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Some fixed and coincidence point theorems for expansive maps in cone metric spaces

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Abstract

In this article, we establish some common fixed and common coincidence point theorems for expansive type mappings in the setting of cone metric spaces. Our results extend some known results in metric spaces to cone metric spaces. Also, we introduce some examples the support the validity of our results.

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Keywords: common fixed point, cone metric space, coincidence fixed point

1. Introduction

Huang and Zhang [1] introduced the notion of cone metric spaces as a generalization of metric spaces. They replacing the set of real numbers by an ordered Banach space. Huang and Zhang [1] presented the notion of convergence of sequences in cone metric spaces and proved some fixed point theorems. Then after, many authors established many fixed point theorems in cone metric spaces. For some fixed point theorems in cone metric spaces we refer the reader to [2-30].

In the present article, E stands for a real Banach space.

Definition 1.1. *Let P be a subset of E with* $\mathrm{Int}(P) \neq \emptyset$. *Then P is called a cone if the following conditions are satisfied:*

(1) *P is closed and $P \neq \{\theta\}$.*
(2) *$a, b \in \mathbf{R}^+$, $x, y \in P$ implies $ax + by \in P$.*
(3) *$x \in P \cap -P$ implies $x = \theta$.*

For a cone P, define a partial ordering $\preccurlyeq$ with respect to P by $x \preccurlyeq y$ if and only if $y - x \in P$. We shall write $x \prec y$ to indicate that $x \preccurlyeq y$ but $x \neq y$, while $x \ll y$ will stand for $y - x \in \mathrm{Int}\, P$. It can be easily shown that $\lambda\mathrm{Int}(P) \subseteq \mathrm{Int}(P)$ for all positive scalar λ.

Definition 1.2. [1]*Let X be a nonempty set. Suppose the mapping $d : X \times X \to E$ satisfies*

(1) *$\theta \prec d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$.*
(2) *$d(x, y) = d(y, x)$ for all $x, y \in X$.*
(3) *$d(x, y) \preccurlyeq d(x, z) + d(y, z)$ for all $x, y \in X$.*

Then d is called a cone metric on X, and (X, d) is called a cone metric space.

Definition 1.3. [1]*Let (X, d) be a cone metric space. Let (x_n) be a sequence in X and $x \in X$. If for every $c \in E$ with $\theta \ll c$, there is an $N \in \mathbf{N}$ such that $d(x_n, x) \ll c$ for all $n \geq N$, then (x_n) is said to be convergent and (x_n) converges to x and x is the limit of (x_n). We denote this by $\lim_{n\to+\infty} x_n = x$ or $x_n \to x$ as $n \to +\infty$. If for every $c \in E$ with $\theta \ll c$ there is an $N \in \mathbf{N}$ such that $d(x_n,x_m) \ll c$ for all $n,m \geq N$, then (x_n) is called a Cauchy sequence in X. The space (X,d) is called a complete cone metric space if every Cauchy sequence is convergent.*

The cone P in a real Banach space E is called normal if there is a number $\lambda > 0$ such that for all $x,y \in E$,

$$\theta \preccurlyeq x \preccurlyeq y \text{ implies } \|x\| \leq \lambda \|y\| .$$

Iiker ⊠ahin and Mustafa Telci [30] studied a theorem on common fixed points of expansive type mappings in cone metric spaces.

Definition 1.4. [30]*Let E and F be real Banach spaces and P and Q be cones in E and F, respectively. Let (X, d) and (Y, ρ) be cone metric spaces, where $d : X \times X \to E$ and $\rho : Y \times Y \to F$. A function $f : X \to Y$ is said to be continuous at $x_0 \in X$, if for every $c \llcorner F$ with $0 \ll c$, there exists $b \in E$ with $0 \ll b$ such that $\rho(fx, fx_0) \ll c$ whenever $x \in X$ and $d(x, x_0) \ll b$.*

Lemma 1.1. [30]*Let (X, d) and (Y, ρ) be cone metric spaces. A function $f : X \to Y$ is continuous at a point $x_0 \in X$ if and only if whenever a sequence (x_n) in X converges to x_0, the sequence (fx_n) converges to fx_0.*

Theorem 1.1. [30]*Let (X, d) be a complete cone metric space and P be a cone. Let f and g be surjective self-mappings of X satisfying the following inequalities*

$$d(g\,fx, fx) \succcurlyeq ad\left(fx, x\right),$$
$$d\left(f\,gx, gx\right) \succcurlyeq bd\left(gx, x\right)$$

for all $x \in X$, where $a,b > 1$. If either f or g is continuous, then f and g have a common fixed point.

The aim of this article is to study new theorems of common fixed and coincidence point for expansive mappings in cone metric spaces under a set of conditions. Our results generalize several well known comparable results in the literature. Also, we introduced some examples to support the validity of our results.

2. Main results

We start with the following theorem

Theorem 2.1. *Let (X, d) be a cone metric space with a solid cone P. Let $T, f : X \to X$ be mappings satisfying:*

$$d\left(Tx, Ty\right) \succcurlyeq ad\left(fx, fy\right) + bd(fx, Tx) + cd(fy, Ty) \tag{1}$$

for all $x, y \in X$ where $a,b,c \geq 0$ with $a + b + c > 1$. Suppose the following hypotheses:

(1) *$b < 1$ or $c < 1$.*
(2) *$fX \subseteq TX$.*
(3) *TX is a complete subspace of X.*

Then T and f have a coincidence point.

Proof. Let $x_0 \in X$. Since $fX \subseteq TX$, we choose $x_1 \in X$ such that $Tx_1 = fx_0$. Again, we can choose $x_2 \in$ X such that $Tx_2 = fx_1$. Continuing in the same way, we construct a sequence (x_n) in X such that $Tx_{n+1} = fx_n$ for all $n \in \mathbf{N} \cup \{\mathbf{0}\}$.

If $fx_{m-1} = fx_m$ for some $m \in \mathbf{N}$, then $Tx_m = fx_m$. Thus x_m is a coincidence point of T and f.

Now, assume that $x_{n-1} \neq x_n$ for all $n \in \mathbf{N}$.

Case (1): Suppose $b < 1$.

By (1), we have

$$\begin{aligned} d\left(fx_{n-1}, fx_n\right) &= d\left(Tx_n, Tx_{n+1}\right) \\ &\succcurlyeq ad\left(fx_n, fx_{n+1}\right) + bd(fx_n, Tx_n) + cd(fx_{n+1}, Tx_{n+1}) \\ &= ad\left(fx_n, fx_{n+1}\right) + bd(fx_n, fx_{n-1}) + cd\left(fx_{n+1}, fx_n\right). \end{aligned}$$

Thus, we have

$$(1 - b)d\left(fx_{n-1}, fx_n\right) \succcurlyeq (a + c)d\left(fx_{n+1}, fx_n\right).$$

Hence

$$d\left(fx_{n+1}, fx_n\right) \preccurlyeq \frac{1-b}{a+c} d\left(fx_{n-1}, fx_n\right). \tag{2}$$

Case (2): Suppose $c < 1$.

By (1), we have

$$\begin{aligned} d(fx_n, fx_{n-1}) &= d\left(Tx_{n+1}, Tx_n\right) \\ &\succcurlyeq ad(fx_{n+1}, fx_n) + bd(fx_{n+1}, Tx_{n+1}) + cd(fx_n, Tx_n) \\ &= ad\left(fx_n, fx_{n+1}\right) + bd(fx_{n+1}, fx_n) + cd(fx_n, fx_{n-1}). \end{aligned}$$

Thus, we have

$$(1 - c)\, d(fx_{n-1}, fx_n) \succcurlyeq (a + b)d\left(fx_{n+1}, fx_n\right).$$

Hence

$$d\left(fx_{n+1}, fx_n\right) \preccurlyeq \frac{1-c}{a+b} d\left(fx_{n-1}, fx_n\right). \tag{3}$$

In Case (1), we let

$$\lambda = \frac{1-b}{a+c}$$

and in Case (2), we let

$$\lambda = \frac{1-c}{a+b}.$$

Thus in both cases, we have $\lambda < 1$ and

$$d\left(fx_{n+1}, fx_n\right) \preccurlyeq \lambda d(fx_{n-1}, fx_n). \tag{4}$$

By (4), we have

$$\begin{aligned} d(fx_{n+1}, fx_n) &\preccurlyeq \lambda d(fx_{n-1}, fx_n) \\ &\preccurlyeq \lambda^2 d(fx_{n-2}, fx_{n-1}) \\ &\vdots \\ &\preccurlyeq \lambda^n d(fx_0, fx_1). \end{aligned}$$

So for $m > n$, we have

$$\begin{aligned} d(fx_n, fx_m) &\preccurlyeq d(fx_n, fx_{n+1}) + d(fx_{n+1}, fx_{n+2}) + \cdots + d(fx_{m-1}, fx_m) \\ &\preccurlyeq (\lambda^n + \lambda^{n+1} + \cdots + \lambda^{m-1}) d(fx_0, fx_1) \\ &\preccurlyeq \lambda^n \sum_{i=0}^{+} \infty \lambda^i d(fx_0, fx_1) \\ &= \frac{\lambda^n}{1-\lambda} d(fx_0, fx_1). \end{aligned}$$

Let $\theta \ll c$ be given, choose $\delta > 0$ such that $c + N_\delta(0) \subseteq P$, where

$$N_\delta(0) = \{y \in E : \|y\| < \delta\}.$$

Also, choose a natural number N_1 such that

$$\frac{\lambda^n}{1-\lambda} d(fx_0, fx_1) \in N_\delta(0),$$

for $m \geq N_1$. Then

$$\frac{\lambda^n}{1-\lambda} d(fx_0, fx_1) \ll c,$$

for all $m \geq N_1$. Thus,

$$d(fx_n, fx_m) \preccurlyeq \frac{\lambda^n}{1-\lambda} d(fx_0, fx_1) \ll c,$$

for all $m > n$. Therefore (Tx_n) is a cauchy sequence in (TX, d). Since (TX, d) is a complete cone metric space, there *is* $u \in X$ such that (Tx_n) converges to Tu as $n \to +\infty$. Hence fx_n converges to Tu as $n \to +\infty$. Since $a + b + c > 1$, we have a, b and c are not all 0. So we have the following cases.

Case 1: If $a \neq 0$, then

$$d(Tx_n, Tu) \succcurlyeq ad(fx_n, fu) + bd(fx_n, Tx_n) + cd(fu, Tu) \succcurlyeq ad(fx_n, fu).$$

Hence

$$d(fx_n, fu) \preccurlyeq \frac{1}{a} d(Tx_n, Tu).$$

Let $\theta \ll c$ be given, choose $\delta > 0$ such that $c + N_\delta(0) \subseteq P$, where

$$N_\delta(0) = \{y \in E : \|y\| < \delta\}.$$

Since $\frac{1}{a}d(Tx_n, Tu) \to \theta$. We choose a natural number $n_0 \in \mathbf{N}$ such that

$$\frac{1}{a}d(Tx_n, Tu) \in N_\delta(0),$$

for $n \geq n_0$. Then

$$\frac{1}{a}d(Tx_n, Tu) \ll c,$$

for all $n \geq n_0$. Thus,

$$d(fx_n, fu) \preccurlyeq \frac{1}{a}d(Tx_n, Tu) \ll c,$$

for all $n \geq n_0$. Thus $fx_n \to fu$ as $n \to +\infty$. By uniqueness of limit, we have $Tu = fu$. Therefore T and f have a coincidence point.

Case 2: If $b \neq 0$, then

$$d(Tu, Tx_n) \succcurlyeq ad(fx_n, fu) + bd(fu, Tu) + cd(fx_n, Tx_n) \succcurlyeq bd(fu, Tu).$$

Hence

$$d(fu, Tu) \preccurlyeq \frac{1}{b}d(Tx_n, Tu).$$

As similar proof of Case (1), we can show that $fu = Tu$. Thus f and T have a coincidence point.

Case 3: If $c \neq 0$, then

$$d(Tx_n, Tu) \succcurlyeq ad(fx_n, fu) + bd(fx_n, Tx_n) + cd(Tu, fu) \succcurlyeq cd(fu, Tu).$$

Hence

$$d(fu, Tu) \preccurlyeq \frac{1}{c}d(Tx_n, Tu).$$

As similar proof of Case (1), we can show that $fu = Tu$. Thus f and T have a coincidence point.

Corollary 2.1. *Let (X,d) be a cone metric space with a solid cone P. Let $T, f : X \to X$ be mappings satisfying:*

$$d(Tx, Ty) \succcurlyeq ad(fx, fy) + bd(fx, Tx)$$

for all $x, y \in X$ where $a, b \geq 0$ with $a + b > 1$ and $b < 1$. Suppose the following hypotheses:

(1) *$fX \subseteq TX$.*
(2) *TX is a complete subspace of X.*

Then T and f have a coincidence point.

Corollary 2.2. *Let (X, d) be a complete cone metric space with a solid cone P. Let $T, f : X \to X$ be mappings satisfying:*

$$d(Tx, Ty) \succcurlyeq ad(fx, fy)$$

for all x, $y \in X$ where $a > 1$. Suppose the following hypotheses:

(1) $fX \subseteq TX$.
(2) TX *is a complete subspace of* X.

Then T and f have a coincidence point.

Corollary 2.3. *Let (X, d) be a complete cone metric space with a solid cone P. Let $T : X \to X$ be a surjective mapping satisfying:*

$$d(Tx, Ty) \succcurlyeq ad(x, y) + bd(x, Tx) + cd(y, Ty)$$

for all x, $y \in X$ where $a,b,c \geq 0$ with $a + b + c > 1$. Suppose $b < 1$ or $c < 1$. Then T has a fixed point.

Proof. Follows from Theorem 2.1 by taking $f = I$, the identity map.

Corollary 2.4. *Let (X, d) be a complete cone metric space with a solid cone P. Let $T : X \to X$ be a surjective mapping satisfying:*

$$d(Tx, Ty) \succcurlyeq ad(x, y)$$

for all x, $y \in X$ where with $a > 1$. Then T has a fixed point.

Putting $E = \mathbf{R}$, $P = \{x \in \mathbf{R} : x \geq 0\}$ and $d : X \times X \to \mathbf{R}$ in Corollaries 2.1 and 2.2, we have the following results:

Corollary 2.5. *Let (X, d) be a complete metric space. Let $T : X \to X$ be a surjective mapping satisfying:*

$$d(Tx, Ty) \geq ad(x, y) + bd(x, Tx)$$

for all x, $y \in X$ where a, $b \geq 0$ with $a + b > 1$ and $b < 1$. Then T has a fixed point.

Corollary 2.6. *Let (X, d) be a complete metric space. Let $T : X \to X$ be a surjective mapping satisfying:*

$$d(Tx, Ty) \geq ad(x, y) + bd(y, Ty)$$

for all x, $y \in X$ where a, $b \geq 0$ with $a + b > 1$ and $b < 1$. Then T has a fixed point.

Now, we present a fixed point theorem for two maps.

Theorem 2.2. *Let T, $S : X \to X$ be two surjective mappings of a complete cone metric space (X, d) with a solid cone P. Suppose that T and S satisfying the following inequalities*

$$d(T(Sx), Sx) + kd(T(Sx), x) \succcurlyeq ad(Sx, x) \tag{5}$$

and

$$d(S(Tx), Tx) + kd(S(Tx), x) \succcurlyeq bd(Tx, x) \tag{6}$$

for all $x \in X$ and some nonnegative real numbers a, b and k with $a > 1 + 2k$ and $b > 1 + 2k$. If T or S is continuous, then T and S have a common fixed point

Proof. Let x_0 be an arbitrary point in X. Since T is surjective, there exists $x_1 \in X$ such that $x_0 = Tx_1$. Also, since S is surjective, there exists $x_2 \in X$ such that $x_2 = Sx_1$. Continuing this process, we construct a sequence (x_n) in X such that $x_{2n} = Tx_{2n+1}$ and $x_{2n+1} = Sx_{2n+2}$ for all $n \in \mathbf{N} \cup \{0\}$. Now, for $n \in \mathbf{N} \cup \{0\}$, we have

$$d(T(Sx_{2n+2}), Sx_{2n+2}) + kd(T(Sx_{2n+2}), x_{2n+2}) \succcurlyeq ad(Sx_{2n+2}, x_{2n+2}).$$

Thus, we have

$$d(x_{2n}, x_{2n+1}) + kd(x_{2n}, x_{2n+2}) \succcurlyeq ad(x_{2n+1}, x_{2n+2}),$$

which implies that

$$d(x_{2n}, x_{2n+1}) + kd(x_{2n}, x_{2n+1}) + kd(x_{2n+1}, x_{2n+2}) \succcurlyeq ad(x_{2n+1}, x_{2n+2}).$$

Hence

$$d(x_{2n+1}, x_{2n+2}) \preccurlyeq \frac{1+k}{a-k} d(x_{2n}, x_{2n+1}) \tag{7}$$

On other hand, we have

$$d(S(Tx_{2n+1}), Tx_{2n+1}) + kd(S(Tx_{2n+1}), x_{2n+1}) \succcurlyeq bd(Tx_{2n+1}, x_{2n+1}).$$

Thus, we have

$$d(x_{2n-1}, x_{2n}) + kd(x_{2n-1}, x_{2n+1}) \succcurlyeq bd(x_{2n}, x_{2n+1}).$$

Since $d(x_{2n-1}, x_{2n}) + d(x_{2n}, x_{2n+1}) \succcurlyeq d(x_{2n-1}, x_{2n+1})$, we have

$$d(x_{2n-1}, x_{2n}) + kd(x_{2n-1}, x_{2n}) + kd(x_{2n}, x_{2n+1}) \succcurlyeq bd(x_{2n}, x_{2n+1}).$$

Hence

$$d(x_{2n}, x_{2n+1}) \preccurlyeq \frac{1+k}{b-k} d(x_{2n-1}, x_{2n}) \tag{8}$$

Let

$$\lambda = \max\left\{\frac{1+k}{a-k}, \frac{1+k}{b-k}\right\}.$$

Then by combining (7) and (8), we have

$$d(x_n, x_{n+1}) \leq \lambda d(x_{n-1}, x_n) \quad \forall n \in \mathbf{N} \cup \{0\}. \tag{9}$$

Repeating (9) n-times, we get

$$d(x_n, x_{n+1}) \preccurlyeq \lambda^n d(x_0, x_1).$$

Thus, for $m > n$, we have

$$\begin{aligned} d(x_n, x_m) &\preccurlyeq d(x_n, x_{n+1}) \cdots + d(x_{m-1}, x_m) \\ &\preccurlyeq (\lambda^n + \cdots + \lambda^{m-1}) d(x_0, x_1) \\ &\preccurlyeq \frac{\lambda^n}{1-\lambda} d(x_0, x_1). \end{aligned}$$

As similar arguments to proof of Theorem 2.1, we can show that (x_n) is a Cauchy sequence in the complete cone metric space (X, d). Then there exists $v \in X$ such that $x_n \to v$ as $n \to +\infty$. Therefore $x_{2n+1} \to v$ and $x_{2n+2} \to v$ as $n \to +\infty$. Without loss of generality, we may assume that T is continuous, then $Tx_{2n+1} \to Tv$ as $n \to +\infty$. But $Tx_{2n+1} = x_{2n} \to v$ as $n \to +\infty$. Thus, we have $Tv = v$. Since S is surjective, there exists $w \in X$ such that $Sw = v$. Now,

$$d(T(Sw), Sw) + kd(T(Sw), w) \succcurlyeq ad(Sw, w),$$

implies that $kd(v,w) \succcurlyeq ad(v,w)$. Thus

$$d(v,w) \preccurlyeq \frac{k}{a} d(v,w).$$

Since $a > k$, we conclude that $d(v, w) = \theta$. So $v = w$. Hence $Tv = Sv = v$. Therefore v is a common fixed point of T and S.

By taking b = a in Theorem 2.2, we have the following result.

Corollary 2.7. *Let T, $S : X \to X$ be two surjective mappings of a complete cone metric space (X, d) with a solid cone P. Suppose that T and S satisfying the following inequalities*

$$d(T(Sx), Sx) + kd(T(Sx), x) \succcurlyeq ad(Sx, x) \tag{10}$$

and

$$d(S(Tx), Tx) + kd(S(Tx), x) \succcurlyeq ad(Tx, x) \tag{11}$$

for all $x \in X$ and some nonnegative real numbers a and k with $a > 1 + 2k$. If T or S is continuous, then T and f have a common fixed point

By taking $S = T$ in Corollary 2.7, we have the following corollary.

Corollary 2.8. *Let $T : X \to X$ be a surjective mapping of a complete cone metric space (X, d) with a solid cone P. Suppose that T satisfying*

$$d(T(Tx), Tx) + kd(T(Tx), x) \succcurlyeq ad(Tx, x) \tag{12}$$

for all $x \in X$ and some nonnegative real number a and k with $a > 1 + 2k$. If T is continuous, then T has a fixed point.

Now, we present some examples to illustrate the useability of our results.

Example 2.1. (The case of normal cone) *Let $X = [0,+\infty)$, $E = \mathbf{R}^2$. Let $P = \{(a, b) : a \geq 0, b \geq 0\}$ be the cone with $d(x, y) = (|x - y|, |x - y|)$. Then (X, d) is a complete cone metric space. Define $T : X \to X$ by $Tx = 2x$. Then T has a fixed point.*

Proof. Note that

$$d(T(Tx), Tx) + d(T(Tx), x) \geq 4d(Tx, x)$$

for all $x \in X$. Thus T satisfies all the hypotheses of Corollary 2.8 and hence T has a fixed point. Here 0 is the fixed point of T.

Example 2.2. (The case of non-normal cone) *Let $X = [0, 1]$, $E = C^1_{\mathbf{R}}([0, 1])$. Let $P = \{\varphi \in E : \varphi(t) \geq 0,\ t \in [0, 1]\}$. Define the mapping $d : X \times X \to E$ by*

$$d(x, y)(t) := |x - y|\, \phi(t),$$

where $\varphi \in P$ is a fixed function, for example $\varphi(t) = e^t$. Define T, $f : X \to X$ by $fx = \frac{1}{16}x$and $fx = \frac{1}{16}x$. Then T and f have a coincidence point.

Proof. Note that

$$\begin{aligned} d(Tx, Ty)(t) &= \left|\frac{1}{4}x - \frac{1}{4}y\right| e^t \\ &= 4\left|\frac{1}{16}x - \frac{1}{16}y\right| e^t \\ &= 4d(fx, fy)(t) \end{aligned}$$

for all x, $y \in X$ and $t \in [0, 1]$. Thus T and f satisfy all the hypotheses of Corollary 2.2 and hence T and f have a coincidence point. Here 0 is the coincidence point of T and f.

Remarks:

(1) Theorem 4.1 of [29] is a special case of Theorem 2.2.
(2) Corollary 4.1 of [29] is a special case of Corollary 2.8.
(3) Theorem 4 of [31] is a special case of Corollary 2.8.

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Authors' contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

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