Arunchai and Plubtieng *Fixed Point Theory and Applications* 2013, **2013**:142
http://www.fixedpointtheoryandapplications.com/content/2013/1/142

Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH Open Access

Random fixed point theorem of Krasnoselskii type for the sum of two operators

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Abstract

In this paper, we prove some random fixed point theorem for the sum of a weakly-strongly continuous random operator and a nonexpansive random operator in Banach spaces. Our results are the random versions of some deterministic fixed point theorems of Edmund (Math. Ann. 174:233-239, 1967), O'Regan (Appl. Math. Lett. 9:1-8, 1996) and some known results in the literature.

Keywords: random fixed point; Krasnoselskii type; weakly-strongly continuous random operator; nonexpansive random operator

1 Introduction

Probabilistic functional analysis has emerged as one of the momentous mathematical disciplines in view of its requirements in analyzing probabilistic models in applied problems. Random fixed point theorems are stochastic versions of classical or deterministic fixed point theorems and are useful in the study of various classes of random equations. The study of random fixed point theorems was initiated by the Prague school of probabilists in 1950s. Since then there have been several interesting results and a lot of activity in this area has appeared. In 1976, Bharucha Reid [1] has given sufficient conditions for a stochastic analogue of Schauder's fixed point theorem for random operators. Recently, Spacek [2] and Hans [3] have proved the stochastic analogue of a Banach fixed point theorem in a separable metric space. Moreover, Itoh [4] extended Spacek and Hans's theorem to a multi-valued contraction random operator. In [5], Xu extended Itoh's theorem to a nonself-random operator T, where T satisfies either weakly inward (see [5]) or the Leray-Schauder condition (see [5]). The interest in the generalizations of a random fixed point theorem from self-maps to nonself-mappings has been revived after the papers by Sehgal and Waters [6], Sehgal and Singh [7], Papageorgiou [8, 9], Lin [10, 11], Xu [5], Tan and Yuan [12] and Beg and Shahzad [13] and has received much attention in recent years (see, *e.g.*, [14–24]).

In 1955, Krasnoselskii [25] proved that a sum of two mappings has a fixed point, when the mappings are a contraction and compact. Recently, Rao [26] obtained a probabilistic version of Krasnoselskii's theorem which is a sum of a contraction random operator and a compact random operator on a closed convex subset of a separable Banach space. Moreover, Itoh [4] extended Rao's result to a sum of a nonexpansive random operator and a completely continuous random operator on a weakly compact convex subset of a separable uniformly convex Banach space. In [10], Lin obtained a sum of a locally almost nonexpansive (see [10]) random operator and a completely continuous random operator on a nonempty closed convex bounded subset of a separable uniformly convex Banach space. In 1996, Shahzad [27] extended Itoh and Lin's results to a sum of two random nonself-random operators, by assuming an additional condition that the sum of these operators satisfies either weakly inward or the Leray-Schauder condition. See [14, 26] and the references therein. Recently, Vijayaraju [28] proved a random fixed point theorem for a sum of a 1-set-contraction and a compact (completely continuous) mapping and has received much attention in recent years (see, *e.g.*, [29–33]). On the other hand, it is known that the fixed point theorem of Krasnoselskii has nice applications to perturbed mixed type of integral and nonlinear differential equations including the allied areas of mathematics for proving the existence theorems under mixed Lipschitz and compactness conditions (see [34, 35] and the references therein).

In this paper, inspired and motivated by [5, 34] and [35], we obtain a random fixed point theorem for the sum of a weakly-strongly continuous random operator and a non-expansive random operator which contains as a special Krasnoselskii type of Edmund and O'Regan via the method of measurable selectors.

2 Preliminaries

Throughout this paper, (Ω, Σ) denotes a measurable space, where Ω is a nonempty set and Σ is a σ-algebra of subsets of Ω. Let X be a Banach space and M be a nonempty subset of X. A multi-valued operator $T : \Omega \to X$ is called $(\Sigma-)$ *measurable* if for any open subset G of X, $T^{-1}(G) \in \Sigma$, where $T^{-1}(G) := \{\omega \in \Omega : T(\omega) \cap G \neq \emptyset\}$. Note that if $T(\omega)$ belongs to a compact subsets of X for all $\omega \in \Omega$, then T is measurable if and only if $T^{-1}(C) \in \Sigma$ for every closed subset C of X. A measurable (single-valued) operator $\xi : \Omega \to X$ is called a *measurable selector* of a measurable operator $T : \Omega \to X$ if $\xi(\omega) \in T(\omega)$ for each $\omega \in \Omega$. An operator $T : \Omega \times M \to X$ is called a *random operator* if for each fixed $x \in M$, the operator $T(\cdot, x) : \Omega \to X$ is measurable. A measurable operator $\xi : \Omega \to X$ is called a *random fixed point* of a random operator $T : \Omega \times M \to X$ if $\xi(\omega) \in M$ and $T(\omega, \xi(\omega)) = \xi(\omega)$ for all $\omega \in \Omega$.

Let S be a nonempty bounded subset of X. In [28] denote the Kuratowski measure of noncompactness of a bounded set S in X is nonnegative real number $\alpha(S)$ defined by

$$\alpha(S) = \inf\left\{ r > 0 : S = \bigcup_{i=1}^{n} S_i, \operatorname{diam}(S_i) \leq r, \forall i \right\}.$$

An operator $T : X \to X$ is called *compact* if $\overline{T(X)}$ is a compact subset of X. An operator T is called an α*-Lipschitzian*, there is a constant $k \geq 0$ with $\alpha(T(B)) \leq k\alpha(B)$ for all bounded subsets B of X. We call T is completely continuous if it is α-Lipschitzian where $k = 0$. Let M be a nonempty subset of a Banach space X, and $T : M \to X$ is called *weakly-strongly continuous* if for each sequence (x_n) in M, which converges weakly to x in M, the sequence (Tx_n) converges strongly to Tx. The operator $T : X \to X$ is called a *nonlinear contraction* if there exists a continuous nondecreasing function $\phi : [0, \infty) \to [0, \infty)$ satisfying $\phi(z) < z$ for $z > 0$ such that $\|Tx - Ty\| \leq \phi(\|x - y\|)$ for all $x, y \in X$. The operator T is called *non-expansive* if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in X$. A random operator $T : \Omega \times M \to X$ is called *nonexpansive* if for arbitrary $x, y \in X$, one has $\|T(\omega, x) - T(\omega, y)\| \leq \|x - y\|$ for each $\omega \in \Omega$. A random operator $T : \Omega \times M \to X$ is called nonexpansive (compact, continuous, completely continuous, *etc.*) if the operator $T(\omega, \cdot) : M \to X$ is so for each fixed $\omega \in \Omega$.

Lemma 2.1 [36] *Let (X, d) be a complete separable metric space and let $F : \Omega \to CL(X)$ be a measurable map. Then F has a measurable selector.*

In [34], Edmunds proved the following fixed point theorem.

Theorem 2.2 [34] *Let M be a nonempty bounded closed convex subset of a Hilbert space X and let A, B be two maps from M into X such that*

(i) *A is weakly-strongly continuous*;
(ii) *B is a nonexpansive mapping*;
(iii) *$Ax + Bx \in M$ for all $x \in M$.*

Then $A + B$ has a fixed point in M.

The following fixed point theorems are crucial for our purposes.

Theorem 2.3 [35] *Let U be an open set in a closed convex set M of a Banach space X. Assume that $0 \in U$, $T(\overline{U})$ is bounded and $T : \overline{U} \to M$ is given by $T = A + B$, where A and B are two maps from $\overline{U}$ into X satisfying*

(i) *A is continuous and completely continuous*;
(ii) *B is a nonlinear contraction.*

Then either

(A1) *T has a fixed point in $\overline{U}$; or*
(A2) *there is a point $u \in \partial U$ and $\lambda \in (0, 1)$ with $u = \lambda T(u)$.*

Theorem 2.4 [35] *Let X be a Banach space and let Q be a closed convex bounded subset of X with $0 \in \operatorname{int}(Q)$. In addition, assume that $T : Q \to X$ is given by $T = A + B$, where A and B are two maps from Q into X satisfying*

(i) *A is continuous and compact*;
(ii) *B is a nonlinear contraction*;
(iii) *if $\{(x_j, \lambda_j)\}_{j=o}^{\infty}$ is a sequence in $\partial Q \times [0, 1]$ converging to (x, λ) with $x = \lambda T(x)$ and $0 < \lambda < 1$, then $\lambda_j T(x_j) \in Q$ for j sufficiently large.*

Then T has a fixed point.

Theorem 2.5 [35] *Let U be a bounded open convex set in a reflexive Banach space X. Assume that $0 \in U$ and $T : \overline{U} \to X$ is given by $T = A + B$, where A and B are two maps from $\overline{U}$ into X satisfying*

(i) *A is continuous and compact*;
(ii) *B is a nonexpansive map*;
(iii) *$I - T : \overline{U} \to X$ is demiclosed on $\overline{U}$.*

Then either

(A1) *T has a fixed point in $\overline{U}$; or*
(A2) *there is a point $u \in \partial U$ and $\lambda \in (0, 1)$ with $u = \lambda T(u)$.*

Theorem 2.6 [35] *Let U be a bounded open convex set in a uniformly convex Banach space X. Assume that $0 \in U$ and $T : \overline{U} \to X$ is given by $T = A + B$, where A and B are two maps from $\overline{U}$ into X satisfying*

(i) *A is continuous and compact*;
(ii) *B is a nonexpansive map*;
(iii) *In addition, suppose $A : \overline{U} \to X$ is strongly continuous.*

Then either

(A1) *T has a fixed point in $\overline{U}$; or*

(A2) *there is a point $u \in \partial U$ and $\lambda \in (0,1)$ with $u = \lambda T(u)$.*

Theorem 2.7 [35] *Let U be an open subset of a Banach space X and let $\overline{U}$ be a weakly compact subset of X. Assume that $0 \in U$ and $T : \overline{U} \to X$ is given by $T = A + B$, where A and B are two maps from $\overline{U}$ into X satisfying*

(i) *A is continuous and compact;*

(ii) *B is a nonexpansive map;*

(iii) *$I - T : \overline{U} \to X$ is demiclosed on $\overline{U}$.*

Then either

(A1) *T has a fixed point in $\overline{U}$; or*

(A2) *there is a point $u \in \partial U$ and $\lambda \in (0,1)$ with $u = \lambda T(u)$.*

Theorem 2.8 [35] *Let X be a reflexive Banach space and let Q be a closed convex bounded subset of X with $0 \in \operatorname{int}(Q)$. In addition, assume that $T : Q \to X$ is given by $T = A + B$, where A and B are two maps from Q into X satisfying*

(i) *A is continuous and compact;*

(ii) *B is a nonexpansive map;*

(iii) *$I - T : \overline{U} \to X$ is demiclosed on Q;*

(iv) *if $\{(x_j, \lambda_j)\}_{j=o}^{\infty}$ is a sequence in $\partial Q \times [0,1]$ converging to (x, λ) with $x = \lambda T(x)$ and $0 < \lambda < 1$, then $\lambda_j T(x_j) \in Q$ for j sufficiently large.*

Then T has a fixed point.

Theorem 2.9 [35] *Let X be a uniformly convex Banach space and let Q be a closed convex bounded subset of X with $0 \in \operatorname{int}(Q)$. In addition, assume that $T : Q \to X$ is given by $T = A + B$, where A and B are two maps from Q into X satisfying*

(i) *A is continuous and compact;*

(ii) *B is a nonexpansive map;*

(iii) *$A : Q \to X$ is strongly continuous;*

(iv) *if $\{(x_j, \lambda_j)\}_{j=o}^{\infty}$ is a sequence in $\partial Q \times [0,1]$ converging to (x, λ) with $x = \lambda T(x)$ and $0 < \lambda < 1$, then $\lambda_j T(x_j) \in Q$ for j sufficiently large.*

Then T has a fixed point.

3 Main result

Lemma 3.1 *Let M be a subset of X and let $A, B : M \to X$ be two operators satisfying:*

(i) *A is weakly-strongly continuous;*

(ii) *B is nonexpansive;*

(iii) *$Ax + Bx \in M$ for every $x \in M$.*

Then the mapping $T : M \to X$ defined by $Tx = Ax + Bx$ is continuous.

Proof Let (x_n) be a sequence in M converging to a point x in M. Since A is weakly-strongly continuous and B is nonexpansive, we obtain that

$$\begin{aligned} \|Tx_n - Tx_0\| &= \big\|(Ax_n + Bx_n) - (Ax + Bx)\big\| \\ &= \big\|(Ax_n - Ax) + (Bx_n - Bx)\big\| \end{aligned}$$

$$\le \|Ax_n - Ax\| + \|Bx_n - Bx\|$$

$$\le \|Ax_n - Ax\| + \|x_n - x\|.$$

Then we have $\|Tx_n - Tx\| \to 0$ as $n \to \infty$. That is, $Tx_n \to Tx$ as $n \to \infty$. Hence T is continuous. □

Lemma 3.2 *Let M be a nonempty bounded closed convex subset of a Banach space X and let A, B be two maps from M into X such that*

(i) *A is weakly-strongly continuous*;

(ii) *B is nonexpansive*;

(iii) *$Ax + Bx \in M$ for every $x \in M$.*

Then the set $F(A+B) = \{x \in M : Ax + Bx = x\}$ is closed.

Proof Define a mapping $T : M \to X$ by $Tx = Ax + Bx$. By Theorem 2.2 then we have

$$F(T) = F(A+B) = \{x \in M : Ax + Bx = x\}$$

is nonempty. To show that $F(T)$ is a closed subset of M, let (x_n) be a sequence of $F(T)$ with $x_n \to x \in M$. Since T is continuous, it follows by Lemma 3.1 that

$$x = \lim_{n\to\infty} x_n = \lim_{n\to\infty} Tx_n = Tx.$$

Hence $x \in F(T)$ and therefore $F(T) = F(A+B)$ is a closed subset of M. □

Theorem 3.3 *Let M be a nonempty bounded closed convex subset of a separable Banach space X and let $A, B : \Omega \times M \to X$ be two random operators satisfying, for each $\omega \in \Omega$,*

(i) *A is weakly-strongly continuous*;

(ii) *B is nonexpansive*;

(iii) *$A(\omega, x) + B(\omega, x) \in \Omega \times M$ for every $x \in M$.*

Then $A + B$ has a random fixed point in $\Omega \times M$.

Proof Define an operator $T : \Omega \times M \to X$ by

$$T(\omega, x) = A(\omega, x) + B(\omega, x).$$

Since A and B are random operators, $A(\cdot, x)$ and $B(\cdot, x)$ are X-valued random variables for all x in M. Since X is a separable Banach space, we have $T(\omega, x)$ is an X-valued random variable. Hence T is a random operator on M. Moreover, by Lemma 3.1 we note that T is a continuous random operator on M. Define a multi-valued map $F : \Omega \to 2^M$ by

$$F(\omega) = \{x \in M : T(\omega, x) = x\}.$$

By Theorem 2.2 and Lemma 3.2, $F(\omega)$ is nonempty and closed for each $\omega \in \Omega$. Thus, to show the measurability of F, let D be a closed subset of X. It is sufficient to show that $F^{-1}(D)$ is measurable. Denote

$$L(D) = \bigcap_{n=1}^{\infty} \bigcup_{x_i \in D_n} \left\{\omega \in \Omega : \left\| T(\omega, x_i) - x_i \right\| < \frac{2}{n}\right\},$$

where $D_n = \{x \in M : d(x, D) < \frac{1}{n}\}$ and $d(x, D) = \inf\{\|x - y\| : y \in D\}$. Obviously, $L(D)$ is a measurable subset of Ω. We will show that $F^{-1}(D) = L(D)$. Obviously, $F^{-1}(D) \subseteq L(D)$. As in Itoh [4], it is proved that $L(D) \subseteq F^{-1}(D)$. Thus $F^{-1}(D) = L(D)$, and so F is measurable on Ω. Since $F(\omega)$ is compact, it has closed values for each $\omega \in \Omega$. By Lemma 2.1, F admits a measurable selector, *i.e.*, there is a measurable mapping $\xi : \Omega \to X$ such that $\xi(\omega) \in F(\omega)$ for all $\omega \in \Omega$. By definition of $F(\omega)$, which implies that $\xi(\omega) = T(\omega, \xi(\omega))$, hence $\xi(\omega) = A(\omega, \xi(\omega)) + B(\omega, \xi(\omega))$. This completes the proof. □

Theorem 3.4 *Let M be a nonempty bounded closed convex separable subset of a reflexive Banach space X and let $T : \Omega \times M \to X$ be given by $T = A + B$, where $A, B : \Omega \times M \to X$ are two operators satisfying*

(i) *A is weakly-strongly continuous random*;
(ii) *B is nonexpansive random*;
(iii) *$A(\omega, x) + B(\omega, x) \in \Omega \times M$ for each $\omega \in \Omega$ and each $x \in M$*;
(iv) *X is strictly convex and $I - T$ is demiclosed at zero.*

Then $T = A + B$ has a random fixed point in $\Omega \times M$.

Proof Define an operator $T : \Omega \times M \to X$ by

$$T(\omega, x) = A(\omega, x) + B(\omega, x).$$

By Theorem 3.3 we have T is a random operator on M. For each ω in Ω, the set

$$F(\omega) = \{x \in M : T(\omega, x) = x\}$$

is nonempty and closed. Since X is strictly convex, $F(\omega)$ is convex. This implies that $F(\omega)$ is weakly compact, we show that F is ω-measurable, *i.e.*, for each x^* in X^*, the dual space of X, the numerically valued function x^*F is measurable. Let for each integer $n \geq 1$, the set

$$F_n(\omega) = \left\{x \in M : \|T(\omega, x) - x\| < \frac{1}{n}\right\}.$$

Thus $F_n(\omega)$ is closed as T is continuous. Moreover, by Itoh [4], each F_n is measurable. Since M is separable, the weak topology on M is metrizable. Let d_ω be a metric on M which induces the weak topology on M and let H_ω be the Hausdorff metric produced by d_ω. We shall show that

$$\lim_{n\to\infty} H_\omega\big(F_n(\omega) - F(\omega)\big) = 0, \quad \omega \in \Omega. \tag{3.1}$$

In fact, since $(F_n(\omega))$ decreases to $F(\omega)$, the limit in (3.1) exists, and we denote it by $h(\omega)$. Then it is easily seen that

$$H_\omega\big(F_n(\omega) - F(\omega)\big) = \sup_{y \in F_n(\omega)} d_\omega\big(y, F(\omega)\big) \geq h(\omega). \tag{3.2}$$

If $h(\omega) > 0$, then there exists, for each $n \geq 1$, a y_n in $F_n(\omega)$ such that

$$d_\omega\big(y_n, F(\omega)\big) \geq \frac{1}{2} h(\omega). \tag{3.3}$$

Since (M, d_ω) is compact, there exists a subsequence (y_{n_i}) of (y_n) such that $d_\omega(y_{n_i}, y) \to 0$ for some $y \in M$, *i.e.*, (y_{n_i}) converges weakly to y. Then we obtain that

$$d_\omega\big(y, F(\omega)\big) \geq \frac{1}{2}h(\omega) > 0. \tag{3.4}$$

On the other hand, since $\|y_{n_i} - T(\omega, y_{n_i})\| \leq \frac{1}{n_i}$ and $I - T(\omega, \cdot)$ is demiclosed at zero, it follows that $x - T(\omega, x) = 0$, *i.e.*, $x \in F(\omega)$. This is a contradiction to (3.4). Hence

$$\lim_{n\to\infty} H_\omega\big(F_n(\omega) - F(\omega)\big) = 0, \quad \omega \in \Omega.$$

Now, by Itoh [4], we have F is ω-measurable. Thus by Lemma 2.1, there exists a ω-measurable selector x of F, *i.e.*, for each $x^* \in X^*$, x^*x is measurable as a numerically-valued function on Ω. Since M is separable, x is measurable. This x is a random fixed point of T. This completes the proof. □

Similarly we can prove the following, which concerns Theorem 3.3, using the result of Theorem 2.3.

Theorem 3.5 *Let U be an open set in a closed, convex set M of a separable Banach space X. Assume that $0 \in U$, $T(\Omega \times \overline{U})$ is bounded and $T : \Omega \times \overline{U} \to M$ is given by $T = A + B$, where A and B two random operators from $\Omega \times \overline{U}$ into X satisfying, for each $\omega \in \Omega$,*
- (i) *A is continuous and completely continuous*;
- (ii) *B is a nonlinear contraction*;
- (iii) *there does not exist a $u \in \partial U$ such that $u = \lambda(\omega)T(\omega, u)$ for any measurable $\lambda : \Omega \to \mathbb{R}$ with $0 < \lambda(\omega) < 1$, where ∂U is a boundary of $\overline{U}$.*

Then T has a random fixed point in $\overline{U}$.

By Theorem 2.4 and the same as in the proof of Theorem 3.3, we obtain the following.

Theorem 3.6 *Let X be a separable Banach space and let Q be a closed, convex, bounded subset of X with $0 \in \operatorname{int}(Q)$. In addition, assume that $T : \Omega \times Q \to X$ is given by $T = A + B$, where A and B two random operators from $\Omega \times Q$ into X satisfying, for each $\omega \in \Omega$,*
- (i) *A is continuous and compact*;
- (ii) *B is a nonlinear contraction*;
- (iii) *if $\{(u_j, \lambda_j(\omega))\}_{j=o}^{\infty}$ is a sequence in $\partial Q \times [0,1]$ converging to $(u, \lambda(\omega))$ with $u = \lambda(\omega)T(\omega, u)$ and for some measurable $\lambda : \Omega \to \mathbb{R}$ with $0 < \lambda(\omega) < 1$, then $\lambda_j(\omega)T(\omega, u_j) \in \Omega \times Q$ for j sufficiently large.*

Then T has a random fixed point.

We can prove the following, which concerns Theorem 3.3, using the result of Theorem 2.5.

Theorem 3.7 *Let U be a bounded open convex set in a separable reflexive Banach space X. Assume that $0 \in U$ and $T : \Omega \times \overline{U} \to X$ is given by $T = A + B$, where A and B are two random operators from $\Omega \times \overline{U}$ into X satisfying, for each $\omega \in \Omega$,*
- (i) *A is continuous and compact*;
- (ii) *B is a nonexpansive map*;

(iii) $I - T : \Omega \times \overline{U} \to X$ *is demiclosed on* $\overline{U}$;
(iv) *there does not exist a* $u \in \partial U$ *such that* $u = \lambda(\omega)T(\omega, u)$ *for any measurable* $\lambda : \Omega \to \mathbb{R}$ *with* $0 < \lambda(\omega) < 1$, *where* ∂U *is a boundary of* $\overline{U}$.

Then T *has a random fixed point in* $\overline{U}$.

The proof of the following is similar to Theorem 3.3 and in this case we invoke Theorem 2.3 instead of Theorem 2.6 in the proof.

Theorem 3.8 *Let* U *be a bounded open convex set in a separable uniformly convex Banach space* X. *Assume that* $0 \in U$ *and* $T : \Omega \times \overline{U} \to X$ *is given by* $T = A + B$, *where* A *and* B *are two random operators from* $\Omega \times \overline{U}$ *into* X *satisfying, for each* $\omega \in \Omega$,

(i) A *is continuous and compact*;
(ii) B *is a nonexpansive map*;
(iii) $A : \Omega \times \overline{U} \to X$ *is strongly continuous*;
(iv) *there does not exist a* $u \in \partial U$ *such that* $u = \lambda(\omega)T(\omega, u)$ *for any measurable* $\lambda : \Omega \to \mathbb{R}$ *with* $0 < \lambda(\omega) < 1$, *where* ∂U *is a boundary of* $\overline{U}$.

Then T *has a random fixed point in* $\overline{U}$.

The proof of the following can be given by using Theorem 3.3 and the result of Theorem 2.7.

Theorem 3.9 *Let* U *be an open subset of a separable Banach space* X *and let* $\overline{U}$ *be a weakly compact subset of* X. *Assume that* $0 \in U$ *and* $T : \Omega \times \overline{U} \to X$ *is given by* $T = A + B$, *where* A *and* B *are two random operators from* $\Omega \times \overline{U}$ *into* X *satisfying, for each* $\omega \in \Omega$,

(i) A *is continuous and compact*;
(ii) B *is a nonexpansive map*;
(iii) $I - T : \Omega \times \overline{U} \to X$ *is demiclosed on* $\overline{U}$;
(iv) *there does not exist a* $u \in \partial U$ *such that* $u = \lambda(\omega)T(\omega, u)$ *for any measurable* $\lambda : \Omega \to \mathbb{R}$ *with* $0 < \lambda(\omega) < 1$, *where* ∂U *is a boundary of* $\overline{U}$.

Then T *has a random fixed point in* $\overline{U}$.

We can prove the following by Theorem 3.3 and using the result of Theorem 2.8.

Theorem 3.10 *Let* X *be a separable reflexive Banach space and let* Q *be a closed, convex, bounded subset of* X *with* $0 \in \operatorname{int}(Q)$. *In addition, assume that* $T : \Omega \times Q \to X$ *is given by* $T = A + B$, *where* A *and* B *are two random operators from* $\Omega \times Q$ *into* X *satisfying, for each* $\omega \in \Omega$,

(i) A *is continuous and compact*;
(ii) B *is a nonexpansive map*;
(iii) $I - T : \Omega \times \overline{U} \to X$ *is demiclosed on* Q;
(iv) *if* $\{(u_j, \lambda_j(\omega))\}_{j=o}^{\infty}$ *is a sequence in* $\partial Q \times [0, 1]$ *converging to* $(u, \lambda(\omega))$ *with* $u = \lambda(\omega)T(\omega, u)$ *and for some measurable* $\lambda : \Omega \to \mathbb{R}$ *with* $0 < \lambda(\omega) < 1$, *then* $\lambda_j(\omega)T(\omega, u_j) \in \Omega \times Q$ *for* j *sufficiently large*.

Then T *has a random fixed point*.

By the same proof of Theorem 3.3 and the result of Theorem 2.9, we have the following.

Theorem 3.11 *Let X be a separable uniformly convex Banach space and let Q be a closed, convex, bounded subset of X with $0 \in \operatorname{int}(Q)$. In addition, assume that $T : \Omega \times Q \to X$ is given by $T = A + B$, where A and B are two random operators from $\Omega \times Q$ into X satisfying, for each $\omega \in \Omega$,*

(i) *A is continuous and compact*;
(ii) *B is a nonexpansive map*;
(iii) *$A : \Omega \times Q \to X$ is strongly continuous*;
(iv) *if $\{(u_j, \lambda_j(\omega))\}_{j=o}^{\infty}$ is a sequence in $\partial Q \times [0,1]$ converging to $(u, \lambda(\omega))$ with $u = \lambda(\omega)T(\omega, u)$ and for some measurable $\lambda : \Omega \to \mathbb{R}$ with $0 < \lambda(\omega) < 1$, then $\lambda_j(\omega)T(\omega, u_j) \in \Omega \times Q$ for j sufficiently large.*

Then T has a random fixed point.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The work presented here was carried out in collaboration between all authors. SP and AA defined the research theme. SP designed theorems and methods of proof and interpreted the results. AA proved the theorems, interpreted the results and wrote the paper. All authors have contributed to, seen and approved the manuscript.

Acknowledgements
The first author would like to thank the Thailand Research Fund for financial support and the second author is also supported by the Royal Golden Jubilee Program under Grant PHD/0282/2550, Thailand.

Received: 10 March 2013 Accepted: 17 May 2013 Published: 3 June 2013

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doi:10.1186/1687-1812-2013-142
Cite this article as: Arunchai and Plubtieng: **Random fixed point theorem of Krasnoselskii type for the sum of two operators.** *Fixed Point Theory and Applications* 2013 **2013**:142.