Shatanawi and Al-Rawashdeh *Fixed Point Theory and Applications* 2012, **2012**:80
http://www.fixedpointtheoryandapplications.com/content/2012/1/80

Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH Open Access

Common fixed points of almost generalized (ψ, φ)-contractive mappings in ordered metric spaces

Wasfi Shatanawi[1] and Ahmed Al-Rawashdeh[2*]

* Correspondence: aalrawashdeh@uaeu.ac.ae
[2]Department of Mathematical Sciences, UAEU, 17551 Al Ain, United Arab Emirates
Full list of author information is available at the end of the article

Abstract

In this article, we introduce the notion of almost generalized (ψ, φ)-contractive mappings in ordered metric spaces and we establish some fixed and common fixed point results in ordered complete metric spaces. Our results generalize several well-known comparable results in the literature. Finally, an example and an application are given in order to support the useability of our results.

Mathematics Subject Classifications 2000: 54H25; 47H10; 54E50

Keywords: fixed point, generalized contractions, complete ordered metric spaces

1 Introduction

A fundamental principle in computer science is iteration. Iterative techniques are used to find roots of equations, solutions of linear and nonlinear systems of equations, and solution of differential equations. So the attraction of the fixed point iteration is understandable to a large number of mathematicians.

The Banach contraction principle (see [1]) is a very popular tool for solving problems in nonlinear analysis. Some authors generalized this interesting theorem in different ways (see for example [2-20]).

Berinde [21-24] initiated the concept of almost contractions and studied many interesting fixed point theorems for a Ćirić strong almost contraction. So, let us recall the following definition.

Definition 1.1. [21]*A single valued mapping* $f : X \times X$ *is called a Ćirić strong almost contraction if there exist a constant* $\alpha \in [0, 1)$ *and some* $L \geq 0$ *such that*

$$d(fx, fy) \leq \alpha M(x, y) + Ld(y, fx)$$

for all $x, y \in X$*, where*

$$M(x, y) = \max\left\{d(x, y), d(x, fx), d(y, fy), \frac{d(x, fy) + d(y, fx)}{2}\right\}.$$

Babu [25] introduced the class of mappings which satisfy "condition (B)".

Definition 1.2. [25]*Let* (X, d) *be a metric space. A mapping* $f : X \to X$ *is said to satisfy "condition (B)" if there exist a constant* $\delta \in (0, 1)$ *and some* $L \geq 0$ *such that*

$$d(fx, fy) \leq \delta d(x, y) + L\min\{d(x, fx), d(x, fy), d(y, fx)\},$$

for all $x, y \in X$.

Moreover, Babu proved in [25] the existence of fixed point theorem for such mappings on complete metric spaces.

Ćirić et al. [26] introduced the concept of almost generalized contractive condition and they proved some existing results.

Definition 1.3. [26]*Let* $(X,\preccurlyeq)$ *be a partially ordered set. Two mappings* $f, g : X \to X$ *are said to be strictly weakly increasing if* $fx \prec gfx$ *and* $gx \prec fgx$, *for all* $x \in X$.

Definition 1.4. [26]*Let f and g be two self mappings on a metric space* (X, d). *Then they are said to satisfy almost generalized contractive condition if there exist a constant* $\delta \in (0, 1)$ *and some* $L \geq 0$ *such that*

$$\begin{aligned} d(fx, fy) \leq \delta \max\left\{d(x,y), d(x,fx), d(y,gy), \frac{d(x,gy)+d(y,fx)}{2}\right\} \\ + L\min\{d(x,fx), d(x,gy), d(y,fx)\}, \end{aligned} \tag{1}$$

for all $x, y \in X$.

Then Ćirić et al. [26] proved the following theorems.

Theorem 1.1. *Let* $(X, \preccurlyeq)$ *be a partially ordered set and suppose that there exists a metric d on X such that the metric space* (X, d) *is complete. Let* $f : X \to X$ *be a strictly increasing continuous mapping with respect to* $\preccurlyeq$. *Suppose that there exist a constant* $\delta \in [0, 1)$ *and some* $L \geq 0$ *such that*

$$d(fx, fy) \leq \delta M(x,y) + L\min\{d(x,fx), d(x,fy), d(y,fx)\},$$

for all comparable $x, y \in X$, *where*

$$M(x,y) = \max\left\{d(x,y), d(x,fx), d(y,fy), \frac{d(x,fy)+d(y,fx)}{2}\right\}.$$

If there exists $x_0 \in X$ *such that* $x_0 \preccurlyeq fx_0$, *then f has a fixed point in X.*

Theorem 1.2. *Let* $(X, \preccurlyeq)$ *be a partially ordered set and suppose that there exists a metric d on X such that the metric space* (X, d) *is complete. Let* $f, g : X \to X$ *be two strictly weakly increasing mappings which satisfy* (1) *with respect to* $\preccurlyeq$, *for all comparable elements* $x, y \in X$. *If either f or g is continuous, then f and g have a common fixed point in X.*

Khan et al. [27] introduced the concept of altering distance function as follows.

Definition 1.5. [27]*The function* $\varphi : [0, +\infty) \to [0, +\infty)$ *is called an altering distance function, if the following properties are satisfied:*

(1) φ *is continuous and non-decreasing.*

(2) $\varphi(t) = 0$ *if and only if* $t = 0$.

Many authors studied fixed point theorems which are based on altering distance functions, see for example [27-36].

In this article, we introduce the notion of almost generalized (ψ, φ)-contractive mapping and we establish some results in complete ordered metric spaces, where ψ and φ are altering distance functions. Our results generalize Theorems 1.1 and 1.2.

2 Main results

In this section, we define the notion of almost generalized (ψ, φ)-contractive mapping, then we present and prove our new results. In particular, we generalize Theorems 2.1, 2.2 and 2.3 of Ćirić et al. [26].

Let $(X, \preccurlyeq, d)$ be an ordered metric space and let $f : X \to X$ be a mapping. Set

$$M(x,y) = \max\left\{d(x,y), d(x,fx), d(y,fy), \frac{d(x,fy)+d(y,fx)}{2}\right\}$$

and

$$N(x,y) = \min\{d(x,fx), d(y,fx)\}.$$

Now, we introduce the following definition.

Definition 2.1. *Let (X, d) be a metric space and let ψ and φ be altering distance functions. We say that a mapping $f : X \to X$ is an almost generalized (ψ, φ)-contractive mapping if there exists $L \geq 0$ such that*

$$\psi(d(fx,fy)) \leq \psi(M(x,y)) - \phi(M(x,y)) + L\psi(N(x,y) \tag{2}$$

for all comparable $x, y \in X$.

Throughout this article, the mappings ψ and φ denote altering distance functions. Now, let us prove our first result.

Theorem 2.1. *Let $(X, \preccurlyeq)$ be a partially ordered set and suppose that there exists a metric d on X such that the metric (X, d) is complete. Let $f : X \to X$ be a non-decreasing continuous mapping with respect to $\preccurlyeq$. Suppose that f is an almost generalized (ψ, φ)-contractive mapping. If there exists $x_0 \in X$ such that $x_0 \preccurlyeq fx_0$, then f has a fixed point.*

Proof. Let $x_0 \in X$. Then, we define a sequences (x_n) in X such that $x_{n+1} = fx_n$. Since $x_0 \preccurlyeq fx_0 = x_1$ and f is non-decreasing, we have $x_1 = fx_0 \preccurlyeq x_2 = fx_1$. Again, as $x_1 \preccurlyeq x_2$ and f is non-decreasing, we have $x_2 = fx_1 \preccurlyeq x_3 = fx_2$. By induction, we show that

$$x_0 \preccurlyeq x_1 \preccurlyeq \ldots \preccurlyeq x_n \preccurlyeq x_{n+1} \preccurlyeq \ldots .$$

If $x_n = x_{n+1}$ for some $n \in \mathbb{N}$, then $x_n = fx_n$ and hence x_n is a fixed point of f. So, we may assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. By (2), we have

$$\begin{aligned}\psi(d(x_n, x_{n+1})) &= \psi(d(fx_{n-1}, fx_n)) \\ &\leq \psi(M(x_{n-1}, x_n)) - \phi(M(x_{n-1}, x_n)) + L\psi(N(x_{n-1}, x_n)),\end{aligned} \tag{3}$$

where

$$\begin{aligned}M(x_{n-1}, x_n) &= \max\left\{d(x_{n-1}, x_n), d(x_{n-1}, fx_{n-1}), d(x_n, fx_n), \frac{d(x_{n-1}, fx_n) + d(x_n, fx_{n-1})}{2}\right\} \\ &= \max\left\{d(x_{n-1}, x_n), d(x_n, x_{n+1}), \frac{d(x_{n-1}, x_{n+1})}{2}\right\} \\ &\leq \max\left\{d(x_{n-1}, x_n), d(x_n, x_{n+1}), \frac{d(x_{n-1}, x_n) + d(x_n, x_{n+1})}{2}\right\} \\ &= \max\left\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\right\}\end{aligned} \tag{4}$$

and

$$\begin{aligned}N(x_{n-1}, x_n) &= \min\{d(x_{n-1}, fx_{n-1}), d(x_n, fx_{n-1})\} \\ &= \min\{d(x_{n-1}, x_n), 0\} \\ &= 0.\end{aligned} \tag{5}$$

From (3)-(5) and the properties of ψ and φ, we get

$$\begin{aligned}\psi(d(x_n, x_{n+1})) &\leq \psi(\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\}) - \phi(\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\}) \\ &\leq \psi(\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\})\end{aligned} \tag{6}$$

If

$$\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\} = d(x_n, x_{n+1}),$$

then by (6) we have

$$\begin{aligned}\psi(d(x_n, x_{n+1})) &\le \psi(d(x_n, x_{n+1})) - \phi(d(x_n, x_{n+1})) \\ &< \psi(d(x_n, x_{n+1})),\end{aligned}$$

which gives a contradiction. Thus,

$$\max\{d(x_{n-1}, x_n), d(x_n, x_{n+1})\} = d(x_{n-1}, x_n).$$

Therefore (6) becomes

$$\psi(d(x_n, x_{n+1})) \le \psi(d(x_n, x_{n-1})) - \phi(d(x_{n-1}, x_n)) < \psi(d(x_n, x_{n-1})). \tag{7}$$

Since ψ is a non-decreasing mapping, we have $\{d(x_n, x_{n+1}): n \in \mathbb{N} \cup \{0\}\}$ is a non-increasing sequence of positive numbers. So, there exists $r \ge 0$ such that

$$\lim_{n\to+\infty} d(x_n, x_{n+1}) = r.$$

Letting $n \to +\infty$ in (7), we get

$$\psi(r) \le \psi(r) - \phi(r) \le \psi(r).$$

Therefore $\varphi(r) = 0$, and hence $r = 0$. Thus, we have

$$\lim_{n\to+\infty} d(x_n, x_{n+1}) = 0. \tag{8}$$

Next, we show that (x_n) is a Cauchy sequence in X. Suppose to the contrary; that is, (x_n) is not a Cauchy sequence. Then there exists $\varepsilon > 0$ for which we can find two subsequences $(x_{m(i)})$ and $(x_{n(i)})$ of (x_n) such that $n(i)$ is the smallest index for which

$$n(i) > m(i) > i, \quad d(x_{m(i)}, x_{n(i)}) \ge \varepsilon. \tag{9}$$

This means that

$$d(x_{m(i)}, x_{n(i)-1}) < \varepsilon. \tag{10}$$

From (9), (10) and the triangular inequality, we get

$$\begin{aligned}\varepsilon &\le d(x_{m(i)}, x_{n(i)}) \\ &\le d(x_{m(i)}, x_{m(i)-1}) + d(x_{m(i)-1}, x_{n(i)}) \\ &\le d(x_{m(i)}, x_{m(i)-1}) + d(x_{m(i)-1}, x_{n(i)-1}) + d(x_{n(i)-1}, x_{n(i)}) \\ &\le 2d(x_{m(i)}, x_{m(i)-1}) + d(x_{m(i)}, x_{n(i)-1}) + d(x_{n(i)-1}, x_{n(i)}) \\ &< 2d(x_{m(i)}, x_{m(i)-1}) + \varepsilon + d(x_{n(i)-1}, x_{n(i)}).\end{aligned}$$

Using (8) and letting $i \to +\infty$, we get

$$\begin{aligned}\lim_{i\to+\infty} d(x_{m(i)}, x_{n(i)}) &= \lim_{i\to+\infty} d(x_{m(i)-1}, x_{n(i)}) \\ &= d(x_{m(i)}, x_{n(i)-1}) \\ &= d(x_{m(i)-1}, x_{n(i)-1}) \\ &= \varepsilon.\end{aligned} \tag{11}$$

From (2), we have

$$\begin{aligned}\psi(d(x_{m(i)}, x_{n(i)})) &= \psi(d(fx_{m(i)-1}, fx_{n(i)-1}) \\ &\leq \psi(M(x_{m(i)-1}, x_{n(i)-1})) - \phi(M(x_{m(i)-1}, x_{n(i)-1})) + L\psi(N(x_{m(i)-1}, x_{n(i)-1})),\end{aligned} \tag{12}$$

where

$$\begin{aligned}M(x_{m(i)-1}, x_{n(i)-1}) &= \max\left\{d(x_{m(i)-1}, x_{n(i)-1}), d(x_{m(i)-1}, fx_{m(i)} - 1), d(x_{n(i)-1}, fx_{n(i)-1}),\right. \\ &\qquad \left.\frac{d(x_{m(i)-1}, fx_{n(i)-1}) + d(fx_{m(i)-1}, x_{n(i)-1})}{2}\right\} \\ &= \max\left\{d(x_{m(i)-1}, x_{n(i)-1}), d(x_{m(i)-1}, x_{m(i)}), d(x_{n(i)-1}, x_{n(i)}),\right. \\ &\qquad \left.\frac{d(x_{m(i)-1}, x_{n(i)}) + M(x_{m(i)}, x_{n(i)-1})}{2}\right\}\end{aligned} \tag{13}$$

and

$$\begin{aligned}N(x_{m(i)-1}, x_{n(i)-1}) &= \min\{d(x_{m(i)-1}, fx_{m(i)-1}), d(fx_{m(i)-1}, x_{n(i)-1})\} \\ &= \min\{d(x_{m(i)-1}, x_{m(i)}), d(x_{m(i)}, x_{n(i)-1})\}.\end{aligned} \tag{14}$$

Letting $i \to +\infty$ in (13) and (14) then using (8) and (11), we get

$$\lim_{i\to+\infty} M(x_{m(i)-1}, x_{n(i)-1}) = \varepsilon \tag{15}$$

and

$$\lim_{i\to+\infty} N(x_{m(i)-1}, x_{n(i)-1}) = 0. \tag{16}$$

Letting $i \to +\infty$ in (12) then using (11), (15) and (16) we have

$$\psi(\varepsilon) \leq \psi(\varepsilon) - \phi(\varepsilon) < \psi(\varepsilon),$$

which gives a contradiction. Thus $(x_{n+1} = fx_n)$ is a Cauchy sequence in X. As X is a complete space, there exists $u \in X$ such that

$$\lim_{n\to+\infty} x_{n+1} = \lim_{n\to+\infty} fx_n = u.$$

Now, suppose that f is continuous, then $fx_n \to fu$. By the uniqueness of limit, we have $fu = u$. Thus u is a fixed point of f. □

Notice that the continuity of f in Theorem 2.1 is not necessary and can be dropped.

Theorem 2.2. *Under the same hypotheses of Theorem* 2.1 *and without assuming the continuity of f, assume that whenever* (x_n) *is a non-decreasing sequence in X such that* $x_n \to x \in X$ *implies* $x_n \preccurlyeq x$, *for all* $n \in \mathbb{N}$. *Then f has a fixed point in X.*

Proof. Following similar arguments to those given in Theorem 2.1, we construct an increasing sequence (x_n) in X such that $x_n \to u$ for some $u \in X$. Using the assumption of X, we have $x_n \preccurlyeq u$ for all $n \in \mathbb{N}$. Now, we show that $fu = u$. By (2), we have

$$\begin{aligned}\psi(d(x_{n+1}, fu)) &= \psi(d(fx_n, fu)) \\ &\leq \psi(M(x_n, u)) - \phi(M(x_n, u)) + L\psi(N(x_n, u)),\end{aligned} \tag{17}$$

where

$$\begin{aligned}M(x_n, u) &= \max\left\{d(x_n, u), d(x_n, fx_n), d(u, fu), \frac{d(x_n, fu) + d(fx_n, u)}{2}\right\} \\ &= \max\left\{d(x_n, u), d(x_n, x_{n+1}), d(u, fu), \frac{d(x_n, fu) + d(x_{n+1}, u)}{2}\right\}\end{aligned} \tag{18}$$

and

$$\begin{aligned} N(x_n, u) &= \min\{d(x_n, fx_n), d(u, fx_n)\} \\ &= \min\{d(x_n, x_{n+1}), d(u, x_{n+1})\}. \end{aligned} \tag{19}$$

Letting $n \to +\infty$ in (18) and (19) then we get $M(x_n, u) \to d(u, fu)$ and $N(x_n, u) \to 0$. Again when $n \to +\infty$ in (17) then we get

$$\psi(d(u, fu)) \leq \psi(d(u, fu)) - \phi(d(u, fu)).$$

Therefore, $d(u, fu) = 0$. Thus $u = fu$ and hence u is a fixed point of f. □

Corollary 2.1. *Let* $(X, \preccurlyeq)$ *be a partially ordered set and suppose that there exists a metric* d *on* X *such that the metric* (X, d) *is complete. Let* $f : X \to X$ *be a non-decreasing continuous mapping with respect to* $\preccurlyeq$. *Suppose that there exist* $k \in [0, 1)$ *and* $L \geq 0$ *such that*

$$d(fx, fy) \leq k \max\left\{d(x, y), d(x, fx), d(y, fy), \frac{d(x, fy) + d(y, fx)}{2}\right\} + L \min\{d(x, fx), d(y, fx)\}$$

for all comparable $x, y \in X$. *If there exists* $x_0 \in X$ *such that* $x_0 \leq fx_0$, *then* f *has a fixed point.*

Proof. Follows from Theorem (2.1) by taking $\psi(t) = t$ and $\varphi(t) = (1\text{-}k)t$ for all $t \in [0, +\infty)$ and noticing that f is an almost generalized (ψ, φ)-contractive mapping. □

The continuity of f in Corollary 2.1 is not necessary and can be dropped.

Corollary 2.2. *Under the hypotheses of Corollary* 2.1 *and without assuming the continuity of* f, *assume that whenever* (x_n) *is a non-decreasing sequence in* X *such that* $x_n \to x \in X$ *implies* $x_n \preccurlyeq x$ *for all* $n \in \mathbb{N}$. *Then* f *has a fixed point in* X.

Proof. Follows from Theorem (2.2) by taking $\psi(t) = t$ and $\varphi(t) = (1 \text{-}k)t$ for all $t \in [0, +\infty)$. □

Now, let $(X, \preccurlyeq, d)$ be an ordered metric space and let $f, g : X \to X$ be two mappings. Set

$$M(x, y) = \max\left\{d(x, y), d(x, fx), d(y, gy), \frac{d(x, gy) + d(y, fx)}{2}\right\}$$

and

$$N(x, y) = \min\{d(x, fx), d(y, fx), d(x, gy)\}.$$

Then we introduce the following definition.

Definition 2.2. *Let* $(X, \preccurlyeq)$ *be a partially ordered set having a metric* d, *and let* ψ *and* φ *be altering distance functions. We say that a mapping* $f : X \to X$ *is an almost generalized* (ψ, φ)*-contractive mapping with respect to a mapping* $g : X \to X$ *if there exists* $L \geq 0$ *such that*

$$\psi(d(fx, gy)) \leq \psi(M(x, y)) - \phi(M(x, y)) + L\psi(N(x, y)) \tag{20}$$

for all comparable $x, y \in X$.

Definition 2.3. *Let* $(X, \preccurlyeq)$ *be a partially ordered set. Then two mappings* $f, g : X \to X$ *are said to be weakly increasing if* $fx \preccurlyeq gfx$ *and* $gx \preccurlyeq fgx$, *for all* $X \in X$.

Note that every strictly weakly increasing mapping is weakly increasing.

Theorem 2.3. *Let* $(X, \preccurlyeq)$ *be a partially ordered set and suppose that there exists a metric* d *on* X *such that the metric* (X, d) *is complete, and let* $f, g : X \to X$ *be two* *weakly increasing mappings with respect to* $\preccurlyeq$. *Suppose that* f *is an almost generalized* (ψ, φ)*-contractive mapping with respect to* g. *If either* f *or* g *is continuous, then* f *and* g *have a common fixed point.*

Proof. Let us divide the proof into two parts:

First part: We prove that u is a fixed point of f if and only if u is a fixed point of g. Now, suppose that u is a fixed point of f, then $fu = u$. As $u \preccurlyeq u$, by (20), we have

$$\begin{aligned}\psi(d(u,gu)) &= \psi(d(fu,gu))\\ &\le \psi\left(\max\left\{d(u,fu),d(u,gu),\frac{1}{2}(d(u,gu)+d(u,fu))\right\}\right)\\ &\quad -\phi\left(\max\left\{d(u,fu),d(u,gu),\frac{1}{2}(d(u,gu)+d(u,fu))\right\}\right)+L\min\{d(u,fu),d(u,gu)\}\\ &= \psi(d(u,gu))-\phi(d(u,gu)).\end{aligned}$$

Thus we have $\varphi(d(u, gu)) = 0$. Therefore $d(u, gu) = 0$ and hence $gu = u$. Similarly, we show that if u is a fixed point of g, then u is a fixed point of f.

Second part (construction a sequence by iterative technique):

Let $x_0 \in X$. We construct a sequence (x_n) in X such that $x_{2n+1} = fx_{2n}$ and $x_{2n+2} = gx_{2n+1}$, for all non-negative integers, i.e. $n \in \mathbb{N} \cup \{0\}$. As f and g are weakly increasing with respect $\preccurlyeq$, we obtain the following:

$$x_1 = fx_0 \preccurlyeq gfx_0 = x_2 = gx_1 \preccurlyeq fgx_1 = x_3 \preccurlyeq \ldots x_{2n+1} = fx_{2n} \preccurlyeq gfx_{2n} = x_{2n+2} \preccurlyeq \ldots .$$

If $x_{2n} = x_{2n+1}$ for some $n \in \mathbb{N}$, then $x_{2n} = fx_{2n}$. Thus x_{2n} is a fixed point of f. By the first part, we conclude that x_{2n} is also a fixed point of g.

If $x_{2n+1} = x_{2n+2}$ for some $n \in \mathbb{N}$, then $x_{2n+1} = gx_{2n+1}$. Thus x_{2n+1} is a fixed point of g. By the first part, we conclude that x_{2n+1} is also a fixed point of f.

Therefore, we may assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Now we complete the proof in the following steps:

Step 1. We will prove that

$$\lim_{n\to+\infty} d(x_n, x_{n+1}) = 0.$$

As x_{2n+1} and x_{2n+2} are comparable, by (20), we have

$$\begin{aligned}\psi(d(x_{2n+1},x_{2n+2})) &= \psi(d(fx_{2n},gx_{2n+1}))\\ &\le \psi(M(x_{2n},x_{2n+1}))-\phi(M(x_{2n},x_{2n+1}))+L\psi(N(x_{2n},x_{2n+1})),\end{aligned}$$

where

$$\begin{aligned}M(x_{2n},x_{2n+1}) &= \max\left\{d(x_{2n},x_{2n+1}),d(x_{2n},fx_{2n}),d(x_{2n+1},gx_{2n+1}),\frac{d(fx_{2n},x_{2n+1})+d(x_{2n},gx_{2n+1})}{2}\right\}\\ &= \max\left\{d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2}),\frac{d(x_{2n},x_{2n+2})}{2}\right\}\\ &\le \max\{d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+2})\},\end{aligned}$$

and

$$\begin{aligned}N(x_{2n},x_{2n+1}) &= \min\{d(x_{2n},fx_{2n}),d(x_{2n+1},fx_{2n}),d(x_{2n},gx_{2n+1})\}\\ &= \min\{d(x_{2n},x_{2n+1}),d(x_{2n+1},x_{2n+1}),d(x_{2n},x_{2n+2})\}\\ &= 0.\end{aligned}$$

So, we have

$$\psi(d(x_{2n+1}, x_{2n+2})) \leq \psi(\max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\}) \\ - \phi(\max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\}) \tag{21}$$

If

$$\max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\} = d(x_{2n+1}, x_{2n+2}),$$

then (21) becomes

$$\psi(d(x_{2n+1}, x_{2n+2})) \leq \psi(d(x_{2n+1}, x_{2n+2})) - \phi(d(x_{2n+1}, x_{2n+2})) < \psi(d(x_{2n+1}, x_{2n+2})),$$

which gives a contradiction. So

$$\max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\} = d(x_{2n}, x_{2n+1}),$$

and hence (21) becomes

$$\psi(d(x_{2n+1}, x_{2n+2})) \leq \psi(d(x_{2n}, x_{2n+1})) - \phi(d(x_{2n}, x_{2n+1})) \leq \psi(d(x_{2n}, x_{2n+1})). \tag{22}$$

Similarly, we show that

$$\psi(d(x_{2n+1}, x_{2n})) \leq \psi(d(x_{2n-1}, x_{2n})) - \phi(d(x_{2n-1}, x_{2n})) \leq \psi(d(x_{2n-1}, x_{2n})). \tag{23}$$

By (22) and (23), we get that $\{d(x_n, x_{n+1}); n \in \mathbb{N}\}$ is a non-increasing sequence of positive numbers. Hence there is $r \geq 0$ such that

$$\lim_{n \to +\infty} d(x_n, x_{n+1}) = r.$$

Letting $n \to +\infty$ in (22), we get

$$\psi(r) \leq \psi(r) - \phi(r) \leq \psi(r),$$

which implies that $\varphi(r) = 0$ and hence $r = 0$.

So, we have

$$\lim_{n \to +\infty} d(x_n, x_{n+1}) = 0. \tag{24}$$

Step 2. We will prove that (x_n) is a Cauchy sequence. It is sufficient to show that (x_{2n}) is a Cauchy sequence. Suppose to the contrary; that is, (x_{2n}) is not a Cauchy sequence. Then there exists $\varepsilon > 0$ for which we can find two subsequences of positive integers $(x_{2m(i)})$ and $(x_{2n(i)})$ such that $n(i)$ is the smallest index for which

$$n(i) > m(i) > i, \ d(x_{2m(i)}, x_{2n(i)}) \geq \varepsilon. \tag{25}$$

This means that

$$d(x_{2m(i)}, x_{2n(i)-2}) < \varepsilon. \tag{26}$$

From (25), (26) and the triangular inequality, we get

$$\begin{aligned} \varepsilon &\leq d(x_{2m(i)}, x_{2n(i)}) \\ &\leq d(x_{2m(i)}, x_{2n(i)-2}) + d(x_{2n(i)-2}, x_{2n(i)-1}) + d(x_{2n(i)-1}, x_{2n(i)}) \\ &< \varepsilon + d(x_{2n(i)-2}, x_{2n(i)-1}) + d(x_{2n(i)-1}, x_{2n(i)}). \end{aligned}$$

By letting $i \to +\infty$ in the above inequality and using (24), we have

$$\lim_{i\to+\infty} d(x_{2m(i)}, x_{2n(i)}) = \varepsilon. \tag{27}$$

Moreover,

$$\begin{aligned}
\varepsilon &\le d(x_{2m(i)}, x_{2n(i)})\\
&\le d(x_{2m(i)}, x_{2m(i)+1}) + d(x_{2m(i)+1}, x_{2n(i)+1}) + d(x_{2n(i)+1}, x_{2n(i)})\\
&\le d(x_{2m(i)}, x_{2m(i)+1}) + d(x_{2m(i)+1}, x_{2n(i)}) + 2d(x_{2n(i)+1}, x_{2n(i)})\\
&\le d(x_{2m(i)}, x_{2m(i)+1}) + d(x_{2m(i)+1}, x_{2m(i)+2}) + d(x_{2m(i)+2}, x_{2n(i)}) + 2d(x_{2n(i)+1}, x_{2n(i)})\\
&\le 2d(x_{2m(i)}, x_{2m(i)+1}) + 2d(x_{2m(i)+1}, x_{2m(i)+2}) + d(x_{2m(i)}, x_{2n(i)}) + 2d(x_{2n(i)+1}, x_{2n(i)}).
\end{aligned}$$

Using (24), (27) and letting $i \to +\infty$, we get

$$\begin{aligned}
\lim_{i\to+\infty} d(x_{2m(i)}, x_{2n(i)}) &= \lim_{i\to+\infty} d(x_{2m(i)+1}, x_{2n(i)+1})\\
&= \lim_{i\to+\infty} d(x_{2m(i)+1}, x_{2n(i)})\\
&= \lim_{i\to+\infty} d(x_{2m(i)+2}, x_{2n(i)}) = \varepsilon.
\end{aligned} \tag{28}$$

Since $x_{2n(i)}$ and $x_{2m(i)+1}$ are comparable, so by (20) we have

$$\begin{aligned}
\psi(d(x_{2n(i)+1}, x_{2m(i)+2})) &= \psi(d(fx_{2n(i)}, gx_{2m(i)+1}))\\
&\le \psi(M(x_{2n(i)}, x_{2m(i)+1})) - \phi(M(x_{2n(i)}, x_{2m(i)+1})) + L\psi(N(x_{2n(i)}, x_{2m(i)+1})),
\end{aligned} \tag{29}$$

where

$$\begin{aligned}
M(x_{2n(i)}, x_{2m(i)+1}) &= \max\Big\{d(x_{2n(i)}, x_{2m(i)+1}), d(x_{2n(i)}, fx_{2n(i)}), d(x_{2m(i)+1}, gx_{2m(i)+1}),\\
&\qquad \frac{d(fx_{2n(i)}, x_{2m(i)+1}) + d(x_{2n(i)}, gx_{2m(i)+1})}{2}\Big\}\\
&= \max\Big\{d(x_{2n(i)}, x_{2m(i)+1}), d(x_{2n(i)}, x_{2n(i)+1}), d(x_{2m(i)+1}, x_{2m(i)+2}),\\
&\qquad \frac{d(x_{2n(i)+1}, x_{2m(i)+1}) + d(x_{2n(i)}, x_{2m(i)+2})}{2}\Big\}
\end{aligned} \tag{30}$$

and

$$\begin{aligned}
N(x_{2n(i)}, x_{2m(i)+1}) &= \max\{d(x_{2n(i)}, fx_{2n(i)}), d(x_{2m(i)+1}, fx_{2n(i)}), d(x_{2n(i)}, gx_{2m(i)+1})\}\\
&= \max\{d(x_{2n(i)}, x_{2n(i)+1}), d(x_{2m(i)+1}, x_{2n(i)+1}), d(x_{2n(i)}, x_{2m(i)+2})\}\\
&= 0.
\end{aligned} \tag{31}$$

By letting $i \to +\infty$ in (30) and (31), we get

$$\lim_{i\to+\infty} M(x_{2n(i)}, x_{2m(i)+1}) = \varepsilon.$$

Now, letting $i \to +\infty$ in (29) we get

$$\psi(\varepsilon) \le \psi(\varepsilon) - \phi(\varepsilon).$$

So $\varphi(\varepsilon) = 0$ and then $\varepsilon = 0$, which is a contradiction. Hence (x_n) is a Cauchy sequence in X.

Step 3. (A common fixed point)

As (x_n) is a Cauchy sequence in X which is a complete space, there exists $u \in X$ such that $x_n \to u$ as $n \to +\infty$. Without loss of generality, we may assume that f is continuous. Since $x_{2n} \to u$ as $n \to +\infty$, we have $x_{2n+1} = fx_{2n} \to fu$ as $n \to +\infty$. By the uniqueness of limit we get $fu = u$. Thus u is a fixed point of f. By the first part, we conclude that u is also a fixed point of g. □

The continuity of one of the functions f or g in Theorem 2.3 is not necessary and can be dropped.

Theorem 2.4. *Under the hypotheses of Theorem* 2.3 *and without assuming the continuity of one of the functions f or g, assume that whenever (x_n) is a non-decreasing sequence in X such that $x_n \to x \in X$ implies $x_n \preccurlyeq x$, for all $n \in \mathbb{N}$. Then f and g have a common fixed point in X.*

Proof. Following similar arguments to those given in Theorem 2.3, we construct an increasing sequence (x_n) in X such that $x_n \to u$ for some $u \in X$. Using the assumption on X, we have $x_n \preccurlyeq u$ for all $n \in \mathbb{N}$. Now, we show that $fu = gu = u$. By (2), we have

$$\begin{aligned} \psi(d(x_{2n+1}, gu)) &= \psi(d(fx_{2n}, gu)) \\ &\leq \psi(M(x_{2n}, u)) - \phi(M(x_{2n}, u)) + L\psi(N(x_{2n}, u)), \end{aligned} \tag{32}$$

where

$$\begin{aligned} M(x_{2n}, u) &= \max\left\{d(x_{2n}, u), d(x_{2n}, fx_{2n}), d(u, gu), \frac{d(x_{2n}, gu) + d(fx_{2n}, u)}{2}\right\} \\ &= \max\left\{d(x_{2n}, u), d(x_{2n}, x_{2n+1}), d(u, gu), \frac{d(x_{2n}, gu) + d(x_{2n+1}, u)}{2}\right\} \end{aligned} \tag{33}$$

and

$$\begin{aligned} N(x_{2n}, u) &= \min\{d(x_{2n}, fx_{2n}), d(u, fx_{2n}), d(x_{2n}, gu)\} \\ &= \min\{d(x_{2n}, x_{2n+1}), d(u, x_{2n+1}), d(x_{2n}, gu)\}. \end{aligned} \tag{34}$$

Letting $n \to +\infty$ in (33) and (34) then we get $M(x_{2n}, u) \to d(u, gu)$ and $N(x_{2n}, u) \to 0$ as $n \to +\infty$. Again when $n \to +\infty$ in (32), we get

$$\psi(d(u, gu)) \leq \psi(d(u, gu)) - \phi(d(u, gu)).$$

Therefore, $d(u, gu) = 0$ thus $u = gu$ and then u is a fixed point of f. Similarly, we may show that $fu = u$. Hence u is a common fixed point of f and g. □

Then we have the following consequence results.

Corollary 2.3. *Let $(X, \preccurlyeq)$ be a partially ordered set and suppose that there exists a metric d on X such that the metric (X, d) is complete. Let $f, g : X \to X$ be two strictly weakly increasing mappings with respect to $\preccurlyeq$. Suppose that there exist $k \in [0, 1)$ and $L \geq 0$ such that*

$$d(fx, gy) \leq k \max\left\{d(x, y), d(x, fx), d(y, gy), \frac{d(x, gy) + d(fx, y)}{2}\right\} + L \min\{d(x, fx), d(y, fx), d(x, gy)\}$$

for all comparable $x, y \in X$. If either f or g is continuous, then f and g have a common fixed point.

Proof. Define $\psi, \varphi : [0, +\infty) \to [0, +\infty)$ by $\psi(t) = t$ and $\varphi(t) = (1\text{-}k)t$. Then f is an almost generalized (ψ, φ)-contractive mapping with respect to g. The proof follows from Theorem 2.3. □

The continuity of one of the functions f or g in Corollary 2.3 is not necessary and can be dropped.

Corollary 2.4. *Under the hypotheses of Corollary* 2.3 *and without assuming the continuity of one of the functions f or g, assume that whenever (x_n) is a non-decreasing sequence in $\times$ such that $x_n \to X \in X$ implies $x_n \preccurlyeq x$, for all $n \in \mathbb{N}$. Then f and g have a common fixed point in X.*

Proof. Follows from Theorem 2.4. □

Now, in order to support the useability of our results, let us introduce the following example.

Example 2.1. *Let* $\times = \{0, 1, 2, \ldots\}$. *Define the function* $f, g : X \to X$ *by*

$$fx = \begin{cases} 0, & x = 0; \\ x - 1, & x \neq 0, \end{cases}$$

and

$$gx = \begin{cases} 0, & x \in \{0, 1\}; \\ x - 2, & x \geq 2. \end{cases}$$

Let $d : X \times X \to \mathbb{R}^+$ *be given by*

$$d(x, y) = \begin{cases} 0, & x = y; \\ x + y, & x \neq y. \end{cases}$$

Define ψ, φ: $[0, +\infty) \to [0, +\infty)$ *by* $\psi(t) = t^2$ *and* $\varphi(t) = t$. *Define a relation* $\preccurlyeq$ *on* X *by* $\times \preccurlyeq y$ *iff* $y \leq x$. *Then we have the following:*

(1) $(X, \preccurlyeq)$ *is a partially ordered set having the metric d, where the metric space* (X, d) *is complete.*

(2) f and g are weakly increasing mappings with respect to $\preccurlyeq$.

(3) f is continuous.

(4) f is an almost generalized (ψ, φ)*-contractive mapping with respect to g, that is,*

$$\psi(d(fx, fy)) \leq \psi(M(x, y)) - \phi(M(x, y)) + L\psi(N(x, y))$$

for $x, y \in X$ *with* $\times \preccurlyeq y$ *and* $L \geq 0$, *where*

$$M(x, y) = \max\left\{d(x, y), d(x, fx), d(y, gy), \frac{d(x, gy) + d(y, fx)}{2}\right\}$$

and

$$N(x, y) = \min\{d(x, fx), d(y, fx), d(x, gy)\}.$$

Proof. The proof of (1) is clear. To prove (2), let $x \in X$. If $x \in \{0, 1, 2\}$, then $fgx = 0 \leq gx = 0$ and $gfx = 0 \leq fx$. So $gx \preccurlyeq fgx$ and $fx \preccurlyeq gfx$. While, if $x \geq 3$, then $fgx = x - 3 \leq x - 2 = gx$ and $gfx = x - 3 \leq x - 1 = fx$. So $gx \preccurlyeq fgx$ and $fx \preccurlyeq gfx$. Hence f and g are weakly increasing mappings with respect to $\preccurlyeq$. To prove that f is continuous, let (x_n) be a sequence in X such that $x_n \to x \in X$. By the definition of the metric d, there exists $k \in \mathbb{N}$ such that $x_n = x$ for all $n \geq k$. So $fx_n = fx$ for all $n \geq k$. Hence $fx_n \to fx$ that is, f is continuous. To prove (4), given $x, y \in X$ with $x \preccurlyeq y$. So $y \leq x$. Thus, we have the following cases:

Case 1. $x \in \{0, 1\}$ and $y \in \{0, 1\}$. Then $d(fx, fy) = 0$ and hence

$$\psi(d(fx, fy)) \leq \psi(M(x, y)) - \phi(M(x, y)) + L\psi(N(x, y))$$

for each $L \geq 0$.

Case 2. $x, y \geq 2$ and $x = y$. Then $d(fx, fy) = 0$ and hence

$$\psi(d(fx, fy)) \leq \psi(M(x, y)) - \phi(M(x, y)) + L\psi(N(x, y))$$

for each $L \geq 0$.

Case 3. $x > y \geq 2$.
If $x = y + 1$, then

$$d(fx,fy) = d(fx,f(x--1) = d(x--1,x--2) = 2x--3,$$

and

$$\begin{aligned} M(x,\ y) &= M(x,\ x-1) \\ &= \max\left\{d(x,\ x-1),\ d(x,\ fx),\ d(x-1,\ g(x-1),\ \frac{d(x,g(x-1)) + d(x-1,fx)}{2}\right\} \\ &= \max\left\{2x-1, 2x-4,\ \frac{2x-3}{2}\right\} = 2x-1. \end{aligned}$$

Since

$$(2x-3)^2 \leq (2x-1)^2 - (2x-1),$$

we have

$$\psi(d(fx,fy)) \leq \psi(M(x,\ y)) - \phi(M(x,y)) + L\psi(N(x,y))$$

for each $L \geq 0$.
If $x > y + 1$, then

$$d(fx,fy) = d(x--1,y--1) = x+y--2$$

and

$$\begin{aligned} M(x,\ y) &= \max\left\{d(x,\ y),\ d(x,\ fx),\ d(y,\ gy),\ \frac{d(x,gy) + d(y,fx)}{2}\right\} \\ &= \max\left\{x+y,\ 2x-1, 2y-2,\ \frac{2x+2y-3}{2}\right\} = 2x-1. \end{aligned}$$

As

$$(x+y-2)^2 \leq (2x-3)^2 \leq (2x-1)^2 - (2x-1),$$

we have

$$\psi(d(fx,\ fy)) \leq \psi(M(x,\ y)) - \phi(M(x,\ y)) + L\psi(N(x,\ y))$$

for each $L \geq 0$. By combining all cases together, we conclude that f is an almost generalized (ψ, φ)-contractive mapping with respect to g. Thus f, g, ψ and φ satisfy all the hypotheses of Theorem 2.3 and hence f and g have a common fixed point. Indeed, 0 is the unique common fixed point of f and g. □

3 Applications

Let Φ denote the set of functions $\varphi : [0, +\infty) \to [0, +\infty)$ satisfying the following hypotheses:

(1) Every $\varphi \in \Phi$ is a Lebesgue integrable function on each compact subset of $[0, +\infty)$,

(2) For any $\varphi \in \Phi$ and $\varepsilon > 0$, $\int_0^\varepsilon \phi(s)ds > 0$.

It is an easy matter, to check that the mapping $\psi : [0, +\infty) \to [0, +\infty)$ defined by

$$\psi(t) = \int_0^t \phi(s)ds$$

is an altering distance function. Therefore, we have the following results.

Corollary 3.1. *Let* $(X, \preccurlyeq)$ *be a partially ordered set having a metric d, such that the metric space* (X, d) *is complete. Let* $f : X \to X$ *be a non-decreasing continuous mapping with respect to* $\preccurlyeq$. *Suppose that there exist* $k \in [0, 1)$ *and* $L \geq 0$ *such that*

$$\int_0^{d(fx,fy)} \phi(s)ds \leq k \int_0^{\max\{d(x,y),d(x,fx),d(y,fy),\frac{1}{2}(d(x,fy)+d(y,fx))\}} \phi(s)ds + L \int_0^{\min\{d(x,fx),d(y,fx)\}} \phi(s)ds$$

for all comparable $x, y \in X$. *If there exists* $x_0 \in X$ *such that* $x_0 \preccurlyeq fx_0$, *then f has a fixed point.*

Proof. Follows from Corollary 2.1 by taking $\psi(t) = \int_0^t \phi(s)ds$. □

Corollary 3.2. *Let* $(X, \preccurlyeq)$ *be a partially ordered set having a metric d, such that the metric space* (X, d) *is complete. Let* $f, g : X \to X$ *be two weakly increasing mappings with respect to* $\preccurlyeq$. *Suppose that there exist* $k \in [0, 1)$ *and* $L \geq 0$ *such that*

$$\int_0^{d(fx,gy)} \phi(s)ds \leq k \int_0^{\max\{d(x,y),d(x,fx),d(y,gy),\frac{1}{2}(d(x,gy)+d(y,fx))\}} \phi(s)ds + L \int_0^{\min\{d(x,fx),d(y,fx),d(x,gy)\}} \phi(s)ds$$

for all comparable $x, y \in X$. *If there exists* $x_0 \in X$ *such that* $x_0 \preccurlyeq fx_0$, *then f has a fixed point.*

Proof. Follows from Corollary 2.3 by taking $\psi(t) = \int_0^t \phi(s)ds$. □

Finally, let us finish this article by noticing the following remarks:

Remark 1. *Theorem* 2.1 *of* [26]*is a special case of Corollary* 2.1

Remark 2. *Theorem* 2.2 *of* [26]*is a special case of Corollary* 2.2

Remark 3. *Theorem* 2.3, *Corollaries* 2.4 *and* 2.6 *of* [26]*are special cases of Corollary* 2.3

Acknowledgement
The authors thank the editor and the referees for their useful comments and suggestions.

Author details
[1]Department of Mathematics, Hashemite University, Zarqa 13115, Jordan [2]Department of Mathematical Sciences, UAEU, 17551 Al Ain, United Arab Emirates

Authors' contributions
All the authors contributed equally. All authors read and approved the final manuscript.

Competing interests
The authors declare that they have no competing interests.

Received: 22 January 2012 Accepted: 9 May 2012 Published: 9 May 2012

References

1. Banach, S: Surles operations dans les ensembles et leur application aux equation sitegrales. Fund Math. **3**, 133–181 (1922)

2. Agarwal, RP, El-Gebeily, MA, O'regan, D: Generalized contractions in partially ordered metric spaces. Appl Anal. **87**, 109–116 (2008). doi:10.1080/00036810701556151
3. Aghajani, A, Radenović, S, Roshan, JR: Common fixed point results for four mappings satisfying almost generalized (*S*, *T*)-contractive condition in partially ordered metric spaces. Appl Math Comput. **218**, 5665–5670 (2012). doi:10.1016/j.amc.2011.11.061
4. Di Bari, C, Vetro, P: φ-paris and common fixed points in cone metric spaces. Rendiconti del Circolo Matematico di Palermo. **57**, 279–285 (2008). doi:10.1007/s12215-008-0020-9
5. Cho, YJ, Saadati, R, Wang, Sh: Common fixed point theorems on generalized distance in ordered cone metric spaces. Comput Math Appl. **61**, 1254–1260 (2011). doi:10.1016/j.camwa.2011.01.004
6. Ćirić, LjB: A generalization of Banach's contraction principle. Proc Am Math Soc. **45**, 265–273 (1974)
7. Ćirić, LjB: On contractive type mappings. Math Balkanica. **1**, 52–57 (1971)
8. Dutta, PN, Choudhury, BS: A generalization of contraction principle in metric spaces. Fixed Point Theory Appl **2008**, 8 (2008). (Article ID 406368), doi:10.1155/2008/406368
9. Gholizadeh, L, Saadati, R, Shatanawi, W, Vaezpour, SM: Contractive mapping in generalized, ordered metric spaces with application in integral equations. Math Probl Eng **2011**, 14 (2011). (Article ID 380784), doi:10.1155/2011/380784
10. Kadelburg, Z, Pavlović, M, Radenović, S: Common fixed point theorems for ordered contractions and quasicontractions in ordered cone metric spaces. Comput Math Appl. **59**, 3148–3159 (2010). doi:10.1016/j.camwa.2010.02.039
11. Luong, NV, Thuan, N: Fixed point theorem for generalized weak contractions satisfying rational expressions in ordered metric spaces. Fixed Point Theory Appl. **2011**, 46 (2011). doi:10.1186/1687-1812-2011-46
12. Nashine, HK, Kadelburg, Z, Radenović, S: Common fixed point theorems for weakly is-tone increasing mappings in ordered partial metric spaces. Math Comput Model (2011). doi:10.1016/j.mcm.2011.12.019
13. Nieto, JJ, Rodŕiguez-López, R: Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations. Order. **22**, 223–239 (2005). doi:10.1007/s11083-005-9018-5
14. Radenović, S, Kadelburg, Z: Generalized weak contractions in partially ordered metric spaces. Comput Math Appl. **60**, 1776–1783 (2010). doi:10.1016/j.camwa.2010.07.008
15. Radenović, S, Kadelburg, Z, Jandrlić, D, Jandrlić, A: Some results on weakly contractive maps. Bull Iran Math Soc. (2012, in press)
16. Rhoades, BE: Some theorems on weakly contractive maps. Nonlinear Anal. **47**, 2683–2693 (2001). doi:10.1016/S0362-546X(01)00388-1
17. Shatanawi, W: Fixed point theorems for nonlinear weakly *C*-contractive mappings in metric spaces. Math Comput Model. **54**, 2816–2826 (2011). doi:10.1016/j.mcm.2011.06.069
18. Shatanawi, W, Mustafa, Z, Tahat, N: Some coincidence point theorems for nonlinear contraction in ordered metric spaces. Fixed Point Theory Appl. **2011**, 68 (2011). doi:10.1186/1687-1812-2011-68
19. Samet, B, Rajović, M, Lazović, R, Stojiljković, R: Common fixed-point results for nonlinear contractions in ordered partial metric spaces. Fixed Point Theory Appl. **2011**, 71 (2011). doi:10.1186/1687-1812-2011-71
20. Shatanawi, W: Fixed point theory for contractive mappings satisfying Φ-maps in *G*-metric spaces. Fixed Point Theory Appl **2010**, 9 (2010). (Article ID 181650), doi:10.1155/2010/181650
21. Berinde, V: Some remarks on a fixed point point theorem for Ćirić-type almost contractions. Carpathian J Math. **25**, 157–162 (2009)
22. Berinde, V: Approximating fixed points of weak contractions using the Picard iteration. Nolinear Anal Forum. **9**, 43–53 (2004)
23. Berinde, V: On the approximation fixed points of weak contractive mappings. Carpathian J Math. **19**, 7–22 (2003)
24. Berinde, V: General contractive fixed point theorems for Ćirić-type almost contraction in metric spaces. Carpathian J Math. **24**, 10–19 (2008)
25. Babu, GVR, Sandhya, ML, Kameswari, MVR: A note on a fixed point theorem of Berinde on weak contractions. Carpathian J Math. **24**, 8–12 (2008)
26. Ćirić, LjB, Abbas, M, Saadati, R, Hussain, N: Common fixed points of almost generalized contractive mappings in ordered metric spaces. Appl Math Comput. **217**, 5784–5789 (2011). doi:10.1016/j.amc.2010.12.060
27. Khan, MS, Swaleh, M, Sessa, S: Fixed point theorems by altering distances between the points. Bull Aust Math Soc. **30**, 1–9 (1984). doi:10.1017/S0004972700001659
28. Aydi, H: Common fixed point results for mappings satisfying (ψ, φ)-weak contractions in ordered partial metric space. Int J Math Stat **12**(2) (2012). (in press)
29. Aydi, H, Postolache, M, Shatanawi, W: Coupled fixed point results for (ψ, φ))-weakly contractive mappings in ordered *G*-metric spaces. Comput Math Appl. **63**, 298–309 (2012). doi:10.1016/j.camwa.2011.11.022
30. Aydi, H, Karapinar, E, Shatanawi, W: Coupled fixed point results for (ψ, φ)-weakly contractive condition in ordered partial metric spaces. Comput Math Appl. **62**, 4449–4460 (2011). doi:10.1016/j.camwa.2011.10.021
31. Dorić, D: Common fixed point for generalized (ψ, φ)-weak contraction. Appl Math Lett. **22**, 1896–1900 (2009). doi:10.1016/j.aml.2009.08.001
32. Karapinar, E, Sadarangani, K: Fixed point theory for cyclic (ψ, φ)-contractions. Fixed Point Theory Appl. **2011**, 69 (2011). doi:10.1186/1687-1812-2011-69
33. Nashine, HK, Samet, B: Fixed point results for mappings satisfying (ψ, φ)-weakly contractive condition in partially ordered metric spaces. Nonlinear Anal. **74**, 2201–2209 (2011). doi:10.1016/j.na.2010.11.024
34. Popescu, O: Fixed points for (ψ, φ)-weak contractions. Appl Math Lett. **24**, 1–4 (2011). doi:10.1016/j.aml.2010.06.024
35. Nashine, HK, Samet B Kim, J: Fixed point results for contractions involving generalized altering distances in ordered metric spaces. Fixed Point Theory Appl. **2011**, 5 (2011). doi:10.1186/1687-1812-2011-5
36. Shatanawi, W, Samet, B: On (ψ, φ)-weakly contractive condition in partially ordered metric spaces. Comput Math Appl. **62**, 3204–3214 (2011). doi:10.1016/j.camwa.2011.08.033

doi:10.1186/1687-1812-2012-80
Cite this article as: Shatanawi and Al-Rawashdeh: **Common fixed points of almost generalized (ψ, φ)-contractive mappings in ordered metric spaces.** *Fixed Point Theory and Applications* 2012 **2012**:80.