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Strong convergence of iterative algorithms with variable coefficients for asymptotically strict pseudocontractive mappings in the intermediate sense and monotone mappings

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Abstract

In this article, we propose some iterative algorithms with variable coefficients for finding a common element of the set of fixed points of a uniformly continuous asymptotically κ-strict pseudocon-tractive mapping in the intermediate sense and the set of solutions of the variational inequality problem for a monotone, Lipschitz-continuous mapping. Some strong convergence theorems of these iterative algorithms are obtained without some boundedness assumptions and without some convergence condition. The results of the article improve and extend the recent results of Ceng and Yao, Nadezhkina and Takahashi, and several others.

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Keywords: fixed point, variational inequality, asymptotically strict pseudocontractive mapping in the intermediate sense, monotone mapping, variable coefficient method

1 Introduction

Let H be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$, respectively. Let C a nonempty closed convex subset of H and let P_C be the metric project from H onto C. $F(S) = \{x \in C : Sx = x\}$ denotes the set of fixed points of a self-mapping S on a set C.

A mapping A of C into H is called monotone if

$$\langle Ax - Ay, x - y \rangle \geq 0, \quad \forall x, y \in C.$$

A mapping A of C into H is said to be L-Lipschitz continuous if there exists a positive constant L such that

$$\|Ax - Ay\| \leq L \|x - y\|, \quad \forall x, y \in C.$$

Let the mapping A from C to H be monotone and Lipschitz continuous. The variational inequality problem (for short, $VI(C, A)$) is to find a $u \in C$ such that

$$\langle Au, v - u \rangle \geq 0, \quad \forall v \in C.$$

The set of solutions of the $VI(C, A)$ is denoted Ω. A mapping A of C into H is said to be α-inverse strongly monotone if there exists a positive constant α such that

$$\langle Ax - Ay, x - y\rangle \geq \alpha \left\|Ax - Ay\right\|^2, \quad \forall x, y \in C.$$

It is obvious that if A is α-inverse-strongly monotone, then A is monotone and Lipschitz continuous. Recall that a mapping $S : C \to C$ is called to be nonexpansive if

$$\left\|Sx - Sy\right\| \leq \left\|x - y\right\|, \quad \forall x, y \in C.$$

A mapping $S : C \to C$ is called to be asymptotically nonexpansive [1], if there exists a sequence $\{k_n\} \subset [1, \infty)$ with $k_n \to 1$ as $n \to \infty$ such that

$$\left\|S^n x - S^n y\right\| \leq k_n \left\|x - y\right\|, \quad \text{for all } x, y \in C, \quad \text{and all integers } n \geq 1.$$

$S : C \to C$ is called to be asymptotically nonexpansive in the intermediate sense [2], if it is continuous and the following inequality holds:

$$\limsup_{n\to\infty} \sup_{x,y\in C} (\left\|S^n x - S^n y\right\| - \left\|x - y\right\|) \leq 0. \tag{1.1}$$

In fact, we see that (1.1) is equivalent to

$$\left\|S^n x - S^n y\right\|^2 \leq \left\|x - y\right\|^2 + c_n, \quad \forall n \geq 1, x, y \in C, \tag{1.2}$$

where $c_n \in [0, \infty)$ with $c_n \to 0$ as $n \to \infty$. S is called to be an asymptotically κ-strict pseudocontractive mapping with sequence $\{\gamma_n\}$ [3], if there exist a constant $\kappa \in [0, 1)$ and a sequence $\{\gamma_n\} \subset [0, \infty)$ with $\gamma_n \to 0$ as $n \to \infty$ such that

$$\left\|S^n x - S^n y\right\|^2 \leq (1 + \gamma_n)\left\|x - y\right\|^2 + \kappa\left\|(I - S^n)x - (I - S^n)y\right\|^2 \tag{1.3}$$

for all $x, y \in C$ and all integers $n \geq 1$. It is very clear that, in a real Hilbert space H, (1.3) is equivalent to

$$\langle S^n x - S^n y, x - y\rangle \leq (1 + \frac{\gamma_n}{2})\left\|x - y\right\|^2 - \frac{1-\kappa}{2}\left\|(I - S^n)x - (I - S^n)y\right\|^2.$$

Recall that S is called to be an asymptotically κ-strict pseudocontraction in the intermediate sense with sequence $\{\gamma_n\}$ [4], if

$$\limsup_{n\to\infty} \sup_{x,y\in C} \{\left\|S^n x - S^n y\right\|^2 - (1 + \gamma_n)\left\|x - y\right\|^2 - \kappa\left\|(I - S^n)x - (I - S^n)y\right\|^2\} \leq 0, \tag{1.4}$$

where $\kappa \in [0, 1)$ *and* $\gamma_n \in [0, \infty)$ such that $\gamma_n \to 0$ as $n \to \infty$. In fact, (1.4) is reduced to the following:

$$\left\|S^n x - S^n y\right\|^2 \leq (1 + \gamma_n)\left\|x - y\right\|^2 + \kappa\left\|(I - S^n)x - (I - S^n)y\right\|^2 + c_n, \forall x, y \in C, \tag{1.5}$$

where $c_n \in [0, \infty)$ with $c_n \to 0$ as $n \to \infty$.

Recently, by combining Korpelevich's extragradient method [5] with Takahashi-Toyoda's iterative algorithm [6], Nadezhkina and Takahashi [7] introduced the following iterative scheme for finding a common element of $F(S) \cap \Omega$, the intersection of the set of fixed points of a nonexpansive mapping and the set of solutions of the variational inequality problem for a monotone, Lipschitz-continuous mapping and proved its weak convergence.

Theorem A (See [[7], Theorem 3.1]). *Let C be a nonempty closed convex subset of a Hilbert space H. Let $A : C \to H$ be a monotone, L-Lipschitz continuous mapping and let $S : C \to C$ be a nonexpansive mapping such that $F(S) \cap \Omega \neq \emptyset$. Let $\{x_n\}$ and $\{y_n\}$ be* *the sequences generated by*

$$\begin{cases} x_0 = x \in C \text{ chosen arbitrary}, \\ y_n = P_C(x_n - \lambda_n A x_n), \\ x_{n+1} = \alpha_n x_n + (1 - \alpha_n) S P_C(x_n - \lambda_n A y_n), \quad \forall n \geq 0, \end{cases}$$

where $\lambda_n \subset [a, b]$ *for some* $a, b \in (0, 1/L)$ *and* $\alpha_n \subset [c, d]$ *for some* $c, d \in (0, 1)$. *Then the sequences* $\{x_n\}$, $\{y_n\}$ *converge weakly to the same point* $z \in F(S) \cap \Omega$, *where* $z = \lim_{n\to\infty} P_{F(S)\cap\Omega} x_n$.

Inspired by Nadezhkina and Takahashi [[7], Theorem 3.1], and Zeng and Yao [8] introduced the following iterative process for finding an element of $F(S)\cap\Omega$ and established the following strong convergence theorem.

Theorem B (See [[8], Theorem 3.1]). *Let C be a nonempty closed convex subset of a Hilbert space H. Let* $A : C \to H$ *be a monotone, L-Lipschitz continuous mapping and let* $S : C \to C$ *be a nonexpansive such that* $F(S) \cap \Omega \neq \emptyset$. *Let* $\{x_n\}$ *and* $\{y_n\}$ *be the sequences generated by any given* $x_0 \in C$ *and*

$$\begin{cases} y_n = P_C(x_n - \lambda_n A x_n), \\ x_{n+1} = \alpha_n x_0 + (1 - \alpha_n) S P_C(x_n - \lambda_n A y_n), \end{cases}$$

for every $n \geq 0$, *where* $\{\lambda_n\}$ *and* $\{\alpha_n\}$ *satisfy the conditions:*

(*a*) $\{\lambda_n L\} \subset (0, 1 - \delta)$ *for some* $\delta \in (0, 1)$

(*b*) $\{\alpha_n\} \subset (0, 1), \quad \sum_{n=0}^{\infty} \alpha_n = \infty, \quad \lim_{n\to\infty} \alpha_n = 0,$,

Then the sequences $\{x_n\}$, $\{y_n\}$ *converge strongly to the same point* $P_{F(S)\cap\Omega}(x_0)$, *provided* $\lim_{n\to\infty} \|x_n - x_{n+1}\| = 0$.

Further, utilizing the combination of hybrid-type method and extragradient-type method, Nadezhkina and Takahashi [9] introduced the following iterative scheme for finding a common element of $F(S) \cap \Omega$, and proved the following strong convergence theorem.

Theorem C (See [[9], Theorem 3.1] or [[10], Theorem NT2]). *Let C be a nonempty closed convex subset of a Hilbert space H. Let* $A : C \to H$ *be a monotone, L-Lipschitz continuous mapping and let* $S : C \to C$ *be a nonexpansive mapping such that* $F(S) \cap \Omega \neq \emptyset$. *Let* $\{x_n\}$, $\{y_n\}$, *and* $\{z_n\}$ *be the sequences generated by*

$$\begin{cases} x_0 = x \in C \text{ chosen arbitrary}, \\ y_n = P_C(x_n - \lambda_n A x_n), \\ z_n = \alpha_n x_n + (1 - \alpha_n) S P_C(x_n - \lambda_n A y_n), \\ C_n = \{z \in C : \|z_n - z\| \leq \|x_n - z\|\}, \\ Q_n = \{z \in C : \langle x_n - z, \ x - x_n \rangle \geq 0\}, \\ x_{n+1} = P_{C_n \cap Q_n} x, \quad \forall n \geq 0, \end{cases}$$

where $\lambda_n \subset [a, b]$ *for some* $a, b \in (0, 1/L)$ *and* $\alpha_n \subset [0, c]$ *for some* $c \in [0, 1)$. *Then the sequences* $\{x_n\}$, $\{y_n\}$, *and* $\{z_n\}$ *converge strongly to the same point* $q = P_{F(S)\cap\Omega} x$.

Very recently, considering a nonexpansive nonself mapping, Buong [11] introduced a new iteration method based on the hybrid-type method and extragradient-type method and proved its strong convergence. Considering a finite family of κ-strict pseudocontractive mappings, Ceng et al. [12] proposed parallel extragradient and cyclic-extragradient algorithms, and derived the weak convergence of these algorithms.

Most recently, considering a uniformly continuous asymptotically κ-strict pseudo-con-tractive mapping in the intermediate sense, Ceng and Yao [10] proposed a modified hybrid Mann iterative scheme with perturbed mapping based on the hybrid-type method and extragradient-type method, and established the following interesting result with the help of some boundedness assumptions

Theorem D (See [[10], Theorem 3.1]). *Let C be a nonempty closed convex subset of a real Hilbert space H. Let $A : C \to H$ be a monotone, L-Lipschitz continuous mapping with $(I - A)(C) \subset C$ and let $S : C \to C$ be a uniformly continuous asymptotically κ-strict pseudocontractive mapping in the intermediate sense with sequence $\{\gamma_n\}$ such that $F(S) \cap \Omega$ is nonempty and bounded. Let $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ be the sequences generated by*

$$\begin{cases} x_1 = x \in C \ \text{ chosen arbitrary}, \\ y_n = (1-\mu_n)x_n + \mu_n(S^n x_n - \lambda_n A(S^n x_n)), \\ t_n = P_C(y_n - \lambda_n A y_n), \\ z_n = (1-\alpha_n-\beta_n)x_n + \alpha_n t_n + \beta_n S^n t_n, \\ C_n = \{z \in C : \|z_n - z\|^2 \le \|x_n - z\|^2 + \theta_n\}, \\ Q_n = \{z \in C : \langle x_n - z,\ x - x_n\rangle \ge 0\}, \\ x_{n+1} = P_{C_n \cap Q_n} x, \quad \forall n \ge 1, \end{cases}$$

where $\theta_n = \mu_n \kappa \|x_n - S^n x_n\|^2 + 2\gamma_n \Delta_n + 2c_n + \lambda_n^2(\|A(S^n x_n)\|^2 + \|A y_n\|^2)$ *and*

$$\Delta_n = \max\left\{ \sup_{p \in F(S)\cap\Omega} \|x_n - p\|^2, \sup_{p \in F(S)\cap\Omega} \|t_n - p\|^2 \right\} < \infty.$$

Assume that $\{\lambda_n\}$ is a sequence in (0, 1) *with* $\lim_{n\to\infty} \lambda_n = 0$, *and $\{\alpha_n\}$, $\{\beta_n\}$, and $\{\mu_n\}$ are three sequences in* [0, 1] *satisfying the conditions:*

(i) $\alpha_n + \beta_n \le 1$ *for all* $n \ge 1$;
(ii) $\kappa \le \alpha_n$ *and* $\lim_{n\to\infty} \mu_n = 0$;
(iii) $\liminf_{n\to\infty} \beta_n > 0$;

Then the sequences $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ converge strongly to the same point $q = P_{F(S)\cap\Omega} x$ if and only if $\liminf_{n\to\infty} \langle Ax_n, y - x_n\rangle \ge 0$ for all $y \in C$.

The following problems arise naturally then: (1) Can we relax the convergence condition $\liminf_{n\to\infty} \langle Ax_n, y - x_n\rangle \ge 0$ for all $y \in C$ in Theorem D by proposing some new algorithm. (2) Can we relax the boundedness assumptions that the intersection $F(S)\cap\Omega$ and the sequence $\{\Delta_n\}$ in Theorem D are both bounded. Actually, in many cases, these assumptions and conditions are not easy to examine in advance. Hence, they are interesting and important problems.

In order to solve these problems, motivated and inspired by Ceng and Yao [10], Nadezhkina and Takahashi [9], and Ge et al. [13], we introduce some new algorithms with variable coefficients based on the hybrid-type method and extragradient-type method for finding a common element of the set of fixed points of a uniformly continuous asymptotically κ-strict pseudocontractive mapping in the intermediate sense and the set of solutions of the variational inequality problem for a monotone, Lipschitz-continuous mapping in real Hilbert spaces. Our results improve and extend the corresponding results of Ceng and Yao [10], Nadezhkina and Takahashi [9], and several others.

2 Preliminaries

Throughout this article,

- $x_n \to x$ means that $\{x_n\}$ converges strongly to x.
- $F(S) = \{x \in C : Sx = x\}$ denotes the set of fixed points of a self-mapping S on a set C.
- $B_r(x_1):=\{x \in H : \|x - x_1\| \leq r\}$.
- $\mathbb{N}$ is the set of positive integers.

For every point $x \in H$, there exists a unique nearest point in C, denoted by P_Cx, such that

$$\|x - P_Cx\| \leq \|x - y\|, \quad \forall y \in C.$$

P_C is called the metric projection of H onto C. We know that P_C is a nonexpansive mapping from H onto C. Recall that the inequality holds

$$\langle x - P_Cx, P_Cx - y\rangle \geq 0, \quad \forall x \in H, y \in C. \tag{2.1}$$

Moreover, it is easy to see that it is equivalent to

$$\|P_Cx - P_Cy\|^2 \leq \langle P_Cx - P_Cy, x - y\rangle, \quad \forall x, y \in H;$$

It is also equivalent to

$$\|x - y\|^2 \geq \|x - P_Cx\|^2 + \|y - P_Cx\|^2, \quad \forall x \in H, y \in C. \tag{2.2}$$

Lemma 2.1. [14]. *Let C be a nonempty closed convex subsets of a real Hilbert space H. Given* $x \in H$ *and* $y \in C$. *Then* $y = P_Cx$ *if and only if there holds the inequality*

$$\langle x - y, y - z\rangle \geq 0, \quad \forall z \in C.$$

Lemma 2.2. [10]. *Let* $A : C \to H$ *be a monotone mapping. In the context of the variational inequality problem the characterization of projection* (2.1) *implies*

$$u \in \Omega \Leftrightarrow u = P_C(u - \lambda Au), \quad \forall \lambda > 0.$$

Lemma 2.3. [15]. *Let C be a nonempty closed convex subsets of a real Hilbert space H. Given* $x, y, z \in H$ *and given also a real number a, the set*

$$\{v \in C : \|y - v\|^2 \leq \|x - v\|^2 + \langle z, v\rangle + a\}$$

is convex and closed.

Lemma 2.4. [16]. *Let H be a real Hilbert space. Then for all* $x, y, z \in H$ *and all* $\alpha, \beta, \gamma \in [0, 1]$ *with* $\alpha + \beta + \gamma = 1$, *we have*

$$\|\alpha x + \beta y + \gamma z\|^2 = \alpha\|x\|^2 + \beta\|y\|^2 + \gamma\|z\|^2 - \alpha\beta\|x - y\|^2 - \alpha\gamma\|x - z\|^2 - \beta\gamma\|y - z\|^2.$$

Lemma 2.5. [4]. *Let C be a nonempty closed convex subsets of a real Hilbert space H. and* $S : C \to C$ *be an asymptotically* κ*-strict pseudocontraction in the intermediate sense with sequence* $\{\gamma_n\}$. *Then*

$$\left\|S^n x - S^n y\right\| \leq \frac{1}{1-\kappa}\left(\kappa \left\|x-y\right\| + \sqrt{(1+(1-\kappa)\gamma_n)\left\|x-y\right\|^2 + (1-\kappa)c_n}\right)$$

for all $x, y \in C$ *and* $n \geq 1$.

By Ibaraki et al. [[17], Theorem 4.1], we have the following lemma.

Lemma 2.6. *Let* $\{K_n\}$ *be a sequence of nonempty closed convex subsets of a real Hilbert space H such that* $K_{n+1} \subset K_n$ *for each* $n \in \mathbb{N}$. *If* $K^* = \cap_{n=0}^{\infty} K_n$ *is nonempty, then for each* $x \in H$, $\{P_{K_n}x\}$ *converges strongly to* $P_{K^*}x$.

A set-valued mapping $T : H \to 2^H$ is called monotone if for all $x, y \in H, f \in Tx$ and $g \in Ty$ imply $\langle x - y, f - g\rangle \geq 0$. A monotone mapping $T : H \to 2^H$ is maximal if its graph $G(T)$ is not properly contained in the graph of any other monotone mapping. It is known that a monotone mapping T is maximal if and only if for $(x, f) \in H \times H$, $\langle x-y, f-g\rangle \geq 0$ for all $(y, g) \in G(T)$ implies $f \in Tx$. Let $A : C \to H$ be a monotone and Lipschitz continuous mapping and let $N_C v$ be the normal cone to C at $v \in C$, i.e., $N_C = \{w \in H : (v - u, w) \geq 0, \forall u \in C\}$. Define

$$Tv = \begin{cases} Av + N_C v, & \text{if } v \in C, \\ \emptyset, & \text{if } v \notin C, \end{cases}$$

It is known that in this case T is maximal monotone, and $0 \in Tv$ if and only if $v \in \Omega$, see [18].

3 Results and proofs

Theorem 3.1. *Let C be a nonempty closed convex subset of a real Hilbert space H. Let* $A : C \to H$ *be a monotone, L-Lipschitz continuous mapping and let* $S : C \to C$ *be a uniformly continuous asymptotically* κ*-strict pseudocontractive mapping in the intermediate sense with sequence* $\{\gamma_n\}$ *such that* $F(S) \cap \Omega$ *is nonempty. Let* $\{x_n\}$, $\{y_n\}$, *and* $\{z_n\}$ *be the sequences generated by the following algorithm with variable coefficients*

$$\begin{cases} x_1 \in C \text{ chosen arbitrary}, \\ y_n = P_C(x_n - \lambda_n A x_n), \\ t_n = P_C(x_n - \lambda_n A y_n), \\ z_n = (1 - \alpha_n - \hat{\beta}_n)x_n + \alpha_n t_n + \hat{\beta}_n S^n t_n, \\ C_0 = C, \\ C_n = \{z \in C_{n-1} : \|z_n - z\|^2 \leq \|x_n - z\|^2 - (\alpha_n - \kappa)\hat{\beta}_n \|t_n - S^n t_n\|^2 + \theta_n\}, \\ x_{n+1} = P_{C_n} x_1, \end{cases} \tag{3.1}$$

for every $n \in \mathbb{N}$, *where* $\hat{\beta}_n = \dfrac{\beta_n}{1 + \|x_n - x_1\|^2}$, $\theta_n = \beta_n(2\gamma_n(1 + r_0^2) + c_n)$, $\{\alpha_n\} \subset (a, 1), \{\beta_n\} \subset (b, 1 - a)$ *and* $\{\lambda_n\} \subset (b/L, (1 - a)/L)$ *for some* $a \in (\kappa, 1)$ *and some* $b \in (0, 1 - a)$, *and* c_n *is as in* (1.5), *the positive real number* r_0 *is chosen so that* $B_{r_0}(x_1) \cap F(S) \cap \Omega \neq \emptyset$. *Then the sequences* $\{x_n\}$, $\{y_n\}$, *and* $\{z_n\}$ *converge strongly to a point of* $F(S) \cap \Omega$.

Proof. We divide the proof into seven steps.

Step 1. We claim C_n is nonempty, convex, and closed for each $n \in \mathbb{N}$.

Indeed, by the assumption, C_0, i.e., C is nonempty, convex, and closed. From the definition of C_n and Lemma 2.3, it is easy to see that C_n is convex and closed for each $n \in \mathbb{N}$. We, therefore, only need to prove that C_n is nonempty for each $n \in \mathbb{N}$.

Indeed, let $p \in B_{r_0}(x_1) \cap F(S) \cap \Omega$ be a arbitrary element. Putting $x = x_n - \lambda_n A y_n$ and $y = p$ in (2.2), we have

$$\begin{aligned}\|t_n - p\|^2 &\le \|x_n - \lambda_n A y_n - p\|^2 - \|x_n - \lambda_n A y_n - t_n\|^2 \\ &= \|x_n - p\|^2 - \|x_n - t_n\|^2 + 2\lambda_n\langle A y_n, p - t_n\rangle \\ &= \|x_n - p\|^2 - \|x_n - t_n\|^2 + 2\lambda_n(\langle A y_n - Ap, p - y_n\rangle + \langle Ap, p - y_n\rangle) \\ &\quad + 2\lambda_n\langle A y_n, y_n - t_n\rangle\end{aligned} \tag{3.2}$$

Since $A : C \to H$ is a monotone mapping and $p \in \Omega = VI(C, A)$, further, we have

$$\begin{aligned}\|t_n - p\|^2 &\le \|x_n - p\|^2 - \|x_n - t_n\|^2 + 2\lambda_n\langle A y_n, y_n - t_n\rangle \\ &= \|x_n - p\|^2 - \|x_n - y_n\|^2 - 2\langle x_n - y_n, y_n - t_n\rangle - \|y_n - t_n\|^2 + 2\lambda_n\langle A y_n, y_n - t_n\rangle \\ &= \|x_n - p\|^2 - \|x_n - y_n\|^2 - \|y_n - t_n\|^2 + 2\langle x_n - \lambda_n A y_n - y_n, t_n - y_n\rangle\end{aligned} \tag{3.3}$$

Since $y_n = P_C(x_n - \lambda_n A x_n)$ and A is L-Lipschitz continuous, we have

$$\begin{aligned}&\langle x_n - \lambda_n A y_n - y_n, t_n - y_n\rangle \\ &= \langle x_n - \lambda_n A x_n - y_n, t_n - y_n\rangle + \lambda_n\langle A x_n - A y_n, t_n - y_n\rangle \\ &\le \lambda_n\langle A x_n - A y_n, t_n - y_n\rangle \\ &\le \lambda_n L\|x_n - y_n\|\,\|t_n - y_n\|.\end{aligned} \tag{3.4}$$

So, it follows from (3.2), (3.3) and $\{\lambda_n\} \subset (b/L, (1 - a)/L)$, we obtain

$$\begin{aligned}\|t_n - p\|^2 &\le \|x_n - p\|^2 - \|x_n - y_n\|^2 - \|y_n - t_n\|^2 + 2\lambda_n L\|x_n - y_n\|\,\|t_n - y_n\| \\ &\le \|x_n - p\|^2 - \|x_n - y_n\|^2 - \|y_n - t_n\|^2 + \lambda_n^2 L^2\|x_n - y_n\|^2 + \|t_n - y_n\|^2 \\ &= \|x_n - p\|^2 + (\lambda_n^2 L^2 - 1)\|x_n - y_n\|^2 \\ &\le \|x_n - p\|^2.\end{aligned} \tag{3.5}$$

Since $S : C \to C$ be an asymptotically κ-strict pseudocontractive mapping in the intermediate sense with sequences $\{\gamma_n\}$, by the definition, we have

$$\|S^n t_n - p\|^2 \le (1 + \gamma_n)\|t_n - p\|^2 + \kappa\|t_n - S^n t_n\|^2 + c_n, \forall x, y \in C, \tag{3.6}$$

where $c_n \in [0, \infty)$ with $c_n \to 0$ as $n \to \infty$. So, from $z_n = (1 - \alpha_n - \hat{\beta}_n)x_n + \alpha_n t_n + \hat{\beta}_n S^n t_n$, (3.5), (3.6) and Lemma 2.4, we deduce that

$$\begin{aligned}\|z_n - p\|^2 &= \|(1 - \alpha_n - \hat{\beta}_n)(x_n - p) + \alpha_n(t_n - p) + \hat{\beta}_n(S^n t_n - p)\|^2 \\ &\le (1 - \alpha_n - \hat{\beta}_n)\|x_n - p\|^2 + \alpha_n\|t_n - p\|^2 + \hat{\beta}_n\|S^n t_n - p\|^2 \\ &\quad - \alpha_n\hat{\beta}_n\|t_n - S^n t_n\|^2 \\ &\le (1 - \alpha_n - \hat{\beta}_n)\|x_n - p\|^2 + \alpha_n\|t_n - p\|^2 \\ &\quad + \hat{\beta}_n((1 + \gamma_n)\|t_n - p\|^2 + \kappa\|t_n - S^n t_n\|^2 + c_n) - \alpha_n\hat{\beta}_n\|t_n - S^n t_n\|^2 \\ &= (1 - \alpha_n - \hat{\beta}_n)\|x_n - p\|^2 + (\alpha_n + \hat{\beta}_n)\|t_n - p\|^2 \\ &\quad + \hat{\beta}_n(\gamma_n\|t_n - p\|^2 + c_n) - (\alpha_n - \kappa)\hat{\beta}_n\|t_n - S^n t_n\|^2 \\ &\le (1 - \alpha_n - \hat{\beta}_n)\|x_n - p\|^2 + (\alpha_n + \hat{\beta}_n)\|x_n - p\|^2 \\ &\quad - (\alpha_n - \kappa)\hat{\beta}_n\|t_n - S^n t_n\|^2 + \beta_n(\gamma_n\|x_n - p\|^2 + c_n) \\ &= \|x_n - p\|^2 - (\alpha_n - \kappa)\hat{\beta}_n\|t_n - S^n t_n\|^2 + \beta_n(\gamma_n\|x_n - p\|^2 + c_n).\end{aligned} \tag{3.7}$$

Further, it follows fron the definition of $\hat{\beta}_n$ that

$$\begin{aligned} \|z_n - p\|^2 &\le \|x_n - p\|^2 - (\alpha_n - \kappa)\hat{\beta}_n \|t_n - S^n t_n\|^2 \\ &\quad + \beta_n \left(\frac{2\gamma_n \|x_n - x_1\|^2 + \|p - x_1\|^2 + c_n}{1 + \|x_n - x_1\|^2} \right) \\ &\le \|x_n - p\|^2 - (\alpha_n - \kappa)\hat{\beta}_n \|t_n - S^n t_n\|^2 + \beta_n(2\gamma_n(1 + r_0^2) + c_n) \\ &\le \|x_n - p\|^2 - (\alpha_n - \kappa)\hat{\beta}_n \|t_n - S^n t_n\|^2 + \theta_n, \end{aligned} \tag{3.8}$$

where $\theta_n = \beta_n(2\gamma_n(1 + r_0^2) + c_n)$. Therefore, we have

$$B_{r_0}(x_1) \cap F(S) \cap \Omega \subset C_n, \ \forall n \in \mathbb{N}. \tag{3.9}$$

Step 2. We claim that the sequence $\{x_n\}$ converges strongly to an element in C, say x^*.

Since $\{C_n\}$ is a decreasing sequence of closed convex subset of H such that $C^* = \cap_{n=0}^{\infty} C_n$ is a nonempty and closed convex subset of H, it follows from Lemma 2.6 that $\{x_{n+1}\} = \{P_{C_n} x_1\}$ converges strongly to $P_{C^*} x_1$, say x^*.

Step 3. We claim that $\lim_{n\to\infty} z_n = x^*$ and $\lim_{n\to\infty} \|t_n - S^n t_n\| = 0$.

Indeed, the definition of x_{n+1} shows that $x_{n+1} \in C_n$, i.e.,

$$\|z_n - x_{n+1}\|^2 \le \|x_n - x_{n+1}\|^2 - (\alpha_n - \kappa)\hat{\beta}_n \|t_n - S^n t_n\|^2 + \theta_n. \tag{3.10}$$

Note that $\gamma_n \to 0$, $c_n \to 0$, $x_n \to x^*$ as $n \to \infty$ and $\alpha_n > a > \kappa$, $\forall n \in \mathbb{N}$, we have $\theta_n \to 0$, $\|z_n - x_{n+1}\|^2 \to 0$, $\|z_n - x_n\|^2 \to 0$ and $z_n \to x^*$ as $n \to \infty$.

Further, it follows from (3.10) that

$$(a - \kappa)\frac{b}{1 + \|x_n - x_1\|^2}\|t_n - S^n t_n\|^2 \le \|x_n - x_{n+1}\|^2 + \theta_n.$$

Thus $\lim_{n\to\infty} \|t_n - S^n t_n\| = 0$.

Step 4. We claim that $x^* \in F(S)$.

Since $z_n = (1 - \alpha_n - \hat{\beta}_n)x_n + \alpha_n t_n + \hat{\beta}_n S^n t_n$, we have

$$z_n - x_n = (\alpha_n + \hat{\beta}_n)(t_n - x_n) + \hat{\beta}_n(S^n t_n - t_n)$$

Considering $0 < a < \alpha_n + \hat{\beta}_n$, by Step 3 we have

$$t_n - x_n \to 0, \quad t_n \to x^*, \quad as\ n \to \infty. \tag{3.11}$$

From (3.11), Lemma 2.5 and Step 3, we have

$$\|S^n x^* - x^*\| \le \|S^n x^* - S^n t_n\| + \|S^n t_n - t_n\| + \|t_n - x^*\| \to 0, \quad as\ n \to \infty. \tag{3.12}$$

Since $S : C \to C$ is uniformly continuous, by (3.12) we have

$$S^{n+1}x^* = S(S^n x^*) \to Sx^*, \quad as\ n \to \infty.$$

Hence, $Sx^* = x^*$, i.e., $x^* \in F(S)$.

Step 5. We claim that $t_n - y_n \to 0$, $y_n \to x^*$, as $n \to \infty$.

By (3.5), for $p \in B_{r_0}(x_1) \cap F(S) \cap \Omega$, we have

$$\|t_n - p\|^2 \le \|x_n - p\|^2 + (\lambda_n^2 L^2 - 1)\|x_n - y_n\|^2$$

Therefore, we have

$$\begin{aligned}(1-\lambda_n^2L^2)\|x_n-y_n\|^2 &\le \|x_n-p\|^2-\|t_n-p\|^2\\ &\le (\|x_n-p\|+\|t_n-p\|)(\|x_n-p\|-\|t_n-p\|)\\ &\le (\|x_n-p\|+\|t_n-p\|)(\|x_n-t_n\|).\end{aligned}$$

This together with (3.11) and $0<1-(1-a)^2<1-\lambda_n^2L^2$ implies that $\|x_n - y_n\| \to 0$, $\|t_n - y_n\| \to 0$ and $y_n \to x^*$, as $n \to \infty$.

Step 6. We claim that $x^* \in \Omega$.

Indeed, let

$$Tv = \begin{cases} Av + N_C v, & \text{if } v \in C,\\ \emptyset, & \text{if } v \notin C,\end{cases}$$

where $N_C v$ be the normal cone to C at $v \in C$. We have already mentioned in Section 2 that in this case T is maximal monotone, and $0 \in Tv$ if and only if $v \in \Omega$, see [18].

Let $(v, w) \in G(T)$, the graph of T. Then, we have $w \in Tv = Av + N_c v$ and hence $w - Av \in N_C v$. So, we have

$$\langle v-t, w-Av\rangle \ge 0,\ \forall t \in C. \tag{3.13}$$

Noticing $t_n = P_C(x_n - \lambda_n A y_n)$ and $v \in C$, by (2.1) we have

$$\langle x_n-\lambda_n Ay_n-t_n, t_n-v\rangle \ge 0,$$

and hence

$$\langle v-t_n, \frac{t_n-x_n}{\lambda_n}+Ay_n\rangle \ge 0. \tag{3.14}$$

From (3.13), (3.14) and $t_n \in C$, we have

$$\begin{aligned}\langle v-t_n, w\rangle &\ge \langle v-t_n, Av\rangle\\ &\ge \langle v-t_n, Av\rangle - \left\langle v-t_n, \frac{t_n-x_n}{\lambda_n}+Ay_n\right\rangle\\ &\ge \langle v-t_n, Av-At_n\rangle + \langle v-t_n, At_n-Ay_n\rangle - \langle v-t_n, \frac{t_n-x_n}{\lambda_n}\rangle\end{aligned} \tag{3.15}$$

Letting $n \to \infty$ in (3.15), considering $A : C \to H$ be a monotone, L-Lipschitz continuous mapping and $\{\lambda_n\} \subset (b/L, (1-a)/L)$, we have $\langle v-x^*, w\rangle \ge 0$. Since T is maxmal monotone, we have $0 \in Tx^*$ and hence $x^* \in \Omega = VI(C, A)$.

Step 7. We claim that the sequences $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ converge strongly to $x^* \in F(S) \cap \Omega$

From Step 4 and Step 6, we have $x^* \in F(S) \cap \Omega$. Therefore, it follows from Step 2, Step 3, and Step 5 that the sequences $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ converge strongly to $x^* \in F(S) \cap \Omega$. This completes the proof.

Remark 3.2. Theorem 3.1 improves and extends Theorems C and D since

(1) the nonexpansive mapping S in Theorem C is extended to be an asymptotically κ-strict pseudocontractive mapping in the intermediate sense.

(2) the convergence condition that $\liminf_{n\to\infty}\langle Ax_n, y-x_n\rangle \ge 0$ for all $y \in C$ in Theorem D is removed.

(3) the boundedness assumptions that the intersection $F(S) \cap \Omega$ and the sequence $\{\Delta_n\}$ are both bounded in Theorem D are dispensed with.

(4) the requirement $(I - A)(C) \subset C$ in Theorem D is dropped off.

Corollary 3.3. *Let C be a nonempty closed convex subset of a real Hilbert space H. Let $S : C \to C$ be a uniformly continuous asymptotically κ-strict pseudocontractive mapping in the intermediate sense with sequence $\{\gamma_n\}$ such that $F(S)$ is nonempty. Let $\{x_n\}$ and $\{z_n\}$ be the sequences generated by the following algorithm with variable coefficients*

$$\begin{cases} x_1 \in C \quad \text{chosen arbitrary}, \\ z_n = (1 - \alpha_n - \hat{\beta}_n)x_n + \alpha_n x_n + \hat{\beta}_n S^n x_n, \\ C_0 = C, \\ C_n = \{z \in C_{n-1} : \|z_n - z\|^2 \le \|x_n - z\|^2 - (\alpha_n - \kappa)\hat{\beta}_n\|x_n - S^n x_n\|^2 + \theta_n\}, \\ x_{n+1} = P_{C_n}x_1, \end{cases}$$

for every $n \in \mathbb{N}$, where $\hat{\beta}_n = \dfrac{\beta_n}{1 + \|x_n - x_1\|^2}$, $\theta_n = \beta_n(2\gamma_n(1 + r_0^2) + c_n)$, $\{\alpha_n\} \subset (a, 1), \{\beta_n\} \subset (b, 1 - a)$, and $\{\lambda_n\} \subset (b, 1 - a)$ for some $a \in (\kappa, 1)$ and some $b \in (0, 1 - a)$, and c_n is as in (1.5), the positive real number r_0 is chosen so that $B_{r_0}(x_1) \cap F(S) \neq \emptyset$. Then the sequences $\{x_n\}$ and $\{z_n\}$ converge strongly to a point of $F(S)$.

Remark 3.4. Corollary 3.3 improves and extends [[4], Theorem 4.1] and Theorem D since the boundedness assumptions that the $F(S)$ and the sequence $\{\Delta_n\}$ are both bounded in [[4], Theorem 4.1] and Theorem D are dispensed with.

Recall that a mapping $T : C \to C$ is called pseudocontractive [19,20] if

$$\langle Tx - Ty, x - y\rangle \le \|x - y\|^2, \ \forall x, y \in C. \tag{3.16}$$

In fact, we see that (3.16) is equivalent to

$$\|Tx - Ty\|^2 \le \|x - y\|^2 + \|(I - T)x - (I - T)y\|^2, \quad \forall x, y \in C.$$

Corollary 3.5. *Let C be a nonempty closed convex subset of a real Hilbert space H. Let $T : C \to C$ be a pseudocontractive, m-Lipschitz continuous mapping and let $S : C \to C$ be a uniformly continuous asymptotically κ-strict pseudocontractive mapping in the intermediate sense with sequence $\{\gamma_n\}$ such that $F(S) \cap F(T)$ is nonempty. Let $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ be the sequences generated by the following algorithm with variable coefficients*

$$\begin{cases} x_1 \in C \quad \text{chosen arbitrary}, \\ y_n = (1 - \lambda_n)x_n + \lambda_n Tx_n, \\ t_n = P_C(x_n - \lambda_n y_n + \lambda_n Ty_n), \\ z_n = (1 - \alpha_n - \hat{\beta}_n)x_n + \alpha_n t_n + \hat{\beta}_n S^n t_n, \\ C_0 = C, \\ C_n = \{z \in C_{n-1} : \|z_n - z\|^2 \le \|x_n - z\|^2 - (\alpha_n - \kappa)\hat{\beta}_n\|t_n - S^n t_n\|^2 + \theta_n\}, \\ x_{n+1} = P_{C_n}x_1, \end{cases} \tag{3.17}$$

for every $n \in \mathbb{N}$, where $\hat{\beta}_n = \dfrac{\beta_n}{1 + \|x_n - x_1\|^2}$, $\theta_n = \beta_n(2\gamma_n(1 + r_0^2) + c_n)$, $\{\alpha_n\} \subset (a, 1), \{\beta_n\} \subset (b, 1 - a)$, and $\{\lambda_n\}$

$\subset (b/L, (1 - a)/L)$ *for some* $a \in (\kappa, 1)$ *and some* $b \in (0, 1 - a)$, *and* c_n *is as in* (1.5), *the positive real number* r_0 *is chosen so that* $B_{r_0}(x_1) \cap F(S) \cap F(T) \neq \emptyset$. *Then the sequences* $\{x_n\}$, $\{y_n\}$, *and* $\{z_n\}$ *converge strongly to a point of* $F(S) \cap F(T)$.

Proof. The following proof of this corollary is similar to that of [[10], Theorem 4.5].

Let $A = I - T$. It is easy to see that $y_n = P_C(x_n - \lambda_n A x_n) = (1 - \lambda_n)x_n + \lambda_n T x_n$. Now let us show that the mapping A is monotone and $(m + 1)$-Lipschitz continuous. Indeed, observe that

$$\langle Ax - Ay, x - y\rangle = \|x - y\|^2 - \langle Tx - Ty, x - y\rangle \geq 0.$$

and

$$\|Ax - Ay\| = \|x - y - (Tx - Ty)\| \leq (m + 1)\|x - y\|.$$

Next, let us show that $F(T) = \Omega = VI(C, A)$. Indeed, we have, for fixed $\lambda_0 \in (0, 1)$,

$$Tu = u \quad \Leftrightarrow \quad Au = 0 \quad \Leftrightarrow \quad u = u - \lambda_0 Au = P_C(u - \lambda_0 Au) \quad \Leftrightarrow \quad \langle Au, y - u\rangle \geq 0, \forall y \in C.$$

By Theorem 3.1, we obtain the desired conclusion. This completes the proof. □

Remark 3.6. Theorem 3.5 improves and extends [[10], Theorem 4.5] since

(1) the convergence condition that $\liminf_{n\to\infty}\langle Ax_n, y - x_n\rangle \geq 0$ for all $y \in C$ in [[10], Theorem 4.5] is removed.

(2) the boundedness assumptions that the intersection $F(S) \cap F(T)$ and the sequence $\{\Delta_n\}$ are both bounded in [[10], Theorem 4.5] are dispensed with.

By the careful analysis of the proof of Theorem 3.1, we can obtain the following result.

Theorem 3.7. *Let* C *be a nonempty closed convex subset of a real Hilbert space* H. *Let* $A : C \to H$ *be a monotone, L-Lipschitz continuous mapping and let* $S : C \to C$ *be a uniformly continuous asymptotically* κ*-strict pseudocontractive mapping in the intermediate sense with sequence* $\{\gamma_n\}$ *such that* $F(S) \cap \Omega$ *is nonempty and bounded. Let* $\{x_n\}$, $\{y_n\}$, *and* $\{z_n\}$ *be the sequences generated by the following algorithm*

$$\begin{cases} x_1 \in C \quad \text{chosen arbitrary}, \\ y_n = P_C(x_n - \lambda_n A x_n), \\ t_n = P_C(x_n - \lambda_n A y_n), \\ z_n = (1 - \alpha_n - \beta_n)x_n + \alpha_n t_n + \beta_n S^n t_n, \\ C_0 = C \\ C_n = \left\{z \in C_{n-1} : \|z_n - z\|^2 \leq \|x_n - z\|^2 - (\alpha_n - \kappa)\beta_n\|t_n - S^n t_n\|^2 + \theta_n\right\}, \\ x_{n+1} = P_{C_n} x_1, \end{cases}$$

for every $n \in \mathbb{N}$, *where* $\theta_n = \beta_n(\gamma_n\Delta_n + c_n)$, $\Delta_n = \sup_{p\in F(S)\cap\Omega}\|x_n - p\|^2$, $\{\alpha_n\} \subset (a, 1)$, $\{\beta_n\} \subset (b, 1 - a)$ *and* $\{\lambda_n\} \subset (b/L, (1 - a)/L)$ *for some* $a \in (\kappa, 1)$ *and some* $b \in (0, 1 - a)$, *and* c_n *is as in* (1.5). *Then the sequences* $\{x_n\}$, $\{y_n\}$ *and* $\{z_n\}$ *converge strongly to a point of* $F(S) \cap \Omega$.

Proof. Following the reasoning in the proof of Theorem 3.1, from (3.7), we take $\theta_n = \beta_n(\gamma_n\Delta_n + c_n)$ and use $F(S) \cap \Omega$ instead of $B_{r_0}(x_1) \cap F(S) \cap \Omega$ in Step 1. From Step 2, we have the sequence $\{\Delta_n\}$ is bounded, and hence $\theta_n = \beta_n(\gamma_n\Delta_n + c_n) \to 0$ as $n \to \infty$.

The remainder of the proof of Theorem 3.7 is similar to Theorem 3.1. The conclusion therefore follows. This completes the proof.

Theorem 3.8. *Let C be a nonempty closed convex subset of a real Hilbert space H. Let $A : C \to H$ be a monotone, L-Lipschitz continuous mapping and let $S : C \to C$ be a uniformly continuous asymptotically nonexpansive mapping in the intermediate sense such that $F(S) \cap \Omega$ is nonempty. Let $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ be the sequences generated by the following algorithm*

$$\begin{cases} x_1 \in C \quad \text{chosen arbitrary}, \\ y_n = P_C(x_n - \lambda_n A x_n), \\ t_n = P_C(x_n - \lambda_n A y_n), \\ z_n = (1 - \alpha_n - \beta_n)x_n + \alpha_n t_n + \beta_n S^n t_n, \\ C_0 = C, \\ C_n = \left\{ z \in C_{n-1} : \|z_n - z\|^2 \leq \|x_n - z\|^2 - \alpha_n \beta_n \|t_n - S^n t_n\|^2 + \theta_n \right\}, \\ x_{n+1} = P_{C_n} x_1, \end{cases}$$

for every $n \in \mathbb{N}$, where $\theta_n = \beta_n c_n$, $\{\alpha_n\} \subset (a, 1)$, $\{\beta_n\} \subset (b, 1 - a)$, and $\{\lambda_n\} \subset (b/L, (1 - a)/L)$ for some $a \in (0, 1)$ and some $b \in (0, 1 - a)$, and c_n is as in (1.2). Then the sequences $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ converge strongly to a point of $F(S) \cap \Omega$.

Proof. In Theorem 3.1, whenever $S : C \to C$ is an asymptotically nonexpansive mapping in the intermediate sense, we have $\gamma_n = 0$, $\kappa = 0$ for all $n \in \mathbb{N}$. From (3.7), we have

$$\|z_n - p\|^2 \leq \|x_n - p\|^2 - (\alpha_n - \kappa)\beta_n \|t_n - S^n t_n\|^2 + \theta_n,$$

where $\theta_n = \beta_n c_n$. Thus, we have

$$F(S) \cap \Omega \subset C_n, \ \forall n \in \mathbb{N},$$

and hence, the result of Step 1 holds.

Next, following the reasoning in the proof of Theorem 3.1 and using $F(S) \cap \Omega$ instead of $B_{r_0}(x_1) \cap F(S) \cap \Omega$, we deduce the conclusion of Theorem 3.8. □

Remark 3.9. Theorem 3.8 improves and extends Theorem C since the nonexpansive mapping S in Theorem C is extended to be an asymptotically nonexpansive mapping in the intermediate sense.

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