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Coupled coincidence point theorems for contractions in generalized fuzzy metric spaces

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Abstract
In this paper, we introduce the concept of a mixed *g*-monotone mapping and prove coupled coincidence and common coupled fixed point theorems for mappings under ϕ-contractive conditions in partially ordered generalized fuzzy metric spaces. We also give an example to illustrate the theorems.

MSC: 47H10; 54H25

Keywords: coupled coincidence point; common coupled fixed point; partially ordered set; mixed monotone mapping; generalized fuzzy metric space

1 Introduction

The theory of fuzzy sets has evolved in many directions after investigation of the notion of fuzzy sets by Zadeh [1]. Many authors have introduced the concept of a fuzzy metric space in different ways [2, 3]. George and Veeramani [4, 5] modified the concept of a fuzzy metric space introduced by Kramosil and Michalek [3] and defined a Hausdorff topology on this fuzzy metric space. They showed also that every metric induces a fuzzy metric. Later, many fixed point theorems in fuzzy metric spaces and probabilistic metric spaces have been obtained by [6–10].

Nieto and Lopez [11], Ran and Reurings [12], Petrusel and Rus [13] presented some new results for contractions in partially ordered metric spaces. The main idea in [11, 12] involves combining the ideas of the iterative technique in the contractive mapping principle with those in the monotone technique, discussing the existence of a solution to first-order ordinary differential equations with periodic boundary conditions and some applications to linear and nonlinear matrix equations.

Bhaskar and Lakshmikantham [14], Lakshmikantham and Ćirić [15] discussed coupled coincidence and coupled fixed point theorems for two mappings F and g, where F has the mixed g-monotone property and F and g commute. The results were used to study the existence of a unique solution to a periodic boundary value problem. In [16], Choudhury and Kundu established a similar result under the condition that F and g are compatible mappings and the function g is monotone increasing. For more details on ordered metric spaces, we refer to [17–19] and references mentioned therein.

Alternatively Mustafa and Sims [20] introduced a new notion of a generalized metric space called G-metric space. Mujahid Abbas *et al.* [23] proved a unique fixed point of four R-weakly commuting maps in G-metric spaces, and Mujahid Abbas *et al.* [24] obtained some common fixed point results of maps satisfying the generalized (φ, ψ)-weak commuting condition in partially ordered G-metric spaces. Rao *et al.* [22] proved two unique common coupled fixed-point theorems for three mappings in symmetric G-fuzzy metric spaces. Sun and Yang [21] introduced the concept of G-fuzzy metric spaces and proved two common fixed-point theorems for four mappings. Some interesting references on G-metric spaces are [22–25].

In this paper, we introduce the concept of a mixed g-monotone mapping, which is a generalization of the mixed monotone mapping, and prove coupled coincidence point and coupled common fixed point theorems for mappings under ϕ-contractive conditions in partially ordered G-fuzzy metric spaces. The work is an extension of the fixed point result in fuzzy metric spaces and the condition is different from [14–16] even in metric spaces. We also give an example to illustrate the theorems.

Recall that if $(X, \leq)$ is a partially ordered set and $F : X \to X$ satisfies that for $x, y \in X$, $x \leq y$ implies $F(x) \leq F(y)$, then a mapping F is said to be non-decreasing. Similarly, a non-increasing mapping is defined.

Before giving our main results, we recall some of the basic concepts and results in G-metric spaces and G-fuzzy metric spaces.

2 Preliminaries

Definition 2.1 [20] Let X be a nonempty set, and let $G : X \times X \times X \to [0, +\infty)$ be a function satisfying the following properties:

(G-1) $G(x, y, z) = 0$ if $x = y = z$,
(G-2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,
(G-3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$,
(G-4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$, symmetry in all three variables,
(G-5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

The function G is called a generalized metric or a G-metric on X and the pair (X, G) is called a G-metric space.

Definition 2.2 [20] The G-metric space (X, G) is called symmetric if $G(x, x, y) = G(x, y, y)$ for all $x, y \in X$.

Definition 2.3 [20] Let (X, G) be a G-metric space, and let $\{x_n\}$ be a sequence in X. A point $x \in X$ is said to be the limit of $\{x_n\}$ if and only if $\lim_{n,m\to\infty} G(x, x_n, x_m) = 0$. In this case, the sequence $\{x_n\}$ is said to be G-convergent to x.

Definition 2.4 [20] Let (X, G) be a G-metric space, and let $\{x_n\}$ be a sequence in X. $\{x_n\}$ is called a G-Cauchy sequence if and only if $\lim_{n,m,l\to\infty} G(x_n, x_m, x_l) = 0$. (X, G) is called G-complete if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).

Proposition 2.1 [20] *In a G-metric space* (X, G), *the following are equivalent*:

(i) *The sequence* $\{x_n\}$ *is G-Cauchy.*
(ii) *For every* $\varepsilon > 0$, *there exists* $N \in \mathbb{N}$ *such that* $G(x_n, x_m, x_m) < \varepsilon$ *for all* $n, m \geq N$.

Proposition 2.2 [20] *Let* (X, G) *be a G-metric space*; *then the function* $G(x, y, z)$ *is jointly continuous in all three of its variables.*

Proposition 2.3 [20] *Let (X, G) be a G-metric space; then for any $x, y, z, a \in X$, it follows that*

(i) *if $G(x, y, z) = 0$, then $x = y = z$,*
(ii) $G(x, y, z) \leq G(x, x, y) + G(x, x, z)$,
(iii) $G(x, y, y) \leq 2G(x, x, y)$,
(iv) $G(x, y, z) \leq G(x, a, z) + G(a, y, z)$,
(v) $G(x, y, z) \leq \frac{2}{3}(G(x, a, a) + G(y, a, a) + G(z, a, a))$.

Let (X, d) be a metric space. One can verify that (X, G) is a G-metric space, where

$$G(x, y, z) = \max\{d(x, y), d(y, z), d(z, x)\}$$

or

$$G(x, y, z) = \frac{1}{3}\big(d(x, y) + d(y, z) + d(z, x)\big).$$

If (X, G) is a G-metric space, it easy to verify that (X, d_G) is a metric space, where $d_G(x, y) = \frac{1}{2}(G(x, x, y) + G(x, y, y))$.

Definition 2.5 [26] A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0, 1]$;
(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.6 [21] A 3-tuple $(X, G, *)$ is said to be a G-fuzzy metric space (denoted by *GF* space) if X is an arbitrary nonempty set, $*$ is a continuous t-norm and G is a fuzzy set on $X^3 \times (0, +\infty)$ satisfying the following conditions for each $t, s > 0$:

(GF-1) $G(x, x, y, t) > 0$ for all $x, y \in X$ with $x \neq y$;
(GF-2) $G(x, x, y, t) \geq G(x, y, z, t)$ for all $x, y, z \in X$ with $y \neq z$;
(GF-3) $G(x, y, z, t) = 1$ if and only if $x = y = z$;
(GF-4) $G(x, y, z, t) = G(p(x, y, z), t)$, where p is a permutation function;
(GF-5) $G(x, a, a, t) * G(a, y, z, s) \leq G(x, y, z, t + s)$ (the triangle inequality);
(GF-6) $G(x, y, z, \cdot) : (0, \infty) \to [0, 1]$ is continuous.

Remark 2.1 Let $x = w$, $y = u$, $z = u$, $a = v$ in (GF-5), we have

$$G(w, u, u, t + s) \geq G(w, v, v, t) * G(v, u, u, s),$$

which implies that

$$G(u, u, w, s + t) \geq G(u, u, v, s) * G(v, v, w, t),$$

for all $u, v, w \in X$ and $s, t > 0$.

A *GF* space is said to be symmetric if $G(x, x, y, t) = G(x, y, y, t)$ for all $x, y \in X$ and for each $t > 0$.

Example 2.1 Let X be a nonempty set, and let G be a G-metric on X. Define the t-norm $a * b = \min\{a, b\}$ and for all $x, y, z \in X$ and $t > 0$, $G(x, y, z, t) = \frac{t}{t+G(x,y,z)}$. Then $(X, G, *)$ is a GF space.

Remark 2.2 If $(X, M, *)$ is a fuzzy metric space [4], then $(X, G, *)$ is a GF space, where

$$G(x, y, z, t) = \min\{M(x, y, t), M(y, z, t), M(z, x, t)\}.$$

In fact, we only need to verify (GF-5). Since

$$G(x, y, z, t) = \min\{M(x, y, t), M(y, z, t), M(z, x, t)\}, \qquad G(x, a, a, t) = M(x, a, t),$$
$$G(a, y, z, s) = \min\{M(a, y, s), M(y, z, s), M(z, a, s)\},$$

we have

$$\begin{aligned}
&G(x, a, a, t) * G(a, y, z, s) \\
&\quad = M(x, a, t) * \min\{M(a, y, s), M(y, z, s), M(z, a, s)\} \\
&\quad \le \min\{M(x, a, t) * M(a, y, s), M(x, a, t) * M(y, z, s), M(x, a, t) * M(z, a, s)\} \\
&\quad \le \min\{M(x, y, t + s), M(y, z, s), M(x, z, t + s)\} \\
&\quad \le \min\{M(x, y, t + s), M(y, z, t + s), M(x, z, t + s)\} \\
&\quad = G(x, y, z, t + s),
\end{aligned}$$

which implies that (GF-5) holds.

Remark 2.3 If $(X, G, *)$ is a symmetric GF space, let $M(x, y, t) = G(x, y, y, t)$, then $(X, M, *)$ is a fuzzy metric space [4].

Let $(X, G, *)$ be a GF space. For $t > 0$, the open ball $B_G(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined by

$$B_G(x, r, t) = \{y \in X : G(x, y, y, t) > 1 - r\}.$$

A subset $A \subset X$ is called an open set if for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B_G(x, r, t) \subset A$.

Definition 2.7 [21] Let $(X, G, *)$ be a GF space, then

(1) a sequence $\{x_n\}$ in X is said to be convergent to x (denoted by $\lim_{n\to\infty} x_n = x$) if

$$\lim_{n\to\infty} G(x_n, x_n, x, t) = 1$$

for all $t > 0$.

(2) a sequence $\{x_n\}$ in X is said to be a Cauchy sequence if

$$\lim_{n,m\to\infty} G(x_n, x_n, x_m, t) = 1,$$

that is, for any $\varepsilon > 0$ and for each $t > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$G(x_n, x_n, x_m, t) > 1 - \varepsilon,$$

for $n, m \geq n_0$.

(3) A *GF* space $(X, G, *)$ is said to be complete if every Cauchy sequence in X is convergent.

Lemma 2.1 [21] *Let $(X, G, *)$ be a GF space. Then $G(x, y, z, t)$ is non-decreasing with respect to t for all $x, y, z \in X$.*

Lemma 2.2 [21] *Let $(X, G, *)$ be a GF space. Then G is a continuous function on $X^3 \times (0, +\infty)$.*

In the rest of the paper, $(X, G, *)$ will denote a *GF* space with a continuous t-norm $*$ defined as $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$, and we assume that

$$\lim_{t\to\infty} G(x, y, z, t) = 1, \quad \forall x, y, z \in X. \tag{P}$$

Define $\Phi = \{\phi : R^+ \to R^+\}$, where $R^+ = [0, +\infty)$ and each $\phi \in \Phi$ satisfies the following conditions:

(Φ-1) ϕ is strict increasing;
(Φ-2) ϕ is upper semi-continuous from the right;
(Φ-3) $\sum_{n=0}^{\infty} \phi^n(t) < +\infty$ for all $t > 0$, where $\phi^{n+1}(t) = \phi(\phi^n(t))$.

Let $\phi_1(t) = \frac{t}{t+1}$, $\phi_2(t) = kt$, where $0 < k < 1$, then $\phi_1, \phi_2 \in \Phi$.
It is easy to prove that if $\phi \in \Phi$, then $\phi(t) < t$ for all $t > 0$.
Using (P), one can prove the following lemma.

Lemma 2.3 *Let $(X, G, *)$ be a GF space. If there exists $\phi \in \Phi$ such that if $G(x, y, z, \phi(t)) \geq G(x, y, z, t)$ for all $t > 0$, then $x = y = z$.*

Lemma 2.4 *Let $(X, G, *)$ be a GF space. If we define $E_\lambda : X \times X \times X \to [0, \infty)$ by*

$$E_\lambda(x, y, z) = \inf\bigl\{t > 0, G(x, y, z, t) > 1 - \lambda\bigr\} \tag{2.1}$$

for all $\lambda \in (0, 1]$ and $x, y, z \in X$, then we have:

(1) *for each $\lambda \in (0, 1]$, there exists $\mu \in (0, 1]$ such that*

$$E_\lambda(x_1, x_1, x_n) \leq \sum_{i=1}^{n-1} E_\mu(x_i, x_i, x_{i+1}), \quad \forall x_1, \ldots, x_n \in X.$$

(2) *The sequence $\{x_n\}_{n\in\mathbb{N}}$ in X is convergent if and only if $E_\lambda(x_n, x_n, x) \to 0$ as $n \to \infty$ for all $\lambda \in (0, 1]$.*

Proof (1) For any $\lambda \in (0,1]$, let $\mu \in (0,1]$ and $\mu < \lambda$, and so, by the triangular inequality (GF-5) and Remark 2.1, for any $\delta > 0$, we have

$$\begin{aligned} G&\left(x_1, x_1, x_n, \sum_{i=1}^{n-1} E_\mu(x_i, x_i, x_{i+1}) + (n-1)\delta\right) \\ &\geq G\bigl(x_1, x_1, x_2, E_\mu(x_1, x_1, x_2) + \delta\bigr) * G\bigl(x_2, x_2, x_3, E_\mu(x_2, x_2, x_3) + \delta\bigr) * \cdots \\ &\quad * G\bigl(x_{n-1}, x_{n-1}, x_n, E_\mu(x_{n-1}, x_{n-1}, x_n) + \delta\bigr) \\ &\geq \min\bigl\{(1-\mu), (1-\mu), \ldots, (1-\mu)\bigr\} \geq 1 - \lambda, \end{aligned}$$

which implies, by Definition 2.1 of E_μ, that

$$E_\lambda(x_1, x_1, x_n) \leq E_\mu(x_1, x_1, x_2) + E_\mu(x_2, x_2, x_3) + \cdots + E_\mu(x_{n-1}, x_{n-1}, x_n) + (n-1)\delta.$$

Since $\delta > 0$ is arbitrary, we have

$$E_\lambda(x_1, x_1, x_n) \leq E_\mu(x_1, x_1, x_2) + E_\mu(x_2, x_2, x_3) + \cdots + E_\mu(x_{n-1}, x_{n-1}, x_n).$$

(2) Since G is continuous in its fourth argument, by Definition 2.1 of E_μ, we have

$$G(x_n, x_n, x, \eta) > 1 - \lambda \quad \text{for all } \eta > 0.$$

This proved the lemma. □

Lemma 2.5 *Let $(X, G, *)$ be a GF space and $\{y_n\}$ be a sequence in X. If there exists $\phi \in \Phi$ such that*

$$G\bigl(y_n, y_n, y_{n+1}, \phi(t)\bigr) \geq G(y_{n-1}, y_{n-1}, y_n, t) * G(y_n, y_n, y_{n+1}, t) \tag{2.2}$$

for all $t > 0$ and $n = 1, 2, \ldots$, then $\{y_n\}$ is a Cauchy sequence in X.

Proof Let $\{E_\lambda(x, y, z)\}_{\lambda \in (0,1]}$ be defined by (2.1). For each $\lambda \in (0,1]$ and $n \in \mathbb{N}$, putting $a_n = E_\lambda(y_{n-1}, y_{n-1}, y_n)$, we will prove that

$$a_{n+1} \leq \phi(a_n), \quad \forall n \in \mathbb{N}. \tag{2.3}$$

Since ϕ is upper semi-continuous from right, for given $\varepsilon > 0$ and each a_n, there exists $p_n > a_n$ such that $\phi(p_n) < \phi(a_n) + \varepsilon$. From the definition of E_λ by (2.1), it follows from $p_n > a_n = E_\lambda(y_{n-1}, y_{n-1}, y_n)$ that $G(y_{n-1}, y_{n-1}, y_n, p_n) > 1 - \lambda$ for all $n \in \mathbb{N}$.

Thus, by (2.2), (2.3) and Lemma 2.1, we get

$$\begin{aligned} G&\bigl(y_n, y_n, y_{n+1}, \phi\bigl(\max\{p_n, p_{n+1}\}\bigr)\bigr) \\ &\geq G\bigl(y_{n-1}, y_{n-1}, y_n, \max\{p_n, p_{n+1}\}\bigr) * G\bigl(y_n, y_n, y_{n+1}, \max\{p_n, p_{n+1}\}\bigr) \\ &\geq G(y_{n-1}, y_{n-1}, y_n, p_n) * G(y_n, y_n, y_{n+1}, p_{n+1}) > 1 - \lambda. \end{aligned}$$

Again by Definition 2.1, we get

$$E_\lambda(y_n,y_n,y_{n+1}) \le \phi\bigl(\max\{p_n,p_{n+1}\}\bigr) = \max\bigl\{\phi(p_n),\phi(p_{n+1})\bigr\} \\ \le \max\bigl\{\phi(a_n),\phi(a_{n+1})\bigr\} + \varepsilon.$$

By the arbitrariness of ε, we have

$$a_{n+1} = E_\lambda(y_n,y_n,y_{n+1}) \le \max\bigl\{\phi(a_n),\phi(a_{n+1})\bigr\}. \tag{2.4}$$

So, we can infer that $a_{n+1} \le \phi(a_n)$. If not, then by (2.4), we have $a_{n+1} \le \phi(a_{n+1}) < a_{n+1}$, which is a contradiction. Hence, (2.4) implies that $a_{n+1} \le \phi(a_n)$, and (2.3) is proved.

Again and again using (2.3), we get

$$E_\lambda(y_n,y_n,y_{n+1}) \le \phi\bigl(E_\lambda(y_{n-1},y_{n-1},y_n)\bigr) \le \cdots \le \phi^n\bigl(E_\lambda(y_0,y_0,y_1)\bigr) \quad \text{for all } n \in \mathbb{N}.$$

By Lemma 2.4, for each $\lambda \in (0,1]$, there exists $\mu \in (0,\lambda]$ such that

$$E_\lambda(y_n,y_n,y_m) \le \sum_{i=n}^{m-1} E_\mu(y_i,y_i,y_{i+1}), \quad \forall m,n \in \mathbb{N} \text{ with } m > n. \tag{2.5}$$

Since $\phi \in \Phi$, by condition (Φ-3) we have $\sum_{n=0}^{\infty} \phi^n(E_\mu(y_0,y_0,y_1)) < +\infty$. So, for given $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $\sum_{i=n_0}^{\infty} \phi^i(E_\mu(y_0,y_0,y_1)) < \varepsilon$. Thus, it follows from (2.5) that

$$E_\lambda(y_n,y_n,y_m) \le \sum_{i=n}^{\infty} \phi^i\bigl(E_\mu(y_0,y_0,y_1)\bigr) < \varepsilon, \quad \forall n \ge n_0,$$

which implies that $G(y_n,y_n,y_m,\varepsilon) > 1-\lambda$ for all $m,n \in \mathbb{N}$ with $m > n \ge n_0$. Therefore, $\{y_n\}$ is a Cauchy sequence in X. □

3 Main results

Definition 3.1 [14] Let $(X,\le)$ be a partially ordered set. The mapping F is said to have the mixed monotone property if F is monotone non-decreasing in its first argument and is monotone non-increasing in its second argument; that is, for any $x,y \in X$,

$$x_1,x_2 \in X, \qquad x_1 \le x_2 \quad \Rightarrow \quad F(x_1,y) \le F(x_2,y), \tag{3.1}$$

and

$$y_1,y_2 \in X, \qquad y_1 \le y_2 \quad \Rightarrow \quad F(x,y_1) \ge F(x,y_2). \tag{3.2}$$

Definition 3.2 [14] An element $(x,y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X$ if

$$F(x,y) = x, \qquad F(y,x) = y.$$

Definition 3.3 [15] Let $(X, \leq)$ be a partially ordered set and $F : X \times X \to X$ and $g : X \to X$. We say F has the mixed g-monotone property if F is monotone g-non-decreasing in its first argument and is monotone g-non-increasing in its second argument; that is, for any $x, y \in X$,

$$x_1, x_2 \in X, \qquad g(x_1) \leq g(x_2) \quad \Longrightarrow \quad F(x_1, y) \leq F(x_2, y), \tag{3.3}$$

and

$$y_1, y_2 \in X, \qquad g(y_1) \leq g(y_2) \quad \Longrightarrow \quad F(x, y_1) \geq F(x, y_2). \tag{3.4}$$

Note that if g is the identity mapping, then Definition 3.3 reduces to Definition 3.1.

Example 3.1 Let $X = [-1, 1]$ with the natural ordering of real numbers. Let $g : X \to X$ and $F : X \times X \to X$ be defined as

$$g(x) = x^4, \qquad F(x, y) = x^2 - y^2.$$

Then F is not mixed monotone but mixed g-monotone.

Definition 3.4 [15] Let X be a nonempty set, $F : X \times X \to X$ and $g : X \to X$, then

(1) An element $(x, y) \in X \times X$ is called a coupled coincidence point of the mappings F and g if

$$F(x, y) = g(x), \qquad F(y, x) = g(y).$$

(2) An element $(x, y) \in X \times X$ is called a common coupled fixed point of the mappings F and g if

$$F(x, y) = g(x) = x, \qquad F(y, x) = g(y) = y.$$

Definition 3.5 The mappings $F : X \times X \to X$ and $g : X \to X$ are said to be compatible if

$$\lim_{n\to\infty} G\bigl(gF(x_n, y_n), gF(x_n, y_n), F\bigl(g(x_n), g(y_n)\bigr), t\bigr) = 1$$

and

$$\lim_{n\to\infty} G\bigl(gF(y_n, x_n), gF(y_n, x_n), F\bigl(g(y_n), g(x_n)\bigr), t\bigr) = 1$$

for all $t > 0$ whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X such that

$$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x, \qquad \lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y$$

for all $x, y \in X$ are satisfied.

Definition 3.6 [16] The mappings $F : X \times X \to X$ and $g : X \to X$ are called w-compatible if

$$g\big(F(x,y)\big) = F(gx, gy), \qquad g\big(F(y,x)\big) = F(gy, gx)$$

whenever $g(x) = F(x,y)$ and $g(y) = F(y,x)$ for some $(x,y) \in X \times X$.

Remark 3.1 It is easy to prove that if F and g are compatible then they are w-compatible.

Theorem 3.1 *Let $(X, \leq)$ be a partially ordered set and $(X, G, *)$ be a complete GF space. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings such that F has the mixed g-monotone property and there exists $\phi \in \Phi$ such that*

$$\begin{aligned} &G\big(F(x,y), F(x,y), F(u,v), \phi(t)\big) \\ &\quad \geq G(gx, gx, gu, t) * G\big(gx, gx, F(x,y), t\big) * G\big(gu, gu, F(u,v), t\big) \end{aligned} \tag{3.5}$$

for all $x, y, u, v \in X$, $t > 0$ for which $g(x) \leq g(u)$ and $g(y) \geq g(v)$, or $g(x) \geq g(u)$ and $g(y) \leq g(v)$.
Suppose $F(X \times X) \subseteq g(X)$, g is continuous and F and g are compatible. Also suppose
(a) *F is continuous or*
(b) *X has the following properties*:

$$\text{(i)} \quad \text{if a non-decreasing sequence } x_n \to x, \text{ then } x_n \leq x \text{ for all } n, \tag{3.6}$$

$$\text{(ii)} \quad \text{if a non-increasing sequence } y_n \to y, \text{ then } y_n \geq y \text{ for all } n. \tag{3.7}$$

If there exists $x_0, y_0 \in X$ such that $g(x_0) \leq F(x_0, y_0)$ and $g(y_0) \geq F(y_0, x_0)$, then there exist $x, y \in X$ such that $g(x) = F(x,y)$ and $g(y) = F(y,x)$; that is, F and g have a coupled coincidence point in X.

Proof Let $x_0, y_0 \in X$ be such that $g(x_0) \leq F(x_0, y_0)$ and $g(y_0) \geq F(y_0, x_0)$. Since $F(X \times X) \subseteq g(X)$, we can choose $x_1, y_1 \in X$ such that $g(x_1) = F(x_0, y_0)$ and $g(y_1) = F(y_0, x_0)$. Continuing in this way, we construct two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$g(x_{n+1}) = F(x_n, y_n), \qquad g(y_{n+1}) = F(y_n, x_n), \quad \text{for all } n \geq 0. \tag{3.8}$$

We shall show that

$$g(x_n) \leq g(x_{n+1}), \tag{3.9}$$

$$g(y_n) \geq g(y_{n+1}) \tag{3.10}$$

for all $n \geq 0$.

We shall use the mathematical induction. Let $n = 0$. Since $g(x_0) \leq F(x_0, y_0)$ and $g(y_0) \geq F(y_0, x_0)$, and as $g(x_1) = F(x_0, y_0)$ and $g(y_1) = F(y_0, x_0)$, we have $g(x_0) \leq g(x_1)$ and $g(y_0) \geq g(y_1)$. Thus, (3.9) and (3.10) hold for $n = 0$. Suppose now that (3.9) and (3.10) hold for some fixed $n \geq 0$. Then since $g(x_n) \leq g(x_{n+1})$ and $g(y_n) \geq g(y_{n+1})$, and as F has the mixed

g-monotone property, from (3.8) and (3.3),

$$\left.\begin{aligned} &g(x_{n+1}) = F(x_n, y_n) \leq F(x_{n+1}, y_n), \\ &F(y_{n+1}, x_n) \leq F(y_n, x_n) = g(y_{n+1}), \end{aligned}\right\} \tag{3.11}$$

and from (3.8) and (3.4),

$$\left.\begin{aligned} &g(x_{n+2}) = F(x_{n+1}, y_{n+1}) \geq F(x_{n+1}, y_n), \\ &F(y_{n+1}, x_n) \geq F(y_{n+1}, x_{n+1}) = g(y_{n+2}). \end{aligned}\right\} \tag{3.12}$$

Now from (3.11) and (3.12), we get $g(x_{n+1}) \leq g(x_{n+2})$ and $g(y_{n+1}) \geq g(y_{n+2})$. Thus, by mathematical induction, we conclude that (3.9) and (3.10) hold for all $n \geq 0$. Therefore,

$$g(x_0) \leq g(x_1) \leq g(x_2) \leq \cdots \leq g(x_n) \leq g(x_{n+1}) \leq \cdots \tag{3.13}$$

and

$$g(y_0) \geq g(y_1) \geq g(y_2) \geq \cdots \geq g(y_n) \geq g(y_{n+1}) \geq \cdots. \tag{3.14}$$

By putting ($x = x_{n-1}$, $y = y_{n-1}$, $u = x_n$, $v = y_n$) in (3.5), we get

$$\begin{aligned} &G\big(F(x_{n-1}, y_{n-1}), F(x_{n-1}, y_{n-1}), F(x_n, y_n), \phi(t)\big) \\ &\quad \geq G(gx_{n-1}, gx_{n-1}, gx_n, t) * G\big(gx_{n-1}, gx_{n-1}, F(x_{n-1}, y_{n-1}), t\big) * G\big(gx_n, gx_n, F(x_n, y_n), t\big). \end{aligned}$$

So, by (3.8), we have

$$\begin{aligned} &G\big(g(x_n), g(x_n), g(x_{n+1}), \phi(t)\big) \\ &\quad \geq G(gx_{n-1}, gx_{n-1}, gx_n, t) * G(gx_{n-1}, gx_{n-1}, gx_n, t) * G(gx_n, gx_n, gx_{n+1}, t) \\ &\quad = G(gx_{n-1}, gx_{n-1}, gx_n, t) * G(gx_n, gx_n, gx_{n+1}, t). \end{aligned}$$

Now, by Lemma 2.5, $\{g(x_n)\}$ is a Cauchy sequence.

By putting ($x = y_n$, $y = x_n$, $u = y_{n-1}$, $v = x_{n-1}$) in (3.5), we get

$$\begin{aligned} &G\big(F(y_{n-1}, x_{n-1}), F(y_{n-1}, x_{n-1}), F(y_n, x_n), \phi(t)\big) \\ &\quad \geq G(gy_{n-1}, gy_{n-1}, gy_n, t) * G\big(gy_{n-1}, gy_{n-1}, F(y_{n-1}, x_{n-1}), t\big) * G\big(gy_n, gy_n, F(y_n, x_n), t\big). \end{aligned}$$

So, by (3.8), we have

$$\begin{aligned} &G\big(gy_n, gy_n, gy_{n+1}, \phi(t)\big) \\ &\quad \geq G(gy_{n-1}, gy_{n-1}, gy_n, t) * G(gy_{n-1}, gy_{n-1}, gy_n, t) * G(gy_n)gy_n, gy_{n+1}, t) \\ &\quad = G(gy_{n-1}, gy_{n-1}, gy_n, t) * G(gy_n, gy_n, gy_{n+1}, t). \end{aligned}$$

Now, by Lemma 2.5, $\{g(y_n)\}$ is also a Cauchy sequence.

Since X is complete, there exist $x, y \in X$ such that

$$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x, \qquad \lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y. \tag{3.15}$$

Since F and g are compatible, we have by (3.15)

$$\lim_{n\to\infty} G\big(g\big(F(x_n, y_n)\big), g\big(F(x_n, y_n)\big), F\big(g(x_n), g(y_n)\big), t\big) = 1 \tag{3.16}$$

and

$$\lim_{n\to\infty} G\big(g\big(F(y_n, x_n)\big), g\big(F(y_n, x_n)\big), F\big(g(y_n), g(x_n)\big), t\big) = 1 \tag{3.17}$$

for all $t > 0$. Next, we prove that $g(x) = F(x, y)$ and $g(y) = F(y, x)$.

Let (a) hold. Since F and g are continuous, by Lemma 2.2, taking limits as $n \to \infty$ in (3.16) and (3.17), we get

$$G\big(g(x), g(x), F(x, y), t\big) = 1, \qquad G\big(g(y), g(y), F(y, x), t\big) = 1$$

for all $t > 0$. We have $g(x) = F(x, y)$, $g(y) = F(y, x)$.

Next, we suppose that (b) holds. By (3.9), (3.10), (3.15), we have for all $n \geq 0$

$$g(x_n) \leq x, \qquad g(y_n) \geq y. \tag{3.18}$$

Since F and g are compatible and g is continuous, by (3.16) and (3.17), we have

$$\lim_{n\to\infty} g(gx_n) = gx = \lim_{n\to\infty} g\big(F(x_n, y_n)\big) = \lim_{n\to\infty} F\big(g(x_n), g(y_n)\big) \tag{3.19}$$

and

$$\lim_{n\to\infty} g(gy_n) = gy = \lim_{n\to\infty} g\big(F(y_n, x_n)\big) = \lim_{n\to\infty} F\big(g(y_n), g(x_n)\big). \tag{3.20}$$

Now, we have

$$\begin{aligned} G\big(gx, gx, F(x, y), \phi(t)\big) \geq{}& G\big(gx, gx, g(gx_{n+1}), \phi(t) - \phi(kt)\big) \\ &* G\big(g(gx_{n+1}), g(gx_{n+1}), F(x, y), \phi(kt)\big) \end{aligned}$$

for all $0 \leq k < 1$. Taking the limit as $n \to \infty$ in the above inequality, by continuity of G, using (3.8) and (3.19), we have

$$\begin{aligned} &G\big(gx, gx, F(x, y), \phi(t)\big) \\ &\quad\geq \lim_{n\to\infty} \big\{G\big(gx, gx, g(gx_{n+1}), \phi(t) - \phi(kt)\big) \\ &\qquad * G\big(g\big(F(x_n, y_n)\big), g\big(F(x_n, y_n)\big), F(x, y), \phi(kt)\big)\big\} \\ &\quad\geq \lim_{n\to\infty} G\big(F(gx_n, gy_n), F(gx_n, gy_n), F(x, y), \phi(kt)\big). \end{aligned}$$

By (3.5), (3.19) and the above inequality, we have that

$$\begin{aligned} &G\bigl(gx, gx, F(x,y), \phi(t)\bigr) \\ &\quad\geq \lim_{n\to\infty}\bigl\{G\bigl(g(gx_n), g(gx_n), gx, kt\bigr) \\ &\qquad * G\bigl(g(gx_n), g(gx_n), F(gx_n, gy_n), kt\bigr) * G\bigl(gx, gx, F(x,y), kt\bigr)\bigr\} \\ &\quad\geq G\bigl(gx, gx, F(x,y), kt\bigr). \end{aligned}$$

Letting $k \to 1$, which implies that $gx = F(x,y)$ by Lemma 2.3, and similarly, by the virtue of (3.8), (3.15) and (3.20), we get $gy = F(y,x)$. Thus, we have proved that F and g have a coupled coincidence point in X.

This completes the proof of Theorem 3.1. □

Taking $g = I$ (the identity mapping) in Theorem 3.1, we get the following consequence.

Corollary 3.1 *Let $(X,\leq)$ be a partially ordered set and $(X, G, *)$ be a complete GF space. Let $F : X \times X \to X$ be a mapping such that F has the mixed monotone property and there exists $\phi \in \Phi$ such that*

$$G\bigl(F(x,y), F(x,y), F(u,v), \phi(t)\bigr) \geq G(x,x,u,t) * G\bigl(x,x,F(x,y),t\bigr) * G\bigl(u,u,F(u,v),t\bigr)$$

for all $x, y, u, v \in X$, $t > 0$ for which $x \leq u$ and $y \geq v$. Suppose

(a) *F is continuous or*

(b) *X has the following properties:*

  (i) *if a non-decreasing sequence $x_n \to x$, then $x_n \leq x$ for all n,*

  (ii) *if a non-increasing sequence $y_n \to y$, then $y_n \geq y$ for all n.*

If there exists $x_0, y_0 \in X$ such that $x_0 \leq F(x_0, y_0)$ and $y_0 \geq F(y_0, x_0)$, then there exist $x, y \in X$ such that $x = F(x,y)$ and $y = F(y,x)$; that is, F has a coupled fixed point in X.

Now, we shall prove the existence and uniqueness theorem of a coupled common fixed point. Note that if $(S,\leq)$ is a partially ordered set, then we endow the product $S \times S$ with the following partial order:

$$\text{for } (x,y),(u,v) \in S \times S, \quad (x,y) \leq (u,v) \quad \Leftrightarrow \quad x \leq u, \qquad y \geq v.$$

Theorem 3.2 *In addition to the hypotheses of Theorem 3.1, suppose that for every $(x,y), (x^\star, y^\star) \in X \times X$, there exists a $(u,v) \in X \times X$ satisfying $g(u) \leq g(v)$ or $g(v) \leq g(u)$ such that $(F(u,v), F(v,u)) \in X \times X$ is comparable to $(F(x,y), F(y,x))$, $(F(x^\star,y^\star), F(y^\star,x^\star))$. Then F and g have a unique common coupled fixed point; that is, there exists a unique $(x,y) \in X \times X$ such that*

$$x = g(x) = F(x,y), \qquad y = g(y) = F(y,x).$$

Proof From Theorem 3.1, the set of coupled coincidence points is nonempty. We shall show that if (x,y) and $(x^\star, y^\star)$ are coupled coincidence points, that is, if $g(x) = F(x,y)$, $g(y) = F(y,x)$ and $g(x^\star) = F(x^\star, y^\star)$, $g(y^\star) = F(y^\star, x^\star)$, then

$$g(x) = g\bigl(x^\star\bigr), \qquad g(y) = g\bigl(y^\star\bigr). \tag{3.21}$$

By assumption, there is $(u,v) \in X \times X$ such that $(F(u,v),F(v,u))$ is comparable with $(F(x,y),F(y,x))$, $(F(x^\star,y^\star),F(y^\star,x^\star))$. Put $u_0 = u$, $v_0 = v$ and choose $u_1, v_1 \in X$ so that $g(u_1) = F(u_0,v_0)$ and $g(v_1) = F(v_0,u_0)$. Then, similarly as in the proof of Theorem 3.1, we can inductively define sequences $\{g(u_n)\}$ and $\{g(v_n)\}$ such that

$$g(u_{n+1}) = F(u_n,v_n), \qquad g(v_{n+1}) = F(v_n,u_n).$$

With the similar proof as in Theorem 3.1, we can prove that the limits of $\{g(u_n)\}$ and $\{g(v_n)\}$ exist.

Since $(F(x,y),F(y,x)) = (g(x_1),g(y_1)) = (g(x),g(y))$ and $(F(u,v),F(v,u)) = (g(u_1),g(v_1))$ are comparable, it is easy to show that $(g(x),g(y))$ and $(g(u_n),g(v_n))$ are comparable for all $n \geq 1$. Thus, from (3.5),

$$\begin{aligned}
&G\bigl(g(x),g(x),g(u_{n+1}),\phi(t)\bigr)\\
&\quad = G\bigl(F(x,y),F(x,y),F(u_n,v_n),\phi(t)\bigr)\\
&\quad \geq G\bigl(g(x),g(x),g(u_n),t\bigr) * G\bigl(g(x),g(x),F(x,y),t\bigr) * G\bigl(g(u_n),g(u_n),F(u_n,v_n),t\bigr)\\
&\quad \geq G\bigl(g(x),g(x),g(u_n),t\bigr) * G\bigl(g(u_n),g(u_n),F(u_n,v_n),t\bigr)\\
&G\bigl(g(y),g(y),g(v_{n+1}),\phi(t)\bigr)\\
&\quad = G\bigl(F(y,x),F(y,x),F(v_n,u_n),\phi(t)\bigr)\\
&\quad \geq G\bigl(g(y),g(y),g(v_n),t\bigr) * G\bigl(g(y),g(y),F(y,x),t\bigr) * G\bigl(g(v_n),g(v_n),F(v_n,u_n),t\bigr)\\
&\quad \geq G\bigl(g(y),g(y),g(v_n),t\bigr) * G\bigl(g(v_n),g(v_n),F(v_n,u_n),t\bigr)
\end{aligned}$$

for each $n \geq 1$. Letting $n \to \infty$, we get

$$\lim_{n\to\infty} g(u_n) = g(x), \qquad \lim_{n\to\infty} g(v_n) = g(y). \tag{3.22}$$

Similarly, one can prove that

$$\lim_{n\to\infty} g(u_n) = g\bigl(x^\star\bigr), \qquad \lim_{n\to\infty} g(v_n) = g\bigl(y^\star\bigr). \tag{3.23}$$

By (3.22) and (3.23), we have

$$G\bigl(gx,gx,gx^\star,t\bigr) \geq G\left(gx,gx,gu_{n+1},\frac{t}{2}\right) * G\left(gu_{n+1},gu_{n+1},gx^\star,\frac{t}{2}\right) \to 1 \quad (n \to \infty),$$

which shows that $g(x) = g(x^\star)$.

Similarly, one can prove that $g(y) = g(y^\star)$. Thus, we proved (3.21).

Since $g(x) = F(x,y)$ and $g(y) = F(y,x)$, by the compatibility of F and g, we can get the w-compatibility of F and g, which implies

$$g\bigl(g(x)\bigr) = g\bigl(F(x,y)\bigr) = F\bigl(g(x),g(y)\bigr), \tag{3.24}$$

and

$$g\bigl(g(y)\bigr) = g\bigl(F(y,x)\bigr) = F\bigl(g(y),g(x)\bigr). \tag{3.25}$$

Denote $g(x) = z$, $g(y) = w$. Then from (3.24) and (3.25),

$$g(z) = F(z,w), \qquad g(w) = F(w,z). \tag{3.26}$$

Thus, (z,w) is a coupled coincidence point. From (3.21) with $x^\star = z$, $y^\star = w$, it also follows $g(z) = g(x)$, $g(w) = g(y)$, that is,

$$g(z) = z, \qquad g(w) = w. \tag{3.27}$$

From (3.26) and (3.27), we get

$$z = g(z) = F(z,w), \qquad w = g(w) = F(w,z).$$

Therefore, (z,w) is a common coupled fixed point of F and g. To prove the uniqueness, assume that (p,q) is another coupled common fixed point. Then by (3.21) we have $p = g(p) = g(z) = z$ and $q = g(q) = g(w) = w$. □

From Remark 2.3, let $(X,G,*)$ be a symmetric GF space. From Theorem 3.1, we get the following

Corollary 3.2 *Let $(X,\leq)$ be a partially ordered set and $(X,F,*)$ be a complete fuzzy metric space. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings such that F has the mixed g-monotone property and there exists $\phi \in \Phi$ such that*

$$M\big(F(x,y),F(u,v),\phi(t)\big) \geq M(gx,gu,t) * M\big(gx,F(x,y),t\big) * M\big(gu,F(u,v),t\big)$$

for all $x,y,u,v \in X$, $t > 0$, for which $g(x) \leq g(u)$ and $g(y) \geq g(v)$, or $g(x) \geq g(u)$ and $g(y) \leq g(v)$.

Suppose $F(X \times X) \subseteq g(X)$, g is continuous and F and g are compatible. Also suppose

(a) *F is continuous or*

(b) *X has the following properties*:

  (i) *if a non-decreasing sequence $x_n \to x$, then $x_n \leq x$ for all n,*

  (ii) *if a non-increasing sequence $y_n \to y$, then $y_n \geq y$ for all n.*

If there exist $x_0, y_0 \in X$ such that $g(x_0) \leq F(x_0,y_0)$ and $g(y_0) \geq F(y_0,x_0)$, then there exist $x,y \in X$ such that $g(x) = F(x,y)$ and $g(y) = F(y,x)$, that is, F and g have a coupled coincidence point in X.

Remark 3.2 Compared with the results in [15, 16], we can find that Theorem 3.1 is different in the following aspects:

(1) We assume that F and g are compatible, which is weaker than the conditions in [15, 16], where Theorem 2.1 in [15] assumes commutation for F and g, and Theorem 3.1 in [16] requires g to be a monotone function.

(2) We have a different contractive condition from [15, 16] even in a metric space.

(3) In our paper, we assume that $\phi \in \Phi$, which is a stronger condition than that in [15, 16]. But we would like to point out that in the case of $\phi(t) = kt$, where $0 < k < 1$, the two conditions are equivalent.

Next, we give an example to demonstrate Theorem 3.1.

Example 3.2 Let $X = [0,1]$, $a * b = \min\{a,b\}$. Then $(X,\leq)$ is a partially ordered set with the natural ordering of real numbers. Let

$$G(x,y,z,t) = \frac{t}{t + |x-y| + |y-z| + |z-x|}$$

for all $x,y,z \in [0,1]$. Then $(X,G,*)$ is a complete GF space.

Let $g : X \to X$ and $F : X \times X \to X$ be defined as

$$g(x) = x^2, \quad \text{for all } x \in X,$$
$$F(x,y) = \begin{cases} \frac{x^2-y^2}{3}, & \text{if } x,y \in [0,1], x \geq y, \\ 0, & \text{if } x < y. \end{cases}$$

F obeys the mixed g-monotone property.

Let $\phi(t) = \frac{t}{3}$ for $t \in [0,\infty)$. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X such that

$$\lim_{n\to\infty} F(x_n,y_n) = a, \qquad \lim_{n\to\infty} g(x_n) = a, \qquad \lim_{n\to\infty} F(y_n,x_n) = b, \qquad \lim_{n\to\infty} g(y_n) = b,$$

then $a = 0$, $b = 0$. Now, for all $n \geq 0$,

$$g(x_n) = x_n^2, \qquad g(y_n) = y_n^2,$$
$$F(x_n,y_n) = \begin{cases} \frac{x_n^2-y_n^2}{3}, & \text{if } x_n \geq y_n, \\ 0, & \text{if } x_n < y_n \end{cases}$$

and

$$F(y_n,x_n) = \begin{cases} \frac{y_n^2-x_n^2}{3}, & \text{if } y_n \geq x_n, \\ 0, & \text{if } y_n < x_n. \end{cases}$$

Then it follows that

$$G\big(g\big(F(x_n,y_n)\big), g\big(F(x_n,y_n)\big), F\big(g(x_n), g(y_n)\big), t\big) \to 1 \quad \text{as } n \to \infty,$$
$$G\big(g\big(F(y_n,x_n)\big), g\big(F(y_n,x_n)\big), F\big(g(y_n), g(x_n)\big), t\big) \to 1 \quad \text{as } n \to \infty.$$

Hence, the mappings F and g are compatible in X. Also, $x_0 = 0$ and $y_0 = c$ are two points in X such that

$$g(x_0) = g(0) = F(0,c) = F(x_0,y_0)$$

and

$$g(y_0) = g(c) = c^2 \geq \frac{c^2}{3} = F(c,0) = F(y_0,x_0).$$

We next verify the inequality of Theorem 3.1. We take $x, y, u, v \in X$ such that $g(x) \leq g(u)$ and $g(y) \geq g(v)$, that is, $x^2 \leq u^2$, $y^2 \geq v^2$.

We consider the following cases:

Case 1: $x \geq y$ and $u \geq v$, then

$$\begin{aligned} G\big(F(x,y),F(x,y),F(u,v),\phi(t)\big) &= G\left(\frac{x^2-y^2}{3},\frac{x^2-y^2}{3},\frac{u^2-v^2}{3},\phi(t)\right) \\ &= \frac{\frac{t}{3}}{\frac{t}{3}+|\frac{(x^2-u^2)-(y^2-v^2)}{3}|} \\ &= \frac{t}{t+|(x^2-u^2)-(y^2-v^2)|} \\ &\geq \frac{t}{t+|u^2-\frac{u^2-v^2}{3}|} \\ &= G\big(g(u),g(u),F(u,v),t\big) \\ &\geq G\big(g(x),g(x),g(u),t\big) * G\big(g(x),g(x),F(x,y),t\big) \\ &\quad * G\big(g(u),g(u),F(u,v),t\big). \end{aligned}$$

Case 2: $x \geq y$, $u < v$. Since $x \leq u$, then $u < v$ cannot happen.

Case 3: $x < y$ and $u \geq v$, then

$$\begin{aligned} G\big(F(x,y),F(x,y),F(u,v),\phi(t)\big) &= G\left(0,0,\frac{u^2-v^2}{3},\phi(t)\right) \\ &= \frac{\frac{t}{3}}{\frac{t}{3}+|\frac{(u^2-v^2)}{3}|} \\ &= \frac{t}{t+|u^2-v^2|} \\ &\geq \frac{t}{t+2|u^2-x^2|} \\ &= G\big(g(x),g(x),g(u),t\big) \\ &\geq G\big(g(x),g(x),g(u),t\big) * G\big(g(x),g(x),F(x,y),t\big) \\ &\quad * G\big(g(u),g(u),F(u,v),t\big). \end{aligned}$$

Case 4: $x < y$ and $u < v$ with $x^2 \leq u^2$ and $y^2 \geq v^2$, then $F(x,y) = 0$ and $F(u,v) = 0$, that is, $G(F(x,y),F(x,y),F(u,v),\phi(t)) = 0$. Obviously, (3.5) is satisfied.

Thus, it is verified that the functions F, g, ϕ satisfy all the conditions of Theorem 3.1. Here $(0,0)$ is the coupled coincidence point of F and g in X, which is also their common coupled fixed point.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

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