0, \infty) \longrightarrow [0,1]$ is continuous;
(IF_7) μ is a non-decreasing function on $\mathbb{R}^+$,

$$\lim_{t\to\infty} \mu(x,t) = 1 \quad \text{and} \quad \lim_{t\to 0} \mu(x,t) = 0, \quad \forall x \in X, t > 0;$$

(IF_8) $\upsilon(x,t) < 1$;
(IF_9) $\upsilon(x,t) = 0$ if and only if $x = 0$;
(IF_{10}) $\upsilon(\alpha x, t) = \upsilon(x, \frac{t}{|\alpha|})$ for all $\alpha \neq 0$;
(IF_{11}) $\upsilon(x,t) \diamond \upsilon(y,s) \geq \upsilon(x+y, t+s)$;
(IF_{12}) $\upsilon(x, \cdot) : (0, \infty) \longrightarrow [0,1]$ is continuous;
(IF_{13}) υ is a non-increasing function on $\mathbb{R}^+$,

$$\lim_{t\to\infty} \upsilon(x,t) = 0 \quad \text{and} \quad \lim_{t\to 0} \upsilon(x,t) = 1, \quad \forall x \in X, t > 0.$$

In this case (μ, υ) is called an intuitionistic fuzzy norm.

Definition 1.7 ([3]) Let $(X, \mu, \upsilon, *, \diamond)$ be an IFNS. A sequence $\{x_n\}$ in X is said to be:

(1) convergent to a point $x \in X$ with respect to an intuitionistic fuzzy norm (μ, υ) if for any $\epsilon > 0$ and $t > 0$, there exists $k \in \mathbb{N}$ such that

$$\mu(x_n - x, t) > 1 - \epsilon \quad \text{and} \quad \upsilon(x_n - x, t) < \epsilon, \quad \forall n \geq k.$$

In this case, we write $(\mu, \upsilon) - \lim_{n\to\infty} x_n = x$.

(2) Cauchy sequence with respect to an intuitionistic fuzzy norm (μ, υ) if for any $\epsilon > 0$ and $t > 0$, there exists $k \in \mathbb{N}$ such that

$$\mu(x_n - x_m, t) > 1 - \epsilon \quad \text{and} \quad \upsilon(x_n - x_m, t) < \epsilon, \quad \forall n, m \geq k.$$

Definition 1.8 ([3]) An IFNS $(X, \mu, \upsilon, *, \diamond)$ is said to be *complete* if every Cauchy sequence in $(X, \mu, \upsilon, *, \diamond)$ is convergent.

Definition 1.9 ([13, 14]) Let X and Y be two IFNS. A function $g : X \longrightarrow Y$ is said to be continuous at a point $x_0 \in X$ if for any sequence $\{x_n\}$ in X converging to a point $x_0 \in X$, the sequence $\{g(x_n)\}$ in Y converges to $g(x_0) \in Y$. If g is continuous at each $x \in X$, then $g : X \longrightarrow Y$ is said to be continuous on X.

Examples 1.10 Let $(X, \|\cdot\|)$ be an ordinary normed space and ϕ be an increasing and continuous function from $\mathbb{R}^+$ into $(0,1)$ such that $\lim_{t\to\infty} \phi(t) = 1$. Four typical examples of these functions are as follows:

$$\phi(t) = \frac{t}{t+1}, \qquad \phi(t) = \sin\left(\frac{\pi t}{2t+1}\right), \qquad \phi(t) = 1 - e^{-t}, \qquad \phi(t) = e^{\frac{-1}{t}}.$$

Let $*$ and $\diamond$ be a continuous t-norm and a continuous t-conorm such that

$$a * b \leq ab \leq a \diamond b \quad \text{for all } a, b \in [0,1].$$

For any $t \in (0, \infty)$, we define

$$\mu(x,t) = \left[\phi(t)\right]^{\|x\|}, \qquad \upsilon(x,t) = 1 - \left[\phi(t)\right]^{\|x\|}, \quad \forall x \in X,$$

then $(X, \mu, \upsilon, *, \diamond)$ is an IFNS.

For further details regarding IFNS, we refer to [3].

Definition 1.11 ([9]) Let $(X, \mu, \upsilon, *, \diamond)$ be an IFNS. (μ, υ) is said to satisfy the *n-property* on $X \times (0, \infty)$ if

$$\lim_{n\to\infty} \left[\mu\left(x, k^n t\right)\right]^{n^p} = 1, \qquad \lim_{n\to\infty} \left[\upsilon\left(x, k^n t\right)\right]^{n^p} = 0,$$

where $x \in X$, $p > 0$, and $k > 1$.

Throughout this paper, we assume that (μ, υ) satisfies the n-property on $X \times (0, \infty)$.

Definition 1.12 ([11]) Let X be a non-empty set. An element $(x, y, z) \in X \times X \times X$ is called a *tripled fixed point* of $F : X \times X \times X \longrightarrow X$ if

$$x = F(x, y, z), \qquad y = F(y, x, y) \quad \text{and} \quad z = F(z, y, x).$$

Definition 1.13 Let X be a non-empty set. An element $(x, y, z) \in X \times X \times X$ is called a *tripled coincidence point* of mappings $F : X \times X \times X \longrightarrow X$ and $g : X \longrightarrow X$ if

$$g(x) = F(x, y, z), \qquad g(y) = F(y, x, y), \quad \text{and} \quad g(z) = F(z, y, x).$$

Definition 1.14 ([11]) Let $(X, \preceq)$ be a partially ordered set. A mapping $F : X \times X \times X \longrightarrow X$ is said to have the *mixed monotone property* if F is monotone non-decreasing in its first and third argument and is monotone non-increasing in its second argument; that is, for any $x, y, z \in X$

$$\begin{aligned} &x_1, x_2 \in X, \quad x_1 \preceq x_2 \quad \Longrightarrow \quad F(x_1, y, z) \preceq F(x_2, y, z), \\ &y_1, y_2 \in X, \quad y_1 \preceq y_2 \quad \Longrightarrow \quad F(x, y_2, z) \preceq F(x, y_1, z) \end{aligned}$$

and

$$z_1, z_2 \in X, \quad z_1 \preceq z_2 \quad \Longrightarrow \quad F(x, y, z_1) \preceq F(x, y, z_2).$$

Definition 1.15 Let $(X, \preceq)$ be a partially ordered set, and $g : X \longrightarrow X$. A mapping $F : X \times X \times X \longrightarrow X$ is said to have the *mixed g-monotone property* if F is monotone g-non-decreasing in its first and third argument and is monotone g-non-increasing in its second argument; that is, for any $x, y, z \in X$,

$$\begin{aligned} &x_1, x_2 \in X, \quad g(x_1) \preceq g(x_2) \quad \Longrightarrow \quad F(x_1, y, z) \preceq F(x_2, y, z), \\ &y_1, y_2 \in X, \quad g(y_1) \preceq g(y_2) \quad \Longrightarrow \quad F(x, y_2, z) \preceq F(x, y_1, z) \end{aligned}$$

and

$$z_1, z_2 \in X, \quad g(z_1) \preceq g(z_2) \quad \Longrightarrow \quad F(x, y, z_1) \preceq F(x, y, z_2).$$

Lemma 1.16 ([15]) *Let X be a non-empty set and $g : X \longrightarrow X$ be a mapping. Then there exists a subset $E \subseteq X$ such that $g(E) = g(X)$ and $g : E \longrightarrow X$ is one-to-one.*

2 Main results

Theorem 2.1 *Let $(X, \mu, \upsilon, *, \diamond)$ be a complete IFNS, $\preceq$ be a partial order on X and suppose that*

$$a * b \geq ab \quad \text{and} \quad a \diamond a = a \tag{2.1}$$

for all $a, b \in [0, 1]$. Suppose that $F : X \times X \times X \longrightarrow X$ has the mixed monotone property and

$$\begin{aligned} &\mu\big(F(x, y, z) - F(u, v, w), kt\big) \geq \mu(x - u, t) * \mu(y - v, t) * \mu(z - w, t), \\ &\upsilon\big(F(x, y, z) - F(u, v, w), kt\big) \leq \upsilon(x - u, t) \diamond \upsilon(y - v, t) \diamond \upsilon(z - w, t) \end{aligned} \tag{2.2}$$

for all those x, y, z, u, v, w in X for which $x \preceq u$, $y \succeq v$, $z \preceq w$, where $0 < k < 1$. If either

(a) *F is continuous or*

(b) *X has the following property:*

  (bi) *if $\{x_n\}$ is a non-decreasing sequence and $(\mu, \upsilon) - \lim_{n \longrightarrow \infty} x_n = x$, then $x_n \preceq x$ for all $n \in \mathbb{N}$,*

  (bii) *if $\{y_n\}$ is a non-decreasing sequence and $(\mu, \upsilon) - \lim_{n \longrightarrow \infty} y_n = y$, then $y_n \succeq y$ for all $n \in \mathbb{N}$,* (biii) *if* $\{z_n\}$ *is a non-decreasing sequence and* $(\mu, \upsilon) - \lim_{n\to\infty} z_n = y$, *then* $z_n \preceq z$ *for all* $n \in \mathbb{N}$,

then F *has a tripled fixed point provided that there exist* $x_0, y_0, z_0 \in X$ *such that*

$$x_0 \preceq F(x_0, y_0, z_0), \qquad y_0 \succeq F(y_0, x_0, y_0), \qquad z_0 \preceq F(z_0, y_0, x_0).$$

Proof Let $x_0, y_0, z_0 \in X$ be such that

$$x_0 \preceq F(x_0, y_0, z_0), \qquad y_0 \succeq F(y_0, x_0, y_0), \qquad z_0 \preceq F(z_0, y_0, x_0).$$

As $F(X \times X \times X) \subseteq X$, so we can construct sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ in X such that

$$\begin{aligned} &x_{n+1} = F(x_n, y_n, z_n), \qquad y_{n+1} = F(y_n, x_n, y_n), \\ &z_{n+1} = F(z_n, y_n, x_n), \quad \forall n \geq 0. \end{aligned} \tag{2.3}$$

Now we show that

$$x_n \preceq x_{n+1}, \qquad y_n \succeq y_{n+1}, \qquad z_n \preceq z_{n+1}, \quad \forall n \geq 0. \tag{2.4}$$

Since

$$x_0 \preceq F(x_0, y_0, z_0), \qquad y_0 \succeq F(y_0, x_0, y_0), \qquad z_0 \preceq F(z_0, y_0, x_0),$$

(2.4) holds for $n = 0$. Suppose that (2.4) holds for any $n \geq 0$. That is,

$$x_n \preceq x_{n+1}, \qquad y_n \succeq y_{n+1}, \qquad z_n \preceq z_{n+1}. \tag{2.5}$$

As F has the *mixed monotone property* so by (2.5) we obtain

$$\left\{\begin{array}{ll} F(x_n, y, z) \preceq F(x_{n+1}, y, z), & \text{(i)} \\ F(x, y_n, z) \preceq F(x, y_{n+1}, z), & \text{(ii)} \\ F(x, y, z_n) \preceq F(x, y, z_{n+1}), & \text{(iii)} \end{array}\right|$$

which on replacing y by y_n and z by z_n in (i) implies that $F(x_n, y_n, z_n) \preceq F(x_{n+1}, y_n, z_n)$; replacing x by x_{n+1} and z by z_n in (ii), we obtain $F(x_{n+1}, y_n, z_n) \preceq F(x_{n+1}, y_{n+1}, z_n)$; replacing y by y_{n+1} and x by x_{n+1} in (iii), we get $F(x_{n+1}, y_{n+1}, z_n) \preceq F(x_{n+1}, y_{n+1}, z_{n+1})$. Thus, we have $F(x_n, y_n, z_n) \preceq F(x_{n+1}, y_{n+1}, z_{n+1})$, that is, $x_{n+1} \preceq x_{n+2}$. Similarly, we have

$$\left\{\begin{array}{ll} F(y, x, y_{n+1}) \preceq F(y, x, y_n), & \text{(iv)} \\ F(y_{n+1}, x, y) \preceq F(y_n, x, y), & \text{(v)} \\ F(y, x_{n+1}, y) \preceq F(y, x_n, y), & \text{(vi)} \end{array}\right|$$

which on replacing y by y_{n+1} and x by x_{n+1} in (iv) implies that $F(y_{n+1}, x_{n+1}, y_{n+1}) \preceq F(y_{n+1}, x_{n+1}, y_n)$; replacing x by x_{n+1} and y by y_{n+1} in (v), we obtain $F(y_{n+1}, x_{n+1}, y_n) \preceq$

$F(y_n, x_{n+1}, y_n)$; replacing y by y_n in (vi), we get $F(y_n, x_{n+1}, y_n) \preceq F(y_n, x_n, y_n)$. Thus, we have $F(y_{n+1}, x_{n+1}, y_{n+1}) \preceq F(y_n, x_n, y_n)$, that is, $y_{n+2} \preceq y_{n+1}$. Similarly, we have

$$\begin{cases} F(z_n, y, x) \preceq F(z_{n+1}, y, x), & \text{(vii)} \\ F(z, y_n, x) \preceq F(z, y_{n+1}, x), & \text{(viii)} \\ F(z, y, x_n) \preceq F(z, y, x_{n+1}), & \text{(xi)} \end{cases}$$

which on replacing y by y_n and x by x_n in (vii) implies that $F(z_n, y_n, x_n) \preceq F(z_{n+1}, y_n, x_n)$; replacing x by x_n and z by z_{n+1} in (viii), we obtain $F(z_{n+1}, y_n, x_n) \preceq F(z_{n+1}, y_{n+1}, x_n)$; replacing y by y_{n+1} and z by z_{n+1} in (xi), we get $F(z_{n+1}, y_{n+1}, x_n) \preceq F(z_{n+1}, y_{n+1}, x_{n+1})$. Thus, we have $F(z_n, y_n, x_n) \preceq F(z_{n+1}, y_{n+1}, x_{n+1})$, that is, $z_{n+1} \preceq z_{n+2}$. So, by induction, we conclude that (2.5) holds for all $n \geq 0$, that is,

$$x_0 \preceq x_1 \preceq x_2 \preceq \cdots \preceq x_n \preceq x_{n+1} \cdots, \tag{2.6}$$

$$y_0 \succeq y_1 \succeq y_2 \succeq \cdots \succeq y_n \succeq y_{n+1} \cdots, \tag{2.7}$$

$$z_0 \preceq z_1 \preceq z_2 \preceq \cdots \preceq z_n \preceq z_{n+1} \cdots. \tag{2.8}$$

Define

$$\alpha_n(t) = \mu(x_n - x_{n+1}, t) * \mu(y_n - y_{n+1}, t) * \mu(z_n - z_{n+1}, t). \tag{2.9}$$

Consider

$$\begin{aligned} \mu(x_n - x_{n+1}, kt) &= \mu\big(F(x_{n-1}, y_{n-1}, z_{n-1}) - F(x_n, y_n, z_n), kt\big) \\ &\geq \mu(x_{n-1} - x_n, t) * \mu(y_{n-1} - y_n, t) * \mu(z_{n-1} - z_n, t) \\ &= \alpha_{n-1}(t). \end{aligned} \tag{2.10}$$

Also,

$$\begin{aligned} \mu(z_n - z_{n+1}, kt) &= \mu\big(F(z_{n-1}, y_{n-1}, x_{n-1}) - F(z_n, y_n, x_n), kt\big) \\ &\geq \mu(z_{n-1} - z_n, t) * \mu(y_{n-1} - y_n, t) * \mu(x_{n-1} - x_n, t) \\ &= \mu(x_{n-1} - x_n, t) * \mu(y_{n-1} - y_n, t) * \mu(z_{n-1} - z_n, t) \\ &= \alpha_{n-1}(t). \end{aligned} \tag{2.11}$$

Now,

$$\begin{aligned} \mu(y_n - y_{n+1}, kt) &= \mu\big(F(y_{n-1}, x_{n-1}, y_{n-1}) - F(y_n, x_n, y_n), kt\big) \\ &\geq \mu(y_{n-1} - y_n, t) * \mu(x_{n-1} - x_n, t) * \mu(y_{n-1} - y_n, t) \\ &= \mu(y_{n-1} - y_n, t) * \mu(x_{n-1} - x_n, t) * \mu(y_{n-1} - y_n, t) * 1 * 1 * 1 \\ &\geq \mu(y_{n-1} - y_n, t) * \mu(x_{n-1} - x_n, t) * \mu(y_{n-1} - y_n, t) \\ &\quad * \mu(z_{n-1} - z_n, t) * \mu(z_{n-1} - z_n, t) * \mu(x_{n-1} - x_n, t) \\ &\geq \alpha_{n-1}(t) * \alpha_{n-1}(t). \end{aligned} \tag{2.12}$$

Using the properties of a t-norm, (2.9)-(2.12) and (2.1), we obtain

$$\begin{aligned}\alpha_n(kt) &= \mu(x_n - x_{n+1}, kt) * \mu(y_n - y_{n+1}, kt) * \mu(z_n - z_{n+1}, kt)\\ &\geq \alpha_{n-1}(t) * \alpha_{n-1}(t) * \alpha_{n-1}(t) * \alpha_{n-1}(t)\\ &\geq \bigl(\alpha_{n-1}(t)\bigr)^4 \quad \forall n \geq 1,\end{aligned}$$

which implies that

$$\alpha_n(t) \geq \left(\alpha_{n-1}\left(\frac{t}{k}\right)\right)^4 \quad \forall n \geq 1.$$

Now, repetition of the above process gives

$$\alpha_n(t) \geq \left(\alpha_{n-1}\left(\frac{t}{k}\right)\right)^4 \geq \cdots \geq \left(\alpha_0\left(\frac{t}{k^n}\right)\right)^{4^n} \quad \forall n \geq 1.$$

Hence,

$$\begin{aligned}&\mu(x_n - x_{n+1}, t) * \mu(y_n - y_{n+1}, t) * \mu(z_n - z_{n+1}, t)\\ &\quad\geq \left[\mu\left(x_0 - x_1, \frac{t}{k^n}\right)\right]^{4^n} * \left[\mu\left(y_0 - y_1, \frac{t}{k^n}\right)\right]^{4^n} * \left[\mu\left(z_0 - z_1, \frac{t}{k^n}\right)\right]^{4^n}.\end{aligned} \tag{2.13}$$

It is obvious to note that

$$t(1-k)\bigl(1 + k + \cdots + k^{m-n-1}\bigr) < t \quad \forall m > n, 0 < k < 1.$$

Consider

$$\begin{aligned}&\mu(x_n - x_m, t) * \mu(y_n - y_m, t) * \mu(z_n - z_m, t)\\ &\quad\geq \mu\bigl(x_n - x_m, t(1-k)\bigl(1 + k + \cdots + k^{m-n-1}\bigr)\bigr)\\ &\qquad * \mu\bigl(y_n - y_m, t(1-k)\bigl(1 + k + \cdots + k^{m-n-1}\bigr)\bigr)\\ &\qquad * \mu\bigl(z_n - z_m, t(1-k)\bigl(1 + k + \cdots + k^{m-n-1}\bigr)\bigr)\\ &\quad\geq \mu\bigl(x_n - x_{n+1}, t(1-k)\bigr) * \mu\bigl(y_n - y_{n+1}, t(1-k)\bigr)\\ &\qquad * \mu\bigl(z_n - z_{n+1}, t(1-k)\bigr)\\ &\qquad * \mu\bigl(x_{n+1} - x_{n+2}, t(1-k)k\bigr) * \mu\bigl(y_{n+1} - y_{n+2}, t(1-k)k\bigr)\\ &\qquad * \mu\bigl(z_{n+1} - z_{n+2}, t(1-k)k\bigr)\\ &\qquad * \cdots\\ &\qquad * \mu\bigl(x_{m-1} - x_m, t(1-k)k^{m-n-1}\bigr) * \mu\bigl(y_{m-1} - y_m, t(1-k)k^{m-n-1}\bigr)\\ &\qquad * \mu\bigl(z_{m-1} - z_m, t(1-k)k^{m-n-1}\bigr)\\ &\quad\geq \left[\mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right] * \left[\mu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right] * \left[\mu\left(z_0 - z_1, (1-k)\frac{t}{k^n}\right)\right]\\ &\qquad * \cdots\end{aligned}$$

$$
\begin{aligned}
&* \left[\mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right] * \left[\mu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right] * \left[\mu\left(z_0 - z_1, (1-k)\frac{t}{k^n}\right)\right] \\
\geq{}& \left[\mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{m-n} * \left[\mu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right]^{m-n} \\
&* \left[\mu\left(z_0 - z_1, (1-k)\frac{t}{k^n}\right)\right]^{m-n} \\
\geq{}& \left[\mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{m} * \left[\mu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right]^{m} \\
&* \left[\mu\left(z_0 - z_1, (1-k)\frac{t}{k^n}\right)\right]^{m} \\
\geq{}& \left[\mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p} * \left[\mu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p} \\
&* \left[\mu\left(z_0 - z_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p},
\end{aligned}
$$

where $p > 0$ such that $m < n^p$. Since (μ, υ) has the n-property on $X \times (0, \infty)$, therefore

$$
\begin{aligned}
&\lim_{n\to\infty}\left[\mu\left(x_0 - x_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p} = 1, \\
&\lim_{n\to\infty}\left[\mu\left(y_0 - y_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p} = 1, \quad \text{and} \\
&\lim_{n\to\infty}\left[\mu\left(z_0 - z_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p} = 1.
\end{aligned}
$$

Hence,

$$
\lim_{n\to\infty} \mu(x_n - x_m, t) * \mu(y_n - y_m, t) * \mu(z_n - z_m, t) = 1. \tag{2.14}
$$

Next, we show that

$$
\lim_{n\to\infty} \upsilon(x_n - x_m, t) \diamond \upsilon(y_n - y_m, t) \diamond \upsilon(z_n - z_m, t) = 0.
$$

Define

$$
\beta_n(t) = \upsilon(x_n - x_{n+1}, t) \diamond \upsilon(y_n - y_{n+1}, t) \diamond \upsilon(z_n - z_{n+1}, t). \tag{2.15}
$$

Note that

$$
\begin{aligned}
\upsilon(x_n - x_{n+1}, kt) &= \upsilon\bigl(F(x_{n-1}, y_{n-1}, z_{n-1}) - F(x_n, y_n, z_n), kt\bigr) \\
&\leq \upsilon(x_{n-1} - x_n, t) \diamond \upsilon(y_{n-1} - y_n, t) \diamond \upsilon(z_{n-1} - z_n, t) \\
&= \beta_{n-1}(t), \qquad (2.16) \\
\upsilon(z_n - z_{n+1}, kt) &= \upsilon\bigl(F(z_{n-1}, y_{n-1}, x_{n-1}) - F(z_n, y_n, x_n), kt\bigr) \\
&\leq \upsilon(z_{n-1} - z_n, t) \diamond \upsilon(y_{n-1} - y_n, t) \diamond \upsilon(x_{n-1} - x_n, t)
\end{aligned}
$$

$$\begin{aligned} &= \upsilon(x_{n-1}-x_n,t)\diamond\upsilon(y_{n-1}-y_n,t)\diamond\upsilon(z_{n-1}-z_n,t) \\ &= \beta_{n-1}(t), \end{aligned} \tag{2.17}$$

and

$$\begin{aligned} \upsilon(y_n-y_{n+1},kt) &= \upsilon\bigl(F(y_{n-1},x_{n-1},y_{n-1})-F(y_n,x_n,y_n),kt\bigr) \\ &\le \upsilon(y_{n-1}-y_n,t)\diamond\mu(x_{n-1}-x_n,t)\diamond\mu(y_{n-1}-y_n,t) \\ &= \upsilon(y_{n-1}-y_n,t)\diamond\upsilon(x_{n-1}-x_n,t)\diamond\upsilon(y_{n-1}-y_n,t) \\ &\quad \diamond 0\diamond 0\diamond 0 \\ &\le \upsilon(y_{n-1}-y_n,t)\diamond\upsilon(x_{n-1}-x_n,t)\diamond\upsilon(y_{n-1}-y_n,t) \\ &\quad \diamond\upsilon(z_{n-1}-z_n,t)\diamond\upsilon(z_{n-1}-z_n,t)\diamond\upsilon(x_{n-1}-x_n,t) \\ &\le \beta_{n-1}(t)\diamond\beta_{n-1}(t). \end{aligned} \tag{2.18}$$

Using the properties of a t-conorm, (2.15)-(2.18) and (2.1), we obtain

$$\begin{aligned} \beta_n(kt) &= \upsilon(x_n-x_{n+1},t)\diamond\upsilon(y_n-y_{n+1},t)\diamond\upsilon(z_n-z_{n+1},t) \\ &\le \beta_{n-1}(t)\diamond\beta_{n-1}(t)\diamond\beta_{n-1}(t)\diamond\beta_{n-1}(t)=\beta_{n-1}(t) \quad \forall n\ge 1, \end{aligned}$$

that is,

$$\beta_n(t)\le\beta_{n-1}\left(\frac{t}{k}\right) \quad \forall n\ge 1.$$

Now, repetition of the above process gives

$$\beta_n(t)\le\beta_{n-1}\left(\frac{t}{k}\right)\le\cdots\le\left(\beta_0\left(\frac{t}{k^n}\right)\right)^n \quad \forall n\ge 1,$$

which further implies that

$$\begin{aligned} &\upsilon(x_n-x_{n+1},t)\diamond\upsilon(y_n-y_{n+1},t)\diamond\upsilon(z_n-z_{n+1},t) \\ &\quad\le\left[\upsilon\left(x_0-x_1,\frac{t}{k^n}\right)\right]^n\diamond\left[\upsilon\left(y_0-y_1,\frac{t}{k^n}\right)\right]^n\diamond\left[\upsilon\left(z_0-z_1,\frac{t}{k^n}\right)\right]^n. \end{aligned} \tag{2.19}$$

Using the properties of a t-conorm, we get

$$\begin{aligned} &\upsilon(x_n-x_m,t)\diamond\upsilon(y_n-y_m,t)\diamond\upsilon(z_n-z_m,t) \\ &\quad\le\upsilon\bigl(x_n-x_m,t(1-k)\bigl(1+k+\cdots+k^{m-n-1}\bigr)\bigr) \\ &\qquad\diamond\upsilon\bigl(y_n-y_m,t(1-k)\bigl(1+k+\cdots+k^{m-n-1}\bigr)\bigr) \\ &\qquad\diamond\upsilon\bigl(z_n-z_m,t(1-k)\bigl(1+k+\cdots+k^{m-n-1}\bigr)\bigr) \\ &\quad\le\upsilon\bigl(x_n-x_{n+1},t(1-k)\bigr)\diamond\upsilon\bigl(y_n-y_{n+1},t(1-k)\bigr) \\ &\qquad\diamond\upsilon\bigl(z_n-z_{n+1},t(1-k)\bigr) \end{aligned}$$

$$
\begin{aligned}
&\diamond \upsilon\big(x_{n+1}-x_{n+2}, t(1-k)k\big) \diamond \upsilon\big(y_{n+1}-y_{n+2}, t(1-k)k\big) \\
&\diamond \upsilon\big(z_{n+1}-z_{n+2}, t(1-k)k\big) \\
&\diamond \cdots \\
&\diamond \upsilon\big(x_{m-1}-x_m, t(1-k)k^{m-n-1}\big) \diamond \upsilon\big(y_{m-1}-y_m, t(1-k)k^{m-n-1}\big) \\
&\diamond \upsilon\big(z_{m-1}-z_m, t(1-k)k^{m-n-1}\big) \\
\le{}& \left[\upsilon\left(x_0-x_1, (1-k)\frac{t}{k^n}\right)\right] \diamond \left[\upsilon\left(y_0-y_1, (1-k)\frac{t}{k^n}\right)\right] \\
&\diamond \left[\upsilon\left(z_0-z_1, (1-k)\frac{t}{k^n}\right)\right] \\
&\diamond \cdots \\
&\diamond \left[\upsilon\left(x_0-x_1, (1-k)\frac{t}{k^n}\right)\right] \diamond \left[\upsilon\left(y_0-y_1, (1-k)\frac{t}{k^n}\right)\right] \\
&\diamond \left[\upsilon\left(z_0-z_1, (1-k)\frac{t}{k^n}\right)\right] \\
\le{}& \left[\upsilon\left(x_0-x_1, (1-k)\frac{t}{k^n}\right)\right]^{m-n} \diamond \left[\upsilon\left(y_0-y_1, (1-k)\frac{t}{k^n}\right)\right]^{m-n} \\
&\diamond \left[\upsilon\left(z_0-z_1, (1-k)\frac{t}{k^n}\right)\right]^{m-n} \\
\le{}& \left[\upsilon\left(x_0-x_1, (1-k)\frac{t}{k^n}\right)\right]^{m} \diamond \left[\upsilon\left(y_0-y_1, (1-k)\frac{t}{k^n}\right)\right]^{m} \\
&\diamond \left[\upsilon\left(z_0-z_1, (1-k)\frac{t}{k^n}\right)\right]^{m} \\
\le{}& \left[\upsilon\left(x_0-x_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p} \diamond \left[\upsilon\left(y_0-y_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p} \\
&\diamond \left[\upsilon\left(z_0-z_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p},
\end{aligned}
$$

where $p > 0$ such that $m > n^p$. Since (μ, υ) has the *n-property* on $X \times (0, \infty)$, we have

$$
\begin{aligned}
&\lim_{n\to\infty}\left[\upsilon\left(x_0-x_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p} = 0, \\
&\lim_{n\to\infty}\left[\upsilon\left(y_0-y_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p} = 0, \quad \text{and} \\
&\lim_{n\to\infty}\left[\upsilon\left(z_0-z_1, (1-k)\frac{t}{k^n}\right)\right]^{n^p} = 0.
\end{aligned}
$$

So,

$$
\lim_{n\to\infty} \upsilon(x_n - x_m, t) \diamond \upsilon(y_n - y_m, t) \diamond \upsilon(z_n - z_m, t) = 0. \tag{2.20}
$$

Now, (2.14) and (2.20) imply that $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ are Cauchy sequences in X. Since X is complete, there exist x, y and z such that $\lim_{n\to\infty} x_n = x$, $\lim_{n\to\infty} y_n = y$ and $\lim_{n\to\infty} z_n = z$.

If the assumption (a) does hold, then we have

$$\begin{aligned} x &= \lim_{n\to\infty} x_{n+1} = \lim_{n\to\infty} F(x_n, y_n, z_n) \\ &= F\Bigl(\lim_{n\to\infty} x_n, \lim_{n\to\infty} y_n, \lim_{n\to\infty} z_n\Bigr) = F(x, y, z), \\ y &= \lim_{n\to\infty} y_{n+1} = \lim_{n\to\infty} F(y_n, x_n, y_n) \\ &= F\Bigl(\lim_{n\to\infty} y_n, \lim_{n\to\infty} x_n, \lim_{n\to\infty} y_n\Bigr) = F(y, x, y), \end{aligned}$$

and

$$\begin{aligned} z &= \lim_{n\to\infty} z_{n+1} = \lim_{n\to\infty} F(z_n, y_n, x_n) \\ &= F\Bigl(\lim_{n\to\infty} z_n, \lim_{n\to\infty} y_n, \lim_{n\to\infty} x_n\Bigr) = F(z, y, x). \end{aligned}$$

Suppose that the assumption (b) holds then

$$\begin{aligned} &\mu\bigl(x_{n+1} - F(x, y, z), kt\bigr) \\ &\quad = \mu\bigl(F(x_n, y_n, z_n) - F(x, y, z), kt\bigr) \geq \mu(x_n - x, y_n - y, z_n - z, t), \end{aligned}$$

which, on taking limit as $n \to \infty$, gives $\mu(x - F(x, y, z), kt) = 1$, $x = F(x, y, z)$. Also,

$$\begin{aligned} &\mu\bigl(y_{n+1} - F(y, x, y), kt\bigr) \\ &\quad = \mu\bigl(F(y_n, x_n, y_n) - F(y, x, y), kt\bigr) \geq \mu(y_n - y, x_n - x, y_n - y, t), \end{aligned}$$

which, on taking limit as $n \to \infty$, implies $\mu(y - F(y, x, y), kt) = 1$, $y = F(y, x, y)$. Finally, we have

$$\begin{aligned} &\mu\bigl(z_{n+1} - F(z, y, x), kt\bigr) \\ &\quad = \mu\bigl(F(z_n, y_n, x_n) - F(z, y, x), kt\bigr) \geq \mu(z_n - z, y_n - y, x_n - x, t), \end{aligned}$$

which, on taking limit as $n \to \infty$, gives $\mu(z - F(z, y, x), kt) = 1$, $z = F(z, y, x)$. □

Theorem 2.2 *Let $(X, \mu, \upsilon, *, \diamond)$ be an IFNS, $\preceq$ be a partial order on X, and suppose that*

$$a * b \geq ab \quad \text{and} \quad a \diamond a = a \tag{2.21}$$

for all $a, b \in [0, 1]$. Let $F : X \times X \times X \longrightarrow X$ and $g : X \longrightarrow X$ be mappings such that F has the mixed g-monotone property and

$$\begin{aligned} \mu\bigl(F(x, y, z) - F(u, v, w), kt\bigr) &\geq \mu(gx - gu, t) * \mu(gy - gv, t) \\ &\quad * \mu(gz - gw, t) \quad \text{and} \\ \upsilon\bigl(F(x, y, z) - F(u, v, w), kt\bigr) &\leq \upsilon(gx - gu, t) \diamond \upsilon(gy - gv, t) \\ &\quad \diamond \upsilon(gz - gw, t) \end{aligned} \tag{2.22}$$

for all those x, y, z, and u, v, w for which $gx \preceq gu$, $gy \succeq gv$, $gz \preceq gw$, where $0 < k < 1$. Assume that $g(X)$ is complete, $F(X \times X \times X) \subseteq g(X)$ and g is continuous. If either

(a) *F is continuous or*

(b) *X has the following property*:

  (bi) *if $\{x_n\}$ is a non-decreasing sequence and $(\mu, \upsilon) - \lim_{n\longrightarrow\infty} x_n = x$, then $x_n \preceq x$ for all $n \in \mathbb{N}$,*

  (bii) *if $\{y_n\}$ is a non-decreasing sequence and $(\mu, \upsilon) - \lim_{n\longrightarrow\infty} y_n = y$, then $y_n \succeq y$ for all $n \in \mathbb{N}$, and*

  (biii) *if $\{z_n\}$ is a non-decreasing sequence and $(\mu, \upsilon) - \lim_{n\longrightarrow\infty} z_n = y$, then $z_n \preceq z$ for all $n \in \mathbb{N}$.*

Then F has a tripled coincidence point provided that there exist $x_0, y_0, z_0 \in X$ such that

$$g(x_0) \preceq F(x_0, y_0, z_0), \qquad g(y_0) \succeq F(y_0, x_0, y_0), \qquad g(z_0) \preceq F(z_0, y_0, x_0).$$

Proof By Lemma 1.16, there exists $E \subseteq X$ such that $g : E \longrightarrow X$ is one-to-one and $g(E) = g(X)$. Now, define a mapping $\mathcal{A} : g(E) \times g(E) \times g(E) \longrightarrow X$ by

$$\mathcal{A}(gx, gy, gz) = F(x, y, z) \quad \forall x, y, z \in X. \tag{2.23}$$

Since g is one-to-one, so $\mathcal{A}$ is well defined. Now, (2.22) and (2.23) imply that

$$\begin{aligned} \mu\big(\mathcal{A}(gx, gy, gz) - \mathcal{A}(gu, gv, gw), kt\big) &\geq \mu(gx - gu, t) * \mu(gy - gv, t) \\ &\quad * \mu(gz - gw, t), \\ \upsilon\big(\mathcal{A}(gx, gy, gz) - \mathcal{A}(gu, gv, gw), kt\big) &\leq \upsilon(gx - gu, t) \diamond \upsilon(gy - gv, t) \\ &\quad \diamond \upsilon(gz - gw, t) \end{aligned} \tag{2.24}$$

for all $x, y, z, u, v, w \in E$ for which $gx \preceq gu$, $gy \succeq gv$, $gz \preceq gw$. Since F has the *mixed g-monotone property* for all $x, y, z \in X$, so we have

$$\begin{aligned} &x_1, x_2 \in X, \quad g(x_1) \preceq g(x_2) \quad \Longrightarrow \quad F(x_1, y, z) \preceq F(x_2, y, z), \\ &y_1, y_2 \in X, \quad g(y_1) \preceq g(y_2) \quad \Longrightarrow \quad F(x, y_2, z) \preceq F(x, y_1, z), \quad \text{and} \\ &z_1, z_2 \in X, \quad g(z_1) \preceq g(z_2) \quad \Longrightarrow \quad F(x, y, z_1) \preceq F(x, y, z_2). \end{aligned} \tag{2.25}$$

Now, from (2.23) and (2.25), we have

$$\begin{aligned} &x_1, x_2 \in X, \quad g(x_1) \preceq g(x_2) \quad \Longrightarrow \quad \mathcal{A}(gx_1, gy, gz) \preceq \mathcal{A}(gx_2, gy, gz), \\ &y_1, y_2 \in X, \quad g(y_1) \preceq g(y_2) \quad \Longrightarrow \quad \mathcal{A}(gx, gy_2, gz) \preceq \mathcal{A}(gx, gy_1, gz), \\ &z_1, z_2 \in X, \quad g(z_1) \preceq g(z_2) \quad \Longrightarrow \quad \mathcal{A}(gx, gy, gz_1) \preceq \mathcal{A}(gx, gy, gz_2). \end{aligned} \tag{2.26}$$

Hence, $\mathcal{A}$ has the mixed monotone property. Suppose that the assumption (a) holds. Since F is continuous, $\mathcal{A}$ is also continuous. By using Theorem 2.1, $\mathcal{A}$ has a tripled fixed point $(u, v, w) \in g(E) \times g(E) \times g(E)$. If the assumption (b) holds, then using the definition of $\mathcal{A}$, following similar arguments to those given in Theorem 2.1, $\mathcal{A}$ has a tripled fixed point

$(u,v,w) \in g(E) \times g(E) \times g(E)$. Finally, we show that F and g have a tripled coincidence point. Since $\mathcal{A}$ has a tripled fixed point $(u,v,w) \in g(E) \times g(E) \times g(E)$, we get

$$u = \mathcal{A}(u,v,w), \qquad v = \mathcal{A}(v,u,v), \qquad w = \mathcal{A}(w,u,v). \tag{2.27}$$

Hence, there exist $u_1, v_1, w_1 \in X \times X \times X$ such that $gu_1 = u$, $gv_1 = v$, and $gw_1 = w$. Now, it follows from (2.27) that

$$\begin{aligned} gu_1 &= \mathcal{A}(gu_1, gv_1, w) = F(u_1, v_1, w_1), \\ gv_1 &= \mathcal{A}(gv_1, gu_1, gv_1) = F(v_1, u_1, v_1), \quad \text{and} \\ gw_1 &= \mathcal{A}(gw_1, gu_1, gv_1) = F(w_1, v_1, u_1). \end{aligned}$$

Thus, $(u_1, v_1, w_1) \in X \times X \times X$ is a tripled coincidence point of F and g. □

Example 2.3 Let $X = \mathbb{R}$ be a usual normed, $* : [0,1] \times [0,1] \to [0,1]$ and $\diamond : [0,1] \times [0,1] \to [0,1]$ be defined by

$$a * b = ab \quad \text{and} \quad a \diamond b = \max\{a, b\}.$$

It is easy to see that $*$ is a continuous t-norm and $\diamond$ is a continuous t-conorm satisfy

$$a * b \leq ab \leq a \diamond b \quad \text{for all } a, b \in [0,1].$$

Let $\phi : \mathbb{R}^+ \to (0,1)$ be defined by $\phi(t) = e^{-\frac{1}{t}}$ for all $t \in \mathbb{R}^+$. Now we have $(X, \mu, \upsilon, *, \diamond)$ is an IFNS, where

$$\mu(x,t) = \left[\phi(t)\right]^{|x|}, \qquad \upsilon(x,t) = 1 - \left[\phi(t)\right]^{|x|}, \quad \forall x \in X,$$

such that (μ, ν) satisfies the n-property on $X \times (0, \infty)$.

If X is endowed with usual order as $x \preceq y \Longleftrightarrow x - y \leq 0$, then $(X, \preceq)$ is a partially ordered set. Define mappings $F : X \times X \times X \longrightarrow X$ and $g : X \longrightarrow X$ by

$$F(x,y,z) = 2x - 2y + 2z + 1 \quad \text{and} \quad g(x) = 7x - 1.$$

Obviously, F and g both are onto maps so $F(X \times X \times X) \subseteq g(X)$. Also, F and g are continuous and F has the mixed g-monotone property. Indeed,

$$\begin{aligned} x_1, x_2 \in X, \quad gx_1 \preceq gx_2 \quad &\Longrightarrow \quad 2x_1 - 2y + 2z + 1 \leq 2x_2 - 2y + 2z + 1 \\ &\Longrightarrow \quad F(x_1, y, z) \preceq F(x_2, y, z). \end{aligned}$$

Similarly, we can prove that

$$y_1, y_2 \in X, \quad g(y_1) \preceq g(y_2) \quad \Longrightarrow \quad F(x, y_2, z) \preceq F(x, y_1, z)$$

and

$$z_1, z_2 \in X, \quad g(z_1) \preceq g(z_2) \quad \Longrightarrow \quad F(x, y, z_1) \preceq F(x, y, z_2).$$

If $x_0 = 0$, $y_0 = \frac{2}{3}$, $z_0 = 0$, then

$$\begin{aligned}
-1 &= g(x_0) \preceq F(x_0, y_0, z_0) = -\frac{1}{3}, \\
\frac{11}{3} &= g(y_0) \succeq F(y_0, x_0, y_0) = \frac{11}{3}, \\
-1 &= g(z_0) \preceq F(z_0, y_0, x_0) = -\frac{1}{3}.
\end{aligned}$$

So, there exist $x_0, y_0, z_0 \in X$ such that

$$g(x_0) \preceq F(x_0, y_0, z_0), \qquad g(y_0) \succeq F(y_0, x_0, y_0), \qquad g(z_0) \preceq F(z_0, y_0, x_0).$$

Now, for all $x, y, z, u, v, w \in X$, for which $gx \preceq gu$, $gy \succeq gv$, $gz \preceq gw$, we have

$$\begin{aligned}
&\mu(gx - gu, t) * \mu(gy - gv, t) * \mu(gz - gw, t) \\
&\quad = \mu\bigl(7(x - u), t\bigr) * \mu\bigl(7(y - v), t\bigr) * \mu\bigl(7(z - w), t\bigr) \\
&\quad = \mu\left((x - u), \frac{t}{7}\right) * \mu\left((y - v), \frac{t}{7}\right) * \mu\left((z - w), \frac{t}{7}\right) \\
&\quad = \mu\left((x - u), \frac{t}{7}\right) * \mu\left((v - y), \frac{t}{7}\right) * \mu\left((z - w), \frac{t}{7}\right) \\
&\quad \leq \mu\left(x - u + v - y + z - w, \frac{3t}{7}\right) \\
&\quad = \bigl(e^{-\frac{7}{3t}}\bigr)^{|(x-u+v-y+z-w)|} \\
&\quad = \bigl(e^{-\frac{3.5}{3t}}\bigr)^{|2(x-u+v-y+z-w)|} \\
&\quad = \bigl(e^{-\frac{3.5}{3t}}\bigr)^{|2(x-u)+2(v-y)+2(z-w)|} \\
&\quad = \bigl(e^{-\frac{3.5}{3t}}\bigr)^{|F(x,y,z)-F(u,v,w)|} \\
&\quad = \mu\bigl(F(x, y, z) - F(u, v, w), kt\bigr)
\end{aligned}$$

for $k = \frac{3}{3.5} < 1$. Hence, there exists $k = \frac{3}{3.5} < 1$ such that

$$\begin{aligned}
&\mu\bigl(F(x, y, z) - F(u, v, w), kt\bigr) \\
&\quad \geq \mu(gx - gu, t) * \mu(gy - gv, t) * \mu(gz - gw, t)
\end{aligned}$$

for all $x, y, z, u, v, w \in X$, for which $gx \preceq gu$, $gy \succeq gv$, $gz \preceq gw$.

Now, for all $x, y, z, u, v, w \in X$, for which $gx \preceq gu$, $gy \succeq gv$, $gz \preceq gw$, we have

$$\begin{aligned}
&\upsilon(gx - gu, t) \diamond \upsilon(gy - gv, t) \diamond \upsilon(gz - gw, t) \\
&\quad = \upsilon(gx - gu, t) \diamond \upsilon(gv - gy, t) \diamond \upsilon(gz - gw, t) \\
&\quad \geq \upsilon(gx - gu + gv - gy + gz - gw, 3t) \\
&\quad = \upsilon\bigl(7\bigl[(x - u) + (v - y) + (z - w)\bigr], 3t\bigr) \\
&\quad = \upsilon\left(\bigl[(x - u) + (v - y) + (z - w)\bigr], \frac{3}{7}t\right)
\end{aligned}$$

$$
\begin{aligned}
&= 1 - \left[e^{-\frac{7}{3t}}\right]^{|x-u+v-y+z-w|} \\
&= 1 - \left[e^{-\frac{3.5}{3t}}\right]^{|2(x-u+v-y+z-w)|} \\
&= 1 - \left[e^{-\frac{3.5}{3t}}\right]^{|2(x-u)+2(v-y)+2(z-w)|} \\
&= 1 - \left[e^{-\frac{3.5}{3t}}\right]^{|F(x,y,z)-F(u,v,w)|} \\
&= \upsilon\big(F(x,y,z) - F(u,v,w), kt\big)
\end{aligned}
$$

for $k = \frac{3}{3.5} < 1$. Hence, there exists $k = \frac{3}{3.5} < 1$ such that

$$
\begin{aligned}
&\upsilon\big(F(x,y,z) - F(u,v,w), kt\big) \\
&\quad \leq \upsilon(gx - gu, t) \diamond \upsilon(gy - gv, t) \diamond \upsilon(gz - gw, t)
\end{aligned}
$$

for all $x, y, z, u, v, w \in X$, for which $gx \preceq gu$, $gy \succeq gv$, $gz \preceq gw$.

Therefore, all the conditions of Theorem 2.2 are satisfied. So, F and g have a tripled coincidence point and here $(\frac{2}{5}, \frac{2}{5}, \frac{2}{5})$ is a tripled coincidence point of F and g.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors read and approved the final manuscript.

Author details

[1]Lahore University of Management Sciences, Lahore, 54792, Pakistan. [2]Mathematics Department, University of Management and Technology, C-II, Johar Town, Lahore, Pakistan. [3]Department of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok, 10140, Thailand.

Acknowledgements

The third author would like to thank the Research Professional Development Project under the Science Achievement Scholarship of Thailand (SAST) and the fourth author would like to thank the National Research University Project of Thailand's Office of the Higher Education Commission for financial support (under the CSEC Project No. 55000613).

Received: 6 April 2012 Accepted: 8 October 2012 Published: 24 October 2012

References

1. Zadeh, LA: Fuzzy sets. Inf. Control **8**, 338-353 (1965)
2. Atanassov, K: Intuitionistic fuzzy sets. In: Sgrev, V (ed.) VII ITKR's Session, Central Sci. and Techn. Library, Bulg. Academy of Science, Sofia, June 1983 (1984)
3. Saadati, R, Park, JH: On the intuitionistic fuzzy topological spaces. Chaos Solitons Fractals **27**, 331-344 (2006)
4. Saadati, R, Sedghi, S, Shobe, N: Modified intuitionistic fuzzy metric spaces and some fixed point theorems. Chaos Solitons Fractals (2008). doi:10.1016/j.chaos.2006.11.008
5. Chauhan, S, Sintunavarat, W, Kumam, P: Common fixed point theorems for weakly compatible mappings in fuzzy metric spaces using (JCLR) property. Appl. Math. **3**(9), 976-982 (2012)
6. Sintunavarat, W, Kumam, P: Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces. J. Appl. Math. **2011**, Article ID 637958 (2011)
7. Sintunavarat, W, Kumam, P: Fixed point theorems for a generalized intuitionistic fuzzy contraction in intuitionistic fuzzy metric spaces. Thai J. Math. **10**(1), 123-135 (2012)
8. Sintunavarat, W, Kumam, P: Common fixed points for *R*-weakly commuting in fuzzy metric spaces. Ann. Univ. Ferrara (2012, in press). doi:10.1007/s11565-012-0150-z
9. Gordji, ME, Baghani, H, Cho, YJ: Coupled fixed point theorems for contractions in intuitionistic fuzzy normed spaces. Math. Comput. Model. **54**, 1897-1906 (2011)
10. Sintunavarat, W, Cho, YJ, Kumam, P: Coupled coincidence point theorems for contractions without commutative condition in intuitionistic fuzzy normed spaces. Fixed Point Theory Appl. **2011**, 81 (2011)
11. Berinde, V, Borcut, M: Tripled fixed point theorems for contractive type mappings in partially ordered metric spaces. Nonlinear Anal. **74**, 4889-4897 (2011)
12. Schweize, B, Sklar, A: Statistical metric spaces. Pac. J. Math. **10**, 314-334 (1960)
13. Mursaleen, M, Mohiuddine, SA: On stability of a cubic functional equation in intuitionistic fuzzy normed spaces. Chaos Solitons Fractals **42**, 2997-3005 (2009)
14. Mursaleen, M, Mohiuddine, SA: Nonlinear operators between intuitionistic fuzzy normed spaces and Fréhet differentiation. Chaos Solitons Fractals **42**, 1010-1015 (2009)

15. Haghi, RH, Rezapour, S, Shahzad, N: Some fixed point generalisations are not real generalizations. Nonlinear Anal. **74**, 1799-1803 (2011)

doi:10.1186/1687-1812-2012-187
Cite this article as: Abbas et al.: Tripled fixed point and tripled coincidence point theorems in intuitionistic fuzzy normed spaces. *Fixed Point Theory and Applications* 2012 **2012**:187.