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On fixed points of α-ψ-contractive multifunctions

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Abstract

Recently Samet, Vetro and Vetro introduced the notion of α-ψ-contractive type mappings and established some fixed point theorems in complete metric spaces. In this paper, we introduce the notion of α_*-ψ-contractive multifunctions and give a fixed point result for these multifunctions. We also obtain a fixed point result for self-maps in complete metric spaces satisfying a contractive condition.

Keywords: α_*-ψ-contractive multifunction; fixed point; partial metric

1 Introduction

Fixed point theory has many applications in different branches of science. During the last few decades, there has been a lot of activity in this area and several well-known fixed point theorems have been extended by a number of authors in different directions (see, for example, [1–38]). Recently Samet, Vetro and Vetro introduced the notion of α-ψ-contractive type mappings [33]. Denote with Ψ the family of nondecreasing functions $\psi : [0,\infty) \to [0,\infty)$ such that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for all $t > 0$, where ψ^n is the nth iterate of ψ. It is known that $\psi(t) < t$ for all $t > 0$ and $\psi \in \Psi$ [33]. Let (X,d) be a metric space, T be a self-map on X, $\psi \in \Psi$ and $\alpha : X \times X \to [0,\infty)$ be a function. Then T is called an α-ψ-contraction mapping whenever $\alpha(x,y)d(Tx,Ty) \le \psi(d(x,y))$ for all $x,y \in X$. Also, we say that T is α-admissible whenever $\alpha(x,y) \ge 1$ implies $\alpha(Tx,Ty) \ge 1$ [33]. Also, we say that α has the property (B) if $\{x_n\}$ is a sequence in X such that $\alpha(x_n,x_{n+1}) \ge 1$ for all $n \ge 1$ and $x_n \to x$, then $\alpha(x_n,x) \ge 1$ for all $n \ge 1$. Let (X,d) be a complete metric space and T be an α-admissible α-ψ-contractive mapping on X. Suppose that there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0) \ge 1$. If T is continuous or T has the property (B), then T has a fixed point (see [33]; Theorems 2.1 and 2.2). Finally, we say that X has the property (H) whenever for each $x,y \in X$ there exists $z \in X$ such that $\alpha(x,z) \ge 1$ and $\alpha(y,z) \ge 1$. If X has the property (H) in Theorems 2.1 and 2.2, then X has a unique fixed point ([33]; Theorem 2.3). It is considerable that the results of Samet *et al.* generalize similar ordered results in the literature (see the results of the third section in [33]). The aim of this paper is to introduce the notion of α_*-ψ-contractive multifunctions and give a fixed point result about the multifunctions. Let (X,d) be a metric space, $T : X \to 2^X$ be a closed-valued multifunction, $\psi \in \Psi$ and $\alpha : X \times X \to [0,\infty)$ be a function. In this case, we say that T is an α_*-ψ-contractive multifunction whenever $\alpha_*(Tx,Ty)H(Tx,Ty) \le \psi(d(x,y))$ for $x,y \in X$, where H is the Hausdorff generalized metric, $\alpha_*(A,B) = \inf\{\alpha(a,b) : a \in A, b \in B\}$ and 2^X denotes the family of all nonempty subsets of X. Also, we say that T is α_*-admissible whenever $\alpha(x,y) \ge 1$ implies $\alpha_*(Tx,Ty) \ge 1$.

Example 1.1 Let $X = [0,\infty)$, $d(x,y) = |x-y|$ and $\delta \in (0,1)$ be a fixed number. Define $T : X \to 2^X$ by $Tx = [0, \delta x]$ for all $x \in X$ and $\alpha : X \times X \to [0,\infty)$ by $\alpha(x,y) = 1$ whenever $x, y \in [0,1]$ and $\alpha(x,y) = 0$ whenever $x \notin [0,1]$ or $y \notin [0,1]$. Now, we show that T is α_*-admissible. If $\alpha(x,y) \geq 1$, then $x, y \in [0,1]$ and so Tx and Ty are subsets of $[0,1]$. Thus, $a, b \in [0,1]$ for all $a \in Tx$ and $b \in Ty$. Hence, $\alpha(a,b) = 1$ for all $a \in Tx$ and $b \in Ty$. This implies that

$$\alpha_*(Tx, Ty) = \inf\{\alpha(a,b) : a \in Tx, b \in Ty\} = 1.$$

Therefore, T is α_*-admissible. Now, we show that T is an α_*-ψ-contractive multifunction, where $\psi(t) = \delta t$ for all $t \geq 0$. If $x \notin [0, \frac{1}{\delta}]$ or $y \notin [0, \frac{1}{\delta}]$, then an easy calculation shows us that $\alpha_*(Tx, Ty) = 0$. If $0 \leq x, y \leq \frac{1}{\delta}$, then $\alpha_*(Tx, Ty) = 1$. By using the definition of the Hausdorff metric, it is easy to see that $H(Tx, Ty) \leq \delta d(x,y)$ for $x, y \in [0, \frac{1}{\delta}]$. Thus, $\alpha_*(Tx, Ty)H(Tx, Ty) \leq \psi(d(x,y))$ for $x, y \in X$. Therefore, T is an α_*-ψ-contractive multifunction.

Let $(X, \preceq, d)$ be an ordered metric space and $A, B \subseteq X$. We say that $A \preceq B$ whenever for each $a \in A$ there exists $b \in B$ such that $a \preceq b$. Also, we say that $A \preceq_r B$ whenever for each $a \in A$ and $b \in B$ we have $a \preceq b$.

2 Main results

Now, we are ready to state and prove our main results. In the following result, we use the argument similar to that in the proof of Theorem 3.1 in [22].

Theorem 2.1 *Let (X,d) be a complete metric space, $\alpha : X \times X \to [0,\infty)$ be a function, $\psi \in \Psi$ be a strictly increasing map and T be a closed-valued, α_*-admissible and α_*-ψ-contractive multifunction on X. Suppose that there exist $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$. Assume that if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x$, then $\alpha(x_n, x) \geq 1$ for all n. Then T has a fixed point.*

Proof If $x_1 = x_0$, then we have nothing to prove. Let $x_1 \neq x_0$. If $x_1 \in Tx_1$, then x_1 is a fixed point of T. Let $x_1 \notin Tx_1$ and $q > 1$ be given. Then

$$0 < d(x_1, Tx_1) \leq \alpha_*(Tx_0, Tx_1)H(Tx_0, Tx_1) < q\alpha_*(Tx_0, Tx_1)H(Tx_0, Tx_1).$$

Hence, there exists $x_2 \in Tx_1$ such that

$$0 < d(x_1, x_2) < q\alpha_*(Tx_0, Tx_1)H(Tx_0, Tx_1) \leq q\psi\big(d(x_0, x_1)\big).$$

It is clear that $x_2 \neq x_1$ and $\alpha(x_1, x_2) \geq 1$. Thus, $\alpha_*(Tx_1, Tx_2) \geq 1$. Now, put $t_0 = d(x_0, x_1)$. Then, $t_0 > 0$ and $d(x_1, x_2) < q\psi(t_0)$. Since ψ is strictly increasing, $\psi(d(x_1, x_2)) < \psi(q\psi(t_0))$. Put $q_1 = \frac{\psi(q\psi(t_0))}{\psi(d(x_1,x_2))}$. Then $q_1 > 1$. If $x_2 \in Tx_2$, then x_2 is a fixed point of T. Assume that $x_2 \notin Tx_2$. Then

$$0 < d(x_2, Tx_2) \leq \alpha_*(Tx_1, Tx_2)H(Tx_1, Tx_2) < q_1\alpha_*(Tx_1, Tx_2)H(Tx_1, Tx_2).$$

Hence, there exists $x_3 \in Tx_2$ such that

$$0 < d(x_2, x_3) < q_1\alpha_*(Tx_1, Tx_2)H(Tx_1, Tx_2) \leq q_1\psi\big(d(x_1, x_2)\big) = \psi\big(q\psi(t_0)\big).$$

It is clear that $x_3 \neq x_2$, $\alpha(x_2,x_3) \geq 1$ and $\psi(d(x_2,x_3)) < \psi^2(q\psi(t_0))$. Now, put $q_2 = \frac{\psi^2(q\psi(t_0))}{\psi(d(x_2,x_3))}$. Then $q_2 > 1$. If $x_3 \in Tx_3$, then x_3 is a fixed point of T. Assume that $x_3 \notin Tx_3$. Then

$$0 < d(x_3, Tx_3) \leq \alpha_*(Tx_2, Tx_3)H(Tx_2, Tx_3) < q_2\alpha_*(Tx_2, Tx_3)H(Tx_2, Tx_3).$$

Thus, there exists $x_4 \in Tx_3$ such that

$$0 < d(x_3,x_4) < q_1\alpha_*(Tx_2, Tx_3)H(Tx_2, Tx_3) \leq q_2\psi\bigl(d(x_2,x_3)\bigr) = \psi^2\bigl(q\psi(t_0)\bigr).$$

By continuing this process, we obtain a sequence $\{x_n\}$ in X such that $x_n \in Tx_{n-1}$, $x_n \neq x_{n-1}$, $\alpha(x_n,x_{n+1}) \geq 1$ and $d(x_n,x_{n+1}) \leq \psi^{n-1}(q\psi(t_0))$ for all n. Now, for each $m > n$, we have

$$d(x_n,x_m) \leq \sum_{i=n}^{m-1} d(x_i,x_{i+1}) \leq \sum_{i=n}^{m-1} \psi^{i-1}\bigl(q\psi(t_0)\bigr).$$

Hence, $\{x_n\}$ is a Cauchy sequence in X. Choose $x^* \in X$ such that $x_n \to x^*$. Since $\alpha(x_n,x^*) \geq 1$ for all n and T is α_*-admissible, $\alpha_*(Tx_n, Tx^*) \geq 1$ for all n, thus

$$\begin{aligned} d\bigl(x^*, Tx^*\bigr) &\leq H\bigl(Tx^*, Tx_n\bigr) + d\bigl(x_{n+1},x^*\bigr) \leq \alpha_*\bigl(Tx_n, Tx^*\bigr)H\bigl(Tx_n, Tx^*\bigr) + d\bigl(x_{n+1},x^*\bigr) \\ &\leq \psi\bigl(d\bigl(x_n,x^*\bigr)\bigr) + d\bigl(x_{n+1},x^*\bigr) \end{aligned}$$

for all n. Therefore, $d(x^*, Tx^*) = 0$ and so $x^* \in Tx^*$. □

Example 2.1 Let $X = [0,\infty)$ and $d(x,y) = |x-y|$. Define $T : X \to 2^X$ by $Tx = [2x - \frac{3}{2},\infty)$ for all $x > 1$, $Tx = [0, \frac{x}{2}]$ for all $0 \leq x \leq 1$ and $\alpha : X \times X \to [0,\infty)$ by $\alpha(x,y) = 1$ whenever $x,y \in [0,1]$ and $\alpha(x,y) = 0$ whenever $x \notin [0,1]$ or $y \notin [0,1]$. Then it is easy to check that T is an α_*-admissible and α_*-ψ-contractive multifunction, where $\psi(t) = \frac{t}{2}$ for all $t \geq 0$. Put $x_0 = 1$ and $x_1 = \frac{1}{2}$. Then $\alpha(x_0,x_1) \geq 1$. Also, if $\{x_n\}$ is a sequence in X such that $\alpha(x_n,x_{n+1}) \geq 1$ for all n and $x_n \to x$, then $\alpha(x_n,x) \geq 1$ for all n. Note that T has infinitely many fixed points.

Corollary 2.2 *Let $(X,\preceq,d)$ be a complete ordered metric space, $\psi \in \Psi$ be a strictly increasing map and T be a closed-valued multifunction on X such that*

$$H(Tx, Ty) \leq \psi\bigl(d(x,y)\bigr)$$

for all $x,y \in X$ with $x \preceq y$. Suppose that there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $x_0 \preceq x_1$. Assume that if $\{x_n\}$ is a sequence in X such that $x_n \preceq x_{n+1}$ for all n and $x_n \to x$, then $x_n \preceq x$ for all n. If $x \preceq y$ implies $Tx \preceq_r Ty$, then T has a fixed point.

Proof Define $\alpha : X \times X \to [0,\infty)$ by $\alpha(x,y) = 1$ whenever $x \preceq y$ and $\alpha(x,y) = 0$ whenever $x \npreceq y$. Since $x \preceq y$ implies $Tx \preceq_r Ty$, $\alpha(x,y) = 1$ implies $\alpha_*(Tx,Ty) = 1$. Thus, it is easy to check that T is an α_*-admissible and α_*-ψ-contractive multifunction on X. Now, by using Theorem 2.1, T has a fixed point. □

Now, we prove the following result for self-maps.

Theorem 2.3 *Let (X,d) be a complete metric space, $\alpha : X \times X \to [0,\infty)$ be a function, $\psi \in \Psi$ and T be a self-map on X such that $\alpha(x,y)d(Tx,Ty) \leq \psi(m(x,y))$ for all $x,y \in X$, where $m(x,y) = \max\{d(x,y), d(x,Tx), d(y,Ty), \frac{1}{2}[d(x,Ty) + d(y,Tx)]\}$. Suppose that T is α-admissible and there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Assume that if $\{x_n\}$ is a sequence in X such that $\alpha(x_n,x_{n+1}) \geq 1$ for all n and $x_n \to x$, then $\alpha(x_n,x) \geq 1$ for all n. Then T has a fixed point.*

Proof Take $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$ and define the sequence $\{x_n\}$ in X by $x_{n+1} = Tx_n$ for all $n \geq 0$. If $x_n = x_{n+1}$ for some n, then $x^* = x_n$ is a fixed point of T. Assume that $x_n \neq x_{n+1}$ for all n. Since T is α-admissible, it is easy to check that $\alpha(x_n,x_{n+1}) \geq 1$ for all natural numbers n. Thus, for each natural number n, we have

$$\begin{aligned} d(x_n,x_{n+1}) &= d(Tx_{n-1},Tx_n) \leq \alpha(x_{n-1},x_n)d(Tx_{n-1},Tx_n) \\ &\leq \psi\left(\max\left\{d(x_n,x_{n-1}), d(x_n,x_{n+1}), d(x_{n-1},x_n), \frac{1}{2}\big[d(x_n,x_n) + d(x_{n-1},x_{n+1})\big]\right\}\right) \\ &\leq \psi\left(\max\left\{d(x_n,x_{n-1}), d(x_n,x_{n+1}), \frac{1}{2}\big[d(x_n,x_{n-1}) + d(x_n,x_{n+1})\big]\right\}\right) \\ &= \psi\big(\max\big\{d(x_n,x_{n-1}), d(x_n,x_{n+1})\big\}\big). \end{aligned}$$

If $\max\{d(x_n,x_{n-1}), d(x_n,x_{n+1})\} = d(x_n,x_{n+1})$, then

$$d(x_{n+1},x_n) \leq \psi\big(d(x_n,x_{n+1})\big) < d(x_{n+1},x_n)$$

which is contradiction. Thus, $\max\{d(x_n,x_{n-1}), d(x_n,x_{n-1})\} = d(x_n,x_{n+1})$ for all n. Hence, $d(x_{n+1},x_n) \leq \psi(d(x_n,x_{n-1}))$ and so $d(x_{n+1},x_n) \leq \psi^n(d(x_1,x_0))$ for all n. It is easy to check that $\{x_n\}$ is a Cauchy sequence. Thus, there exists $x^* \in X$ such that $x_n \to x^*$. By using the assumption, we have $\alpha(x_n,x^*) \geq 1$ for all n. Thus,

$$\begin{aligned} d\big(Tx^*,x^*\big) &\leq d\big(Tx^*,Tx_n\big) + d\big(x_{n+1},x^*\big) \leq \alpha\big(x_n,x^*\big)d\big(Tx^*,Tx_n\big) + d\big(x_{n+1},x^*\big) \\ &\leq \psi\left(\max\left\{d\big(x_n,x^*\big), d(x_n,x_{n+1}), d\big(x^*,Tx^*\big),\right.\right. \\ &\qquad \left.\left.\frac{1}{2}\big[d\big(x_n,Tx^*\big) + d\big(x^*,x_{n+1}\big)\big]\right\}\right) + d\big(x_{n+1},x^*\big) \\ &\leq \psi\big(d\big(x^*,Tx^*\big)\big) + d\big(x_{n+1},x^*\big) \end{aligned}$$

for sufficiently large n. Hence, $d(Tx^*,x^*) = 0$ and so $Tx^* = x^*$. □

Example 2.2 Let $X = [0,\infty)$ and $d(x,y) = |x-y|$. Define the self-map T on X by $Tx = 2x - \frac{5}{3}$ for $x > 1$, $Tx = \frac{x}{3}$ for $0 \leq x \leq 1$ and $\alpha : X \times X \to [0,\infty)$ by $\alpha(x,y) = 1$ whenever $x,y \in [0,1]$ and $\alpha(x,y) = 0$ whenever $x \notin [0,1]$ or $y \notin [0,1]$. Then it is easy to check that T is α-admissible and $\alpha(x,y)d(Tx,Ty) \leq \psi(m(x,y))$ for all $x,y \in X$, where $\psi(t) = \frac{t}{3}$ for all $t \geq 0$. Also, $\alpha(1,T1) = 1$ and if $\{x_n\}$ is a sequence in X such that $\alpha(x_n,x_{n+1}) \geq 1$ for all n and $x_n \to x$, then $\alpha(x_n,x) \geq 1$ for all n. Note that, T has two fixed points.

Corollary 2.4 *Let $(X,\preceq,d)$ be a complete ordered metric space, $\psi \in \Psi$ and T be a self-map on X such that $d(Tx,Ty) \leq \psi(m(x,y))$ for all $x,y \in X$ with $x \preceq y$. Suppose that there* *exists $x_0 \in X$ such that $x_0 \preceq Tx_0$. If $\{x_n\}$ is a sequence in X such that $x_n \preceq x_{n+1}$ for all n and $x_n \to x$, then $x_n \preceq x$ for all n. If $x \preceq y$ implies $Tx \preceq Ty$, then T has a fixed point.*

If we substitute a partial metric ρ for the metric d in Theorem 2.3, it is easy to check that a similar result holds for the partial metric case as follows.

Theorem 2.5 *Let (X, ρ) be a complete partial metric space, $\alpha : X \times X \to [0, \infty)$ be a function, $\psi \in \Psi$ and T be a self-map on X such that $\alpha(x,y)\rho(Tx,Ty) \leq \psi(m(x,y))$ for all $x, y \in X$, where $m(x,y) = \max\{\rho(x,y), \rho(x,Tx), \rho(y,Ty), \frac{1}{2}[\rho(x,Ty) + \rho(y,Tx)]\}$. Suppose that T is α-admissible and there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Assume that if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all n and $x_n \to x$, then $\alpha(x_n, x) \geq 1$ for all n. Then T has a fixed point.*

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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