Fixed Point Theory and Applications
a SpringerOpen Journal

RESEARCH Open Access

Some new common fixed point results for three pairs of mappings in generalized metric spaces

Feng Gu[1*] and Zhongzhi Yang[2]

*Correspondence:
gufeng99@sohu.com
[1]Institute of Applied Mathematics and Department of Mathematics, Hangzhou Normal University, Hangzhou, Zhejiang 310036, China
Full list of author information is available at the end of the article

Abstract
In this paper, we use weakly commuting and weakly compatible conditions of self-mapping pairs, prove some new common fixed point theorems for three pairs of self-mappings in *G*-metric spaces. An example is provided to support our result. The results presented in this paper extend and improve several well-known comparable results.

MSC: 47H10; 54H25; 54E50

Keywords: *G*-metric space; weakly commuting mapping pairs; weakly compatible mapping pairs; common fixed point

1 Introduction and preliminaries

The metric fixed point theory is very important and useful in mathematics. It can be applied in various areas, for instance, approximation theory, optimization and variational inequalities. Many authors have introduced the generalizations of metric spaces, for example, Gähler [1, 2] (called 2-metric spaces) and Dhage [3, 4] (called *D*-metric spaces). In 2003, Mustafa and Sims [5] found that most of the claims concerning the fundamental topological properties of *D*-metric spaces are incorrect. Therefore, they [6] introduced a new structure of generalized metric spaces, which are called *G*-metric spaces, as a generalization of metric spaces, to develop and introduce a new fixed point theory for various mappings in this new structure. Later, several fixed point and common fixed point theorems in *G*-metric spaces were obtained by [6–51].

The purpose of this paper is to use the concept of weakly commuting mappings and weakly compatible mappings to discuss some new common fixed point problem for six self-mappings in *G*-metric spaces. The results presented in this paper extend and improve the corresponding results of Abbas *et al.* [7], Mustafa and Sims [8], Abbas and Rhoades [9], Mustafa *et al.* [10], Mustafa *et al.* [11], Abbas *et al.* [12], Chugh and Kadian [13], Manro *et al.* [14], Vats *et al.* [15].

We now recall some definitions and properties in *G*-metric spaces.

Definition 1.1 [6] Let X be a nonempty set and let $G : X \times X \times X \to R^+$ be a function satisfying the following properties:

(G_1) $G(x,y,z) = 0$ if $x = y = z$;
(G_2) $0 < G(x,x,y)$ for all $x,y \in X$ with $x \neq y$;

(G_3) $G(x,x,y) \leq G(x,y,z)$ for all $x,y,z \in X$ with $z \neq y$;
(G_4) $G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots$, symmetry in all three variables;
(G_5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for all $x,y,z,a \in X$.

Then the function G is called a generalized metric, or, more specifically, a G-metric on X, and the pair (X,G) is called a G-metric space.

Definition 1.2 [6] Let (X,G) be a G-metric space and let (x_n) be a sequence of points of X. A point $x \in X$ is said to be the limit of the sequence (x_n) if $\lim_{n,m\to+\infty} G(x,x_n,x_m) = 0$, and we say that the sequence (x_n) is G-convergent to x or (x_n) G-convergent to x.

Thus, $x_n \to x$ in a G-metric space (X,G) if, for any $\epsilon > 0$, there exists $k \in N$ such that $G(x,x_n,x_m) < \epsilon$ for all $m,n \geq k$.

Proposition 1.1 [6] *Let (X,G) be a G-metric space, then the following are equivalent*:

1. *(x_n) is G-convergent to x.*
2. *$G(x_n,x_n,x) \to 0$ as $n \to +\infty$.*
3. *$G(x_n,x,x) \to 0$ as $n \to +\infty$.*
4. *$G(x_n,x_m,x) \to 0$ as $n,m \to +\infty$.*

Definition 1.3 [6] Let (X,G) be a G-metric space. A sequence (x_n) is called G-Cauchy if, for every $\epsilon > 0$, there is $k \in N$ such that $G(x_n,x_m,x_l) < \epsilon$ for all $m,n,l \geq k$; that is, $G(x_n,x_m,x_l) \to 0$ as $n,m,l \to +\infty$.

Proposition 1.2 [6] *Let (X,G) be a G-metric space. Then the following are equivalent*:

1. *The sequence (x_n) is G-Cauchy.*
2. *For every $\epsilon > 0$, there is $k \in N$ such that $G(x_n,x_m,x_m) < \epsilon$ for all $m,n \geq k$.*

Definition 1.4 [6] Let (X,G) and (X',G') be G-metric spaces and let $f : (X,G) \to (X',G')$ be a function. Then f is said to be G-continuous at a point $a \in X$ if and only if, for every $\epsilon > 0$, there is $\delta > 0$ such that $x,y \in X$ and $G(a,x,y) < \delta$ imply $G'(f(a),f(x),f(y)) < \epsilon$. A function f is G-continuous at X if only if it is G-continuous at $a \in X$.

Proposition 1.3 [6] *Let (X,G) be a G-metric space. Then the function $G(x,y,z)$ is jointly continuous in all three of its variables.*

Definition 1.5 [6] A G-metric space (X,G) is G-complete if every G-Cauchy sequence in (X,G) is G-convergent in X.

Definition 1.6 [16] Two self-mappings f and g of a G-metric space (X,G) are said to be weakly commuting if $G(fgx,gfx,gfx) \leq G(fx,gx,gx)$ for all x in X.

Definition 1.7 [16] Let f and g be two self-mappings from a G-metric space (X,G) into itself. Then the mappings f and g are said to be weakly compatible if $G(fgx,gfx,gfx) = 0$ whenever $G(fx,gx,gx) = 0$.

Proposition 1.4 [6] *Let (X,G) be a G-metric space. Then, for all x, y, z, a in X, it follows that*:

(i) *If $G(x,x,y) = 0$, then $x = y = z$;*
(ii) $G(x,y,z) \leq G(x,x,y) + G(x,x,z)$;

(iii) $G(x,y,y) \leq 2G(y,x,x)$;
(iv) $G(x,y,z) \leq G(x,a,z) + G(a,y,z)$;
(v) $G(x,y,z) \leq \frac{2}{3}(G(x,y,a) + G(x,a,z) + G(a,y,z))$;
(vi) $G(x,y,z) \leq (G(x,a,a) + G(y,a,a) + G(z,a,a))$.

2 Common fixed point theorems

Theorem 2.1 *Let (X,G) be a complete G-metric space, and let f, g, h, A, B and C be six mappings of X into itself satisfying the following conditions*:

(i) $f(X) \subset B(X)$, $g(X) \subset C(X)$, $h(X) \subset A(X)$;
(ii) $\forall x,y,z \in X$,

$$G(fx,gy,hz) \leq k \max \left\{ \begin{array}{c} G(Ax,By,Cz), G(Ax,fx,fx), \\ G(By,gy,gy), G(Cz,hz,hz), \\ G(Ax,gy,gy), G(Ax,hz,hz), \\ G(By,fx,fx), G(By,hz,hz), \\ G(Cz,fx,fx), G(Cz,gy,gy) \end{array} \right\} \tag{2.1}$$

or

$$G(fx,gy,hz) \leq k \max \left\{ \begin{array}{c} G(Ax,By,Cz), G(Ax,Ax,fx), \\ G(By,By,gy), G(Cz,Cz,hz), \\ G(Ax,Ax,gy), G(Ax,Ax,hz), \\ G(By,By,fx), G(By,By,hz), \\ G(Cz,Cz,fx), G(Cz,Cz,gy) \end{array} \right\}, \tag{2.2}$$

where $k \in [0, \frac{1}{2})$. If one of the following conditions is satisfied:

(a) *Either f or A is G-continuous, the pair (f,A) is weakly commuting, the pairs (g,B) and (h,C) are weakly compatible*;
(b) *Either g or B is G-continuous, the pair (g,B) is weakly commuting, the pairs (f,A) and (h,C) are weakly compatible*;
(c) *Either h or C is G-continuous, the pair (h,C) is weakly commuting, the pairs (f,A) and (g,B) are weakly compatible.*

Then

(I) *one of the pairs (f,A), (g,B) and (h,C) has a coincidence point in X*;
(II) *the mappings f, g, h, A, B and C have a unique common fixed point in X.*

Proof Suppose that mappings f, g, h, A, B and C satisfy condition (2.1).

Let x_0 in X be an arbitrary point since $f(X) \subset B(X)$, $g(X) \subset C(X)$, $h(X) \subset A(X)$. There exist the sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{3n} = fx_{3n} = Bx_{3n+1}, \qquad y_{3n+1} = gx_{3n+1} = Cx_{3n+2}, \qquad y_{3n+2} = hx_{3n+2} = Ax_{3n+3}$$

for all $n = 0,1,2,\ldots$.

If there exists $n_0 \in \mathbb{N}$ such that $y_{n_0} = y_{n_0+1}$, then the conclusion (I) of Theorem 2.1 holds. In fact, if there exists $p \in \mathbb{N}$ such that $y_{3p+2} = y_{3p+3}$, then $fu = Au$, where $u = x_{3p+3}$. Hence the pair (f,A) has a coincidence point $u \in X$. If $y_{3p} = y_{3p+1}$, then $gu = Bu$, where $u = x_{3p+1}$.

Therefore, the pair (g,B) has a coincidence point $u \in X$. If $y_{3p+1} = y_{3p+2}$, then $hu = Cu$, where $u = x_{3p+2}$. And so the pair (h,C) has a coincidence point $u \in X$.

On the other hand, if there exists $n_0 \in \mathbb{N}$ such that $y_{n_0} = y_{n_0+1} = y_{n_0+2}$, then $y_n = y_{n_0}$ for any $n \geq n_0$. This implies that $\{y_n\}$ is a G-Cauchy sequence.

Actually, if there exists $p \in \mathbb{N}$ such that $y_{3p} = y_{3p+1} = y_{3p+2}$, then applying the contractive condition (2.1) with $x = y_{3p+3}$, $y = y_{3p+1}$ and $z = y_{3p+2}$, we get

$$\begin{aligned}
&G(y_{3p+1}, y_{3p+2}, y_{3p+3}) \\
&\quad = G(fx_{3p+3}, gx_{3p+1}, hx_{3p+2}) \\
&\quad \leq k \max \left\{ \begin{array}{c} G(Ax_{3p+3}, Bx_{3p+1}, Cx_{3p+2}), G(Ax_{3p+3}, fx_{3p+3}, fx_{3p+3}), \\ G(Bx_{3p+1}, gx_{3p+1}, gx_{3p+1}), G(Cx_{3p+2}, hx_{3p+2}, hx_{3p+2}), \\ G(Ax_{3p+3}, gx_{3p+1}, gx_{3p+1}), G(Ax_{3p+3}, hx_{3p+2}, hx_{3p+2}), \\ G(Bx_{3p+1}, fx_{3p+3}, fx_{3p+3}), G(Bx_{3p+1}, hx_{3p+2}, hx_{3p+2}), \\ G(Cx_{3p+2}, fx_{3p+3}, fx_{3p+3}), G(Cx_{3p+2}, gx_{3p+1}, gx_{3p+1}) \end{array} \right\} \\
&\quad = k \max \left\{ \begin{array}{c} G(y_{3p+2}, y_{3p}, y_{3p+1}), G(y_{3p+2}, y_{3p+3}, y_{3p+3}), \\ G(y_{3p}, y_{3p+1}, y_{3p+1}), G(y_{3p+1}, y_{3p+2}, y_{3p+2}), \\ G(y_{3p+2}, y_{3p+1}, y_{3p+1}), G(y_{3p+2}, y_{3p+2}, y_{3p+2}), \\ G(y_{3p}, y_{3p+3}, y_{3p+3}), G(y_{3p}, y_{3p+2}, y_{3p+2}), \\ G(y_{3p+1}, y_{3p+3}, y_{3p+3}), G(y_{3p+1}, y_{3p+1}, y_{3p+1}) \end{array} \right\} \\
&\quad \leq k \max \bigl\{ G(y_{3p+2}, y_{3p+3}, y_{3p+3}), G(y_{3p}, y_{3p+3}, y_{3p+3}), G(y_{3p+1}, y_{3p+3}, y_{3p+3}) \bigr\} \\
&\quad = kG(y_{3p+2}, y_{3p+3}, y_{3p+3}).
\end{aligned}$$

If $y_{3p+3} \neq y_{3p+1}$, then from condition (G_3) and Proposition 1.4(iii), we get

$$0 < G(y_{3n+1}, y_{3n+2}, y_{3n+3}) \leq 2kG(y_{3n+2}, y_{3n+2}, y_{3n+3}) \leq 2kG(y_{3n+1}, y_{3n+2}, y_{3n+3}),$$

which implies that $k \geq \frac{1}{2}$, that is a contradiction, since $0 \leq k < \frac{1}{2}$. So, we find $y_n = y_{3p}$ for any $n \geq 3p$. This implies that $\{y_n\}$ is a G-Cauchy sequence. The same conclusion holds if $y_{3p+1} = y_{3p+2} = y_{3p+3}$, or $y_{3p+2} = y_{3p+3} = y_{3p+4}$ for some $p \in \mathbb{N}$.

Assume for the rest of the paper that $y_n \neq y_m$ for any $n \neq m$. Applying again (2.1) with $x = y_{3n}$, $y = y_{3n+1}$ and $z = y_{3n+2}$ and using conditions (G_3) and (G_5), we get that

$$\begin{aligned}
&G(y_{3n}, y_{3n+1}, y_{3n+2}) \\
&\quad = G(fx_{3n}, gx_{3n+1}, hx_{3n+2}) \\
&\quad \leq k \max \left\{ \begin{array}{c} G(Ax_{3n}, Bx_{3n+1}, Cx_{3n+2}), G(Ax_{3n}, fx_{3n}, fx_{3n}), \\ G(Bx_{3n+1}, gx_{3n+1}, gx_{3n+1}), G(Cx_{3n+2}, hx_{3n+2}, hx_{3n+2}), \\ G(Ax_{3n}, gx_{3n+1}, gx_{3n+1}), G(Ax_{3n}, hx_{3n+2}, hx_{3n+2}), \\ G(Bx_{3n+1}, fx_{3n}, fx_{3n}), G(Bx_{3n+1}, hx_{3n+2}, hx_{3n+2}), \\ G(Cx_{3n+2}, fx_{3n}, fx_{3n}), G(Cx_{3n+2}, gx_{3n+1}, gx_{3n+1}) \end{array} \right\} \\
&\quad = k \max \left\{ \begin{array}{c} G(y_{3n-1}, y_{3n}, y_{3n+1}), G(y_{3n-1}, y_{3n}, y_{3n}), \\ G(y_{3n}, y_{3n+1}, y_{3n+1}), G(y_{3n+1}, y_{3n+2}, y_{3n+2}), \\ G(y_{3n-1}, y_{3n+1}, y_{3n+1}), G(y_{3n-1}, y_{3n+2}, y_{3n+2}), \\ G(y_{3n}, y_{3n}, y_{3n}), G(y_{3n}, y_{3n+2}, y_{3n+2}), \\ G(y_{3n+1}, y_{3n}, y_{3n}), G(y_{3n+1}, y_{3n+1}, y_{3n+1}) \end{array} \right\}
\end{aligned}$$

$$\leq k\max\left\{\begin{matrix} G(y_{3n-1},y_{3n},y_{3n+1}),G(y_{3n-1},y_{3n},y_{3n+1}),G(y_{3n-1},y_{3n},y_{3n+1}),\\ G(y_{3n},y_{3n+1},y_{3n+2}),G(y_{3n-1},y_{3n},y_{3n+1}),\\ G(y_{3n-1},y_{3n+1},y_{3n+1})+G(y_{3n+1},y_{3n+2},y_{3n+2}),\\ 0,G(y_{3n},y_{3n+1},y_{3n+2}),G(y_{3n-1},y_{3n},y_{3n+1}),0 \end{matrix}\right\}$$

$$\leq k\max\left\{\begin{matrix} G(y_{3n-1},y_{3n},y_{3n+1}),G(y_{3n-1},y_{3n},y_{3n+1}),G(y_{3n-1},y_{3n},y_{3n+1}),\\ G(y_{3n},y_{3n+1},y_{3n+2}),G(y_{3n-1},y_{3n},y_{3n+1}),\\ G(y_{3n-1},y_{3n},y_{3n+1})+G(y_{3n},y_{3n+1},y_{3n+2}),\\ 0,G(y_{3n},y_{3n+1},y_{3n+2}),G(y_{3n-1},y_{3n},y_{3n+1}),0 \end{matrix}\right\}$$

$$= k\max\left\{\begin{matrix} G(y_{3n-1},y_{3n},y_{3n+1}),G(y_{3n},y_{3n+1},y_{3n+2}),\\ G(y_{3n-1},y_{3n},y_{3n+1})+G(y_{3n},y_{3n+1},y_{3n+2}) \end{matrix}\right\}$$

$$= k\bigl[G(y_{3n-1},y_{3n},y_{3n+1})+G(y_{3n},y_{3n+1},y_{3n+2})\bigr].$$

From $k\in[0,\frac{1}{2})$ we obtain

$$G(y_{3n},y_{3n+1},y_{3n+2})\leq\lambda G(y_{3n-1},y_{3n},y_{3n+1}), \tag{2.3}$$

where $\lambda=\frac{k}{1-k}\in[0,1)$. Similarly it can be shown that

$$G(y_{3n+1},y_{3n+2},y_{3n+3})\leq\lambda G(y_{3n},y_{3n+1},y_{3n+2}) \tag{2.4}$$

and

$$G(y_{3n+1},y_{3n+2},y_{3n+3})\leq\lambda G(y_{3n},y_{3n+1},y_{3n+2}). \tag{2.5}$$

It follows from (2.3), (2.4) and (2.5) that, for all $n\in\mathbb{N}$,

$$G(y_n,y_{n+1},y_{n+2})\leq\lambda G(y_{n-1},y_n,y_{n+1})\leq\lambda^2G(y_{n-2},y_{n-1},y_n)\leq\cdots\leq\lambda^nG(y_0,y_1,y_2).$$

Therefore, for all $n,m\in\mathbb{N}$, $n<m$, by (G_3) and (G_5), we have

$$\begin{aligned} G(y_n,y_m,y_m)&\leq G(y_n,y_{n+1},y_{n+1})+G(y_{n+1},y_{n+2},y_{n+2})+G(y_{n+2},y_{n+3},y_{n+3})\\ &\quad+\cdots+G(y_{m-1},y_m,y_m)\\ &\leq G(y_n,y_{n+1},y_{n+2})+G(y_{n+1},y_{n+2},y_{n+3})+\cdots+G(y_{m-1},y_m,y_{m+1})\\ &\leq\bigl(\lambda^n+\lambda^{n+1}+\lambda^{n+2}+\cdots+\lambda^{m-1}\bigr)G(y_0,y_1,y_2)\\ &\leq\frac{\lambda^n}{1-\lambda}G(y_0,y_1,y_2)\to0,\quad\text{as }n\to\infty. \end{aligned}$$

Hence $\{y_n\}$ is a G-Cauchy sequence in X. Since X is a complete G-metric space, there exists a point $u\in X$ such that $y_n\to u$ $(n\to\infty)$.

Since the sequences $\{fx_{3n}\}=\{Bx_{3n+1}\}$, $\{gx_{3n+1}\}=\{Cx_{3n+2}\}$ and $\{hx_{3n-1}\}=\{Ax_{3n}\}$ are all subsequences of $\{y_n\}$, then they all converge to u

$$\begin{aligned} &y_{3n}=fx_{3n}=Bx_{3n+1}\to u,\qquad y_{3n+1}=gx_{3n+1}=Cx_{3n+2}\to u,\\ &y_{3n-1}=hx_{3n-1}=Ax_{3n}\to u\quad(n\to\infty). \end{aligned} \tag{2.6}$$

Now we prove that u is a common fixed point of f, g, h, A, B and C under condition (a).

First, we suppose that A is continuous, the pair (f,A) is weakly commuting, the pairs (g,B) and (h,C) are weakly compatible.

Step 1. We prove that $u = fu = Au$.

By (2.6) and a weakly commuting of mapping pair (f,A), we have

$$G(fAx_{3n},Afx_{3n},Afx_{3n}) \leq G(fx_{3n},Ax_{3n},Ax_{3n}) \to 0 \quad (n \to \infty). \tag{2.7}$$

Since A is continuous, then $A^2x_{3n} \to Au$ $(n \to \infty)$, $Afx_{3n} \to Au$ $(n \to \infty)$. By (2.7) we know that $fAx_{3n} \to Au$ $(n \to \infty)$.

From condition (2.1) we know

$$\begin{aligned} &G(fAx_{3n},gx_{3n+1},hx_{3n+2}) \\ &\quad \leq k\max\left\{\begin{matrix} G(A^2x_{3n},Bx_{3n+1},Cx_{3n+2}),G(A^2x_{3n},fAx_{3n},fAx_{3n}), \\ G(Bx_{3n+1},gx_{3n+1},gx_{3n+1}),G(Cx_{3n+2},hx_{3n+2},hx_{3n+2}), \\ G(A^2x_{3n},gx_{3n+1},gx_{3n+1}),G(A^2x_{3n},hx_{3n+2},hx_{3n+2}), \\ G(Bx_{3n+1},fAx_{3n},fAx_{3n}),G(Bx_{3n+1},hx_{3n+2},hx_{3n+2}), \\ G(Cx_{3n+2},fx_{3n},fx_{3n}),G(Cx_{3n+2},gx_{3n+1},gx_{3n+1}) \end{matrix}\right\}. \end{aligned}$$

Letting $n \to \infty$ and using Proposition 1.4(iii), we have

$$\begin{aligned} G(Au,u,u) &\leq k\max\left\{\begin{matrix} G(Au,u,u),G(Au,Au,Au),G(u,u,u),G(u,u,u),G(Au,u,u), \\ G(Au,u,u),G(u,Au,Au),G(u,u,u),G(u,Au,Au),G(u,u,u) \end{matrix}\right\} \\ &= k\max\bigl\{G(Au,u,u),G(u,Au,Au)\bigr\} \\ &\leq 2kG(Au,u,u), \end{aligned}$$

which implies that $G(Au,u,u) = 0$, and so $Au = u$ since $0 \leq k < \frac{1}{2}$.

Again, by use of condition (2.1), we have

$$G(fu,gx_{3n+1},hx_{3n+2}) \leq k\max\left\{\begin{matrix} G(Au,Bx_{3n+1},Cx_{3n+2}),G(Au,fu,fu), \\ G(Bx_{3n+1},gx_{3n+1},gx_{3n+1}),G(Cx_{3n+2},hx_{3n+2},hx_{3n+2}), \\ G(Au,gx_{3n+1},gx_{3n+1}),G(Au,hx_{3n+2},hx_{3n+2}), \\ G(Bx_{3n+1},fu,fu),G(Bx_{3n+1},hx_{3n+2},hx_{3n+2}), \\ G(Cx_{3n+2},fu,fu),G(Cx_{3n+2},gx_{3n+1},gx_{3n+1}) \end{matrix}\right\}.$$

Letting $n \to \infty$, using (2.6), $u = Au$ and Proposition 1.4(iii), we obtain

$$\begin{aligned} G(fu,u,u) &\leq k\max\left\{\begin{matrix} G(u,u,u),G(u,fu,fu),G(u,u,u),G(u,u,u),G(u,u,u), \\ G(u,u,u),G(u,fu,fu),G(u,u,u),G(u,fu,fu),G(u,u,u) \end{matrix}\right\} \\ &= kG(u,fu,fu) \leq 2kG(fu,u,u). \end{aligned}$$

This implies that $G(fu,u,u) = 0$ and so $fu = u$. Thus we have $u = Au = fu$.

Step 2. We prove that $u = gu = Bu$.

Since $f(X) \subset B(X)$ and $u = fu \in f(X)$, there is a point $v \in X$ such that $u = fu = Bv$. Again, by use of condition (2.1), we have

$$G(fu, gv, hx_{3n+2}) \leq k \max \left\{ \begin{array}{c} G(Au, Bv, Cx_{3n+2}), G(Au, fu, fu), \\ G(Bv, gv, gv), G(Cx_{3n+2}, hx_{3n+2}, hx_{3n+2}), \\ G(Au, gv, gv), G(Au, hx_{3n+2}, hx_{3n+2}), \\ G(Bv, fu, fu), G(Bv, hx_{3n+2}, hx_{3n+2}), \\ G(Cx_{3n+2}, fu, fu), G(Cx_{3n+2}, gv, gv) \end{array} \right\}.$$

Letting $n \to \infty$, using $u = Au = fu = Bv$ and Proposition 1.4(iii), we have

$$\begin{aligned} G(u, gv, u) &\leq k \max \left\{ \begin{array}{c} G(u,u,u), G(u,u,u), G(u,gv,gv), G(u,u,u), G(u,gv,gv), \\ G(u,u,u), G(u,u,u), G(u,u,u), G(u,u,u), G(u,gv,gv) \end{array} \right\} \\ &= kG(u,gv,gv) \leq 2kG(u,gv,u), \end{aligned}$$

which implies that $G(u, gv, u) = 0$, and so $gv = u = Bv$.

Since the pair (g, B) is weakly compatible, we have

$$gu = gBv = Bgv = Bu.$$

Again, by use of condition (2.1), we have

$$G(fu, gu, hx_{3n+2}) \leq k \max \left\{ \begin{array}{c} G(Au, Bu, Cx_{3n+2}), G(Au, fu, fu), \\ G(Bu, gu, gu), G(Cx_{3n+2}, hx_{3n+2}, hx_{3n+2}), \\ G(Au, gu, gu), G(Au, hx_{3n+2}, hx_{3n+2}), \\ G(Bu, fu, fu), G(Bu, hx_{3n+2}, hx_{3n+2}), \\ G(Cx_{3n+2}, fu, fu), G(Cx_{3n+2}, gu, gu) \end{array} \right\}.$$

Letting $n \to \infty$, using $u = Au = fu$ and $gu = Bu$ and Proposition 1.4(iii), we have

$$\begin{aligned} G(u, gu, u) &\leq k \max \left\{ \begin{array}{c} G(u,gu,u), G(u,u,u), G(gu,gu,gu), G(u,u,u), G(u,gu,gu), \\ G(u,u,u), G(gu,u,u), G(gu,u,u), G(u,u,u), G(u,gu,gu) \end{array} \right\} \\ &= k \max\{G(u,gu,u), G(u,gu,gu)\} \leq 2kG(u,gu,u). \end{aligned}$$

This implies that $G(u, gu, u) = 0$, and so $u = gu = Bu$.

Step 3. We prove that $u = hu = Cu$.

Since $g(X) \subset C(X)$ and $u = gu \in g(X)$, there is a point $w \in X$ such that $u = gu = Cw$. Again, by use of condition (2.1), we have

$$G(fu, gu, hw) \leq k \max \left\{ \begin{array}{c} G(Au, Bu, Cw), G(Au, fu, fu), \\ G(Bu, gu, gu), G(Cw, hw, hw), \\ G(Au, gu, gu), G(Au, hw, hw), \\ G(Bu, fu, fu), G(Bu, hw, hw), \\ G(Cw, fu, fu), G(Cw, gu, gu) \end{array} \right\}.$$

Using $u = Au = fu$, $u = gu = Bu = Cw$ and Proposition 1.4(iii), we obtain

$$\begin{aligned} G(u,u,hw) &\le k\max\left\{\begin{array}{c} G(u,u,u), G(u,u,u), G(u,u,u), G(u,hw,hw), G(u,u,u), \\ G(u,hw,hw), G(u,u,u), G(u,hw,hw), G(u,u,u), G(u,u,u) \end{array}\right\} \\ &= kG(u,hw,hw) \le 2kG(u,u,hw). \end{aligned}$$

Hence $G(u,u,hw) = 0$, and so $hw = u = Cw$.

Since the pair (h,C) is weakly compatible, we have

$$hu = hCw = Chw = Cu.$$

Again, by use of condition (2.1), we have

$$G(fu,gu,hu) \le k\max\left\{\begin{array}{c} G(Au,Bu,Cu), G(Au,fu,fu), \\ G(Bu,gu,gu), G(Cu,hu,hu), \\ G(Au,gu,gu), G(Au,hu,hu), \\ G(Bu,fu,fu), G(Bu,hu,hu), \\ G(Cu,fu,fu), G(Cu,gu,gu) \end{array}\right\}.$$

Using $u = Au = fu$, $u = gu = Bu$, $Cu = hu$ and Proposition 1.4(iii), we have

$$G(u,u,hu) \le k\max\bigl\{G(u,u,hu), G(u,hu,hu)\bigr\} \le 2kG(u,u,hu).$$

Thus $G(u,u,hu) = 0$, and so $u = hu = Cu$.

Therefore u is the common fixed point of f, g, h, A, B and C when A is continuous and the pair (f,A) is weakly commuting, the pairs (g,B) and (h,C) are weakly compatible.

Next, we suppose that f is continuous, the pair (f,A) is weakly commuting, the pairs (g,B) and (h,C) are weakly compatible.

Step 1. We prove that $u = fu$.

By (2.6) and a weakly commuting mapping pair (f,A), we have

$$G(fAx_{3n}, Afx_{3n}, Afx_{3n}) \le G(fx_{3n}, Ax_{3n}, Ax_{3n}) \to 0 \quad (n \to \infty). \tag{2.8}$$

Since f is continuous, then $f^2x_{3n} \to fu$ $(n \to \infty)$, $fAx_{3n} \to fu$ $(n \to \infty)$. By (2.6) we know $Afx_{3n} \to fu$ $(n \to \infty)$.

From condition (2.1) we know

$$\begin{aligned} &G\bigl(f^2x_{3n}, gx_{3n+1}, hx_{3n+2}\bigr) \\ &\quad\le k\max\left\{\begin{array}{c} G(Afx_{3n}, Bx_{3n+1}, Cx_{3n+2}), G(Afx_{3n}, f^2x_{3n}, f^2x_{3n}), \\ G(Bx_{3n+1}, gx_{3n+1}, gx_{3n+1}), G(Cx_{3n+2}, hx_{3n+2}, hx_{3n+2}), \\ G(Afx_{3n}, gx_{3n+1}, gx_{3n+1}), G(Afx_{3n}, hx_{3n+2}, hx_{3n+2}), \\ G(Bx_{3n+1}, f^2x_{3n}, f^2x_{3n}), G(Bx_{3n+1}, hx_{3n+2}, hx_{3n+2}), \\ G(Cx_{3n+2}, f^2x_{3n}, f^2x_{3n}), G(Cx_{3n+2}, gx_{3n+1}, gx_{3n+1}) \end{array}\right\}. \end{aligned}$$

Letting $n \to \infty$ and noting Proposition 1.4(iii), we have

$$\begin{aligned} G(fu,u,u) &\le k\max\left\{\begin{matrix} G(fu,u,u), G(fu,fu,fu), G(u,u,u), G(u,u,u), G(fu,u,u), \\ G(fu,u,u), G(u,fu,fu), G(u,u,u), G(u,fu,fu), G(u,u,u) \end{matrix}\right\} \\ &= k\max\bigl\{G(fu,u,u), G(u,fu,fu)\bigr\} \\ &\le 2kG(fu,u,u), \end{aligned}$$

which implies that $G(fu,u,u) = 0$, and so $fu = u$.

Step 2. We prove that $u = gu = Bu$.

Since $f(X) \subset B(X)$ and $u = fu \in f(X)$, there is a point $z \in X$ such that $u = fu = Bz$. Again, by use of condition (2.1), we have

$$G\bigl(f^2x_{3n}, gz, hx_{3n+2}\bigr) \le k\max\left\{\begin{matrix} G(Afx_{3n}, Bz, Cx_{3n+2}), G(Afx_{3n}, f^2x_{3n}, f^2x_{3n}), \\ G(Bz, gz, gz), G(Cx_{3n+2}, hx_{3n+2}, hx_{3n+2}), \\ G(Afx_{3n}, gz, gz), G(Afx_{3n}, hx_{3n+2}, hx_{3n+2}), \\ G(Bz, f^2x_{3n}, f^2x_{3n}), G(Bz, hx_{3n+2}, hx_{3n+2}), \\ G(Cx_{3n+2}, f^2x_{3n}, f^2x_{3n}), G(Cx_{3n+2}, gz, gz) \end{matrix}\right\}.$$

Letting $n \to \infty$, using $u = fu = Bz$ and Proposition 1.4(iii), we have

$$\begin{aligned} G(u,gz,u) &\le k\max\left\{\begin{matrix} G(u,u,u), G(u,u,u), G(u,gz,gz), G(u,u,u), G(u,gz,gz), \\ G(u,u,u), G(u,u,u), G(u,u,u), G(u,u,u), G(u,gz,gz) \end{matrix}\right\} \\ &= kG(u,gz,gz) \le 2kG(u,gu,u). \end{aligned}$$

This implies that $G(u,gz,u) = 0$, and so $gz = u = Bz$.

Since the pair (g,B) is weakly compatible, we have

$$gu = gBz = Bgz = Bu.$$

Again, by use of condition (2.1), we have

$$G(fx_{3n}, gu, hx_{3n+2}) \le k\max\left\{\begin{matrix} G(Ax_{3n}, Bu, Cx_{3n+2}), G(Ax_{3n}, fx_{3n}, fx_{3n}), \\ G(Bu, gu, gu), G(Cx_{3n+2}, hx_{3n+2}, hx_{3n+2}), \\ G(Ax_{3n}, gu, gu), G(Ax_{3n}, hx_{3n+2}, hx_{3n+2}), \\ G(Bu, fx_{3n}, fx_{3n}), G(Bu, hx_{3n+2}, hx_{3n+2}), \\ G(Cx_{3n+2}, fx_{3n}, fx_{3n}), G(Cx_{3n+2}, gu, gu) \end{matrix}\right\}.$$

Letting $n \to \infty$, using $u = fu$, $gu = Bu$ and Proposition 1.4(iii), we have

$$\begin{aligned} G(u,gu,u) &\le k\max\left\{\begin{matrix} G(u,gu,u), G(u,u,u), G(gu,gu,gu), G(u,u,u), G(u,gu,gu), \\ G(u,u,u), G(gu,u,u), G(gu,u,u), G(u,u,u), G(u,gu,gu) \end{matrix}\right\} \\ &= k\max\bigl\{G(u,gu,u), G(u,gu,gu)\bigr\} \le 2kG(u,gu,u). \end{aligned}$$

Therefore, $G(u,gu,u) = 0$, and so $gu = u = Bu$.

Step 3. We prove that $u = hu = Cu$.

Since $g(X) \subset C(X)$ and $u = gu \in g(X)$, there is a point $t \in X$ such that $u = gu = Ct$. Again, by use of condition (2.1), we have

$$G(fx_{3n}, gu, ht) \le k \max \left\{ \begin{matrix} G(Ax_{3n}, Bu, Ct), G(Ax_{3n}, fx_{3n}, fx_{3n}), \\ G(Bu, gu, gu), G(Ct, ht, ht), \\ G(Ax_{3n}, gu, gu), G(Ax_{3n}, ht, ht), \\ G(Bu, fx_{3n}, fx_{3n}), G(Bu, ht, ht), \\ G(Ct, fx_{3n}, fx_{3n}), G(Ct, gu, gu) \end{matrix} \right\}.$$

Letting $n \to \infty$, using $u = gu = Bu = Ct$ and Proposition 1.4(iii), we obtain

$$\begin{aligned} G(u, u, ht) &\le k \max \left\{ \begin{matrix} G(u,u,u), G(u,u,u), G(u,u,u), G(u,ht,ht), G(u,u,u), \\ G(u,ht,ht), G(u,u,u), G(u,ht,ht), G(u,u,u), G(u,u,u) \end{matrix} \right\} \\ &= kG(u, ht, ht) \le 2kG(u, u, ht). \end{aligned}$$

Thus $G(u, u, ht) = 0$, and so $ht = u = Ct$.

Since the pair (h, C) is weakly compatible, we have

$$hu = hCt = Cht = Cu.$$

Again, by use of condition (2.1), we have

$$G(fx_{3n}, gu, hu) \le k \max \left\{ \begin{matrix} G(Ax_{3n}, Bu, Cu), G(Ax_{3n}, fx_{3n}, fx_{3n}), \\ G(Bu, gu, gu), G(Cu, hu, hu), \\ G(Ax_{3n}, gu, gu), G(Ax_{3n}, hu, hu), \\ G(Bu, fx_{3n}, fx_{3n}), G(Bu, hu, hu), \\ G(Cu, fx_{3n}, fx_{3n}), G(Cu, gu, gu) \end{matrix} \right\}.$$

Letting $n \to \infty$, using $u = fu = gu = Bu$, $Cu = hu$ and Proposition 1.4(iii), we have

$$\begin{aligned} G(u, u, hu) &\le k \max \left\{ \begin{matrix} G(u,u,hu), G(u,u,u), G(u,u,u), G(hu,hu,hu), G(u,u,u), \\ G(u,hu,hu), G(u,u,u), G(u,hu,hu), G(hu,u,u), G(hu,u,u) \end{matrix} \right\} \\ &= k \max\{G(u,u,hu), G(u,hu,hu)\} \le 2kG(u,u,hu), \end{aligned}$$

which implies that $G(u, u, hu) = 0$, and so $hu = u = Cu$.

Step 4. We prove that $u = Au$.

Since $h(X) \subset A(X)$ and $u = hu \in h(X)$, there is a point $p \in X$ such that $u = hu = Ap$. Again, by use of condition (2.1), we have

$$G(fp, gu, hu) \le k \max \left\{ \begin{matrix} G(Ap, Bu, Cu), G(Ap, fp, fp), \\ G(Bu, gu, gu), G(Cu, hu, hu), \\ G(Ap, gu, gu), G(Ap, hu, hu), \\ G(Bu, fp, fp), G(Bu, hu, hu), \\ G(Cu, fp, fp), G(Cu, gu, gu) \end{matrix} \right\}.$$

Using $u = gu = Bu$, $u = hu = Cu$ and Proposition 1.4(iii), we obtain

$$G(fp,u,u) \leq kG(u,fp,fp) \leq 2\alpha G(fp,u,u).$$

Hence $G(fp,u,u) = 0$, and so $fp = u = Ap$.

Since the pair (f,A) is weakly compatible, we have

$$fu = fAp = Afp = Au = u.$$

Therefore u is the common fixed point of f, g, h, A, B and C when S is continuous and the pair (f,A) is weakly commuting, the pairs (g,B) and (h,C) are weakly compatible.

Similarly we can prove the result that u is a common fixed point of f, g, h, A, B and C under the condition of (b) or (c).

Finally, we prove the uniqueness of a common fixed point u.

Let u and q be two common fixed points of f, g, h, A, B and C. By use of condition (2.1), we have

$$\begin{aligned} G(q,u,u) &= G(fq,gu,hu) \\ &\leq k\max\left\{\begin{array}{c} G(Aq,Bu,Cu), G(Aq,fq,fq), \\ G(Bu,gu,gu), G(Cu,hu,hu), \\ G(Aq,gu,gu), G(Aq,hu,hu), \\ G(Bu,fq,fq), G(Bu,hu,hu), \\ G(Cu,fq,fq), G(Cu,gu,gu) \end{array}\right\} \\ &= k\max\{G(q,u,u), G(u,q,q)\} \\ &\leq 2kG(q,u,u). \end{aligned}$$

This implies that $G(q,u,u) = 0$, and so $q = u$. Thus the common fixed point is unique.

The proof using (2.2) is similar. This completes the proof. □

Now we introduce an example to support Theorem 2.1.

Example 2.2 Let $X = [0,1]$ and let (X,G) be a G-metric space defined by $G(x,y,z) = |x-y| + |y-z| + |z-x|$ for all x,y,z in X. Let f, g, h, A, B and C be self-mappings defined by

$$fx = \begin{cases} 1, & x \in [0,\frac{1}{2}], \\ \frac{5}{6}, & x \in (\frac{1}{2},1], \end{cases} \qquad gx = \begin{cases} \frac{7}{8}, & x \in [0,\frac{1}{2}], \\ \frac{5}{6}, & x \in (\frac{1}{2},1], \end{cases} \qquad hx = \begin{cases} \frac{6}{7}, & x \in [0,\frac{1}{2}], \\ \frac{5}{6}, & x \in (\frac{1}{2},1], \end{cases}$$

$$Ax = x, \qquad Bx = \begin{cases} 1, & x \in [0,\frac{1}{2}], \\ \frac{5}{6}, & x \in (\frac{1}{2},1), \\ 0, & x = 1, \end{cases} \qquad Cx = \begin{cases} 1, & x \in [0,\frac{1}{2}], \\ \frac{5}{6}, & x \in (\frac{1}{2},1), \\ \frac{7}{8}, & x = 1. \end{cases}$$

Note that A is G-continuous in X, and f, g, h, B and C are not G-continuous in X.

(i) Clearly we can get $f(X) \subset B(X)$, $g(X) \subset C(X)$, $h(X) \subset A(X)$.

Actually, because $fX = \{\frac{5}{6}, 1\}$, $BX = \{0, \frac{5}{6}, 1\}$, $gX = \{\frac{5}{6}, \frac{7}{8}\}$, $CX = \{0, \frac{5}{6}, \frac{7}{8}\}$, $hX = \{\frac{5}{6}, \frac{6}{7}\}$, $AX = X = [0,1]$, so we know $f(X) \subset B(X)$, $g(X) \subset C(X)$ and $h(X) \subset A(X)$.

(ii) By the definition of the mappings of f and A, for all $x \in [0,1]$, $G(fAx, Afx, Afx) = G(fx, fx, fx) = 0 \leq G(fx, Ax, Ax)$, so we can get the pair (f, A) is weakly commuting.

By the definition of the mappings of g and B, only for $x \in (\frac{1}{2}, 1)$, $gx = Bx = \frac{5}{6}$, at this time $gBx = T(\frac{5}{6}) = \frac{5}{6} = B(\frac{5}{6}) = Bgx$, so $gBx = Bgx$, so we can obtain the pair (g, B) is weakly compatible. Similarly we can prove the pair (h, C) is also weakly compatible.

(iii) Now we prove the mappings f, g, h, A, B and C satisfy condition (2.1) of Theorem 2.1 with $k = \frac{2}{5}$

$$M(x,y,z) = \max \begin{Bmatrix} G(Ax,By,Cz), G(Ax,fx,fx), \\ G(By,gy,gy), G(Cz,hz,hz), \\ G(Ax,gy,gy), G(Ax,hz,hz), \\ G(By,fx,fx), G(By,hz,hz), \\ G(Cz,fx,fx), G(Cz,gy,gy) \end{Bmatrix}.$$

Case 1. If $x, y, z \in [0, \frac{1}{2}]$, then

$$G(fx,gy,hz) = G\left(1, \frac{7}{8}, \frac{6}{7}\right) = \frac{2}{7},$$

$$G(Ax,fx,fx) = G(x,1,1) = 2|x-1| \geq 1.$$

Thus we have

$$G(fx,gy,hz) = \frac{2}{7} < \frac{2}{5} \times 1 \leq \frac{2}{5} G(Ax,fx,fx) \leq \frac{2}{5} M(x,y,z).$$

Case 2. If $x, y \in [0, \frac{1}{2}]$, $z \in (\frac{1}{2}, 1]$, then

$$G(fx,gy,hz) = G\left(1, \frac{7}{8}, \frac{5}{6}\right) = \frac{1}{3},$$

$$G(Ax,fx,fx) = G(x,1,1) = 2|x-1| \geq 1.$$

Hence we get

$$G(fx,gy,hz) = \frac{1}{3} < \frac{2}{5} \times 1 \leq \frac{2}{5} G(Ax,fx,fx) \leq \frac{2}{5} M(x,y,z).$$

Case 3. If $x, z \in [0, \frac{1}{2}]$, $y \in (\frac{1}{2}, 1]$, then

$$G(fx,gy,hz) = G\left(1, \frac{5}{6}, \frac{6}{7}\right) = \frac{1}{3},$$

$$G(Ax,fx,fx) = G(x,1,1) = 2|x-1| \geq 1.$$

Therefore we obtain

$$G(fx,gy,hz) = \frac{1}{3} < \frac{2}{5} \times 1 \leq \frac{2}{5} G(Ax,fx,fx) \leq \frac{2}{5} M(x,y,z).$$

Case 4. If $y, z \in [0, \frac{1}{2}]$, $x \in (\frac{1}{2}, 1]$, then

$$G(fx, gy, hz) = G\left(\frac{5}{6}, \frac{7}{8}, \frac{6}{7}\right) = \frac{1}{12},$$
$$G(By, gy, gy) = G\left(1, \frac{7}{8}, \frac{7}{8}\right) = \frac{1}{4}.$$

Thus we have

$$G(fx, gy, hz) = \frac{1}{12} < \frac{2}{5} \times \frac{1}{4} = \frac{2}{5} G(By, gy, gy) \leq \frac{2}{5} M(x, y, z).$$

Case 5. If $x \in [0, \frac{1}{2}]$, $y, z \in (\frac{1}{2}, 1]$, then

$$G(fx, gy, hz) = G\left(1, \frac{5}{6}, \frac{5}{6}\right) = \frac{1}{3},$$
$$G(Ax, fx, fx) = G(x, 1, 1) = 2|x - 1| \geq 1.$$

Hence we obtain

$$G(fx, gy, hz) = \frac{1}{3} < \frac{2}{5} \times 1 \leq \frac{2}{5} G(Ax, fx, fx) \leq \frac{2}{5} M(x, y, z).$$

Case 6. If $y \in [0, \frac{1}{2}]$, $x, z \in (\frac{1}{2}, 1]$, then

$$G(fx, gy, hz) = G\left(\frac{5}{6}, \frac{7}{8}, \frac{5}{6}\right) = \frac{1}{12},$$
$$G(By, gy, gy) = G\left(1, \frac{7}{8}, \frac{7}{8}\right) = \frac{1}{4}.$$

So we have

$$G(fx, gy, hz) = \frac{1}{12} < \frac{2}{5} \times \frac{1}{4} = \frac{2}{5} G(By, gy, gy) \leq \frac{2}{5} M(x, y, z).$$

Case 7. If $z \in [0, \frac{1}{2}]$, $x, y \in (\frac{1}{2}, 1]$, then

$$G(fx, gy, hz) = G\left(\frac{5}{6}, \frac{5}{6}, \frac{6}{7}\right) = \frac{1}{21},$$
$$G(Cz, hz, hz) = G\left(1, \frac{6}{7}, \frac{6}{7}\right) = \frac{2}{7}.$$

Thus we get

$$G(fx, gy, hz) = \frac{1}{21} < \frac{2}{5} \times \frac{2}{7} = \frac{2}{5} G(Cz, hz, hz) \leq \frac{2}{5} M(x, y, z).$$

Case 8. If $x, y, z \in (\frac{1}{2}, 1]$, then

$$G(fx, gy, hz) = G\left(\frac{5}{6}, \frac{5}{6}, \frac{5}{6}\right) = 0 \leq \frac{2}{5} M(x, y, z).$$

Then in all the above cases, the mappings f, g, h, A, B and C satisfy condition (2.1) of Theorem 2.1 with $k = \frac{2}{5}$. So that all the conditions of Theorem 2.1 are satisfied. Moreover, $\frac{5}{6}$ is the unique common fixed point for all of the mappings f, g, h, A, B and C.

In Theorem 2.1, if we take $A = B = C = I$ (I is identity mapping, the same below), then we have the following corollary.

Corollary 2.3 *Let (X, G) be a complete G-metric space and let f, g and h be three mappings of X into itself satisfying the following conditions*:

$$G(fx, gy, hz) \leq k \max \left\{ \begin{matrix} G(x,y,z), G(x,fx,fx), G(y,gy,gy), G(z,hz,hz), G(x,gy,gy), \\ G(x,hz,hz), G(y,fx,fx), G(y,hz,hz), G(z,fx,fx), G(z,gy,gy) \end{matrix} \right\} \tag{2.9}$$

or

$$G(fx, gy, hz) \leq k \max \left\{ \begin{matrix} G(x,y,z), G(x,x,fx), G(y,y,gy), G(z,z,hz), G(x,x,gy), \\ G(x,x,hz), G(y,y,fx), G(y,y,hz), G(z,z,fx), G(z,z,gy) \end{matrix} \right\} \tag{2.10}$$

$\forall x, y, z \in X$, where $k \in [0, \frac{1}{2})$. Then f, g and h have a unique common fixed point in X.

Remark 2.4 Corollary 2.3 generalizes and extends the corresponding results in Abbas *et al.* [7, Theorem 2.1].

Also, if we take $f = g = h$ and $A = B = C = I$ in Theorem 2.1, then we get the following.

Corollary 2.5 *Let (X, G) be a complete G-metric space and let f be a mapping of X into itself satisfying the following conditions*:

$$G(fx, fy, fz) \leq k \max \left\{ \begin{matrix} G(x,y,z), G(x,fx,fx), G(y,fy,fy), G(z,fz,fz), G(x,fy,fy), \\ G(x,fz,fz), G(y,fx,fx), G(y,fz,fz), G(z,fx,fx), G(z,fy,fy) \end{matrix} \right\} \tag{2.11}$$

or

$$G(fx, fy, fz) \leq k \max \left\{ \begin{matrix} G(x,y,z), G(x,x,fx), G(y,y,fy), G(z,z,fz), G(x,x,fy), \\ G(x,x,fz), G(y,y,fx), G(y,y,fz), G(z,z,fx), G(z,z,fy) \end{matrix} \right\} \tag{2.12}$$

$\forall x, y, z \in X$, where $k \in [0, \frac{1}{2})$. Then f has a unique fixed point in X.

Remark 2.6 Corollary 2.5 generalizes and extends the corresponding results in Mustafa and Sims [8, Theorem 2.1].

Remark 2.7 Theorem 2.1, Corollaries 2.3 and 2.5 in this paper also improve and generalize the corresponding results of Abbas and Rhoades [9, Theorems 2.4 and 2.5], Mustafa *et al.* [10, Theorems 2.3, 2.5, 2.8 and Corollary 2.6], Mustafa *et al.* [11, Theorem 2.5], Abbas *et al.* [12, Theorem 2.1, Corollaries 2.3-2.6] and Chugh and Kadian [13, Theorem 2.2].

Remark 2.8 In Theorem 2.1, we have taken: (1) $f = g = h$; (2) $A = B = C$; (3) $g = h$ and $B = C$; (4) $g = h$, $B = C = I$, several new results can be obtained.

Theorem 2.9 *Let* (X, G) *be a complete G-metric space and let* f, g, h, A, B *and* C *be six mappings of* X *into itself satisfying the following conditions*:

(i) $f(X) \subset B(X)$, $g(X) \subset C(X)$, $h(X) \subset A(X)$;

(ii) *The pairs* (f, A), (g, B) *and* (h, C) *are commuting mappings*;

(iii) $\forall x, y, z \in X$,

$$G\big(f^m x, g^m y, h^m z\big) \le k \max \left\{\begin{matrix} G(Ax, By, Cz), G(Ax, f^m x, f^m x), \\ G(By, g^m y, g^m y), G(Cz, h^m z, h^m z), \\ G(Ax, g^m y, g^m y), G(Ax, h^m z, h^m z), \\ G(By, f^m x, f^m x), G(By, h^m z, h^m z), \\ G(Cz, f^m x, f^m x), G(Cz, g^m y, g^m y) \end{matrix}\right\} \tag{2.13}$$

or

$$G\big(f^m x, g^m y, h^m z\big) \le k \max \left\{\begin{matrix} G(Ax, By, Cz), G(Ax, Ax, f^m x), \\ G(By, By, g^m y), G(Cz, Cz, h^m z), \\ G(Ax, Ax, g^m y), G(Ax, Ax, h^m z), \\ G(By, By, f^m x), G(By, By, h^m z), \\ G(Cz, Cz, f^m x), G(Cz, Cz, g^m y) \end{matrix}\right\}, \tag{2.14}$$

where $\alpha \in [0, \frac{1}{2})$, $m \in \mathbb{N}$, *then* f, g, h, A, B *and* C *have a unique common fixed point in* X.

Proof Suppose that mappings f, g, h, A, B and C satisfy condition (2.13). Since $f^m X \subset f^{m-1} X \subset \cdots \subset fX$, $fX \subset BX$, so that $f^m X \subset BX$. Similar, we can show that $g^m X \subset CX$ and $h^m X \subset AX$. From Theorem 2.1, we see that f^m, g^m, h^m, A, B and C have a unique common fixed point u.

It follows from (2.13) that

$$G\big(f^m fu, g^m u, h^m u\big) \le \alpha \max \left\{\begin{matrix} G(Afu, Bu, Cu), G(Afu, f^m fu, f^m fu), \\ G(Bu, g^m u, g^m u), G(Cu, h^m u, h^m u), \\ G(Afu, g^m u, g^m u), G(Afu, h^m u, h^m u), \\ G(Bu, f^m fu, f^m fu), G(Bu, h^m u, h^m u), \\ G(Cu, f^m fu, f^m fu), G(Cu, g^m u, g^m u) \end{matrix}\right\}.$$

By condition (ii) we have $Afu = fAu = fu$, note that $fu = f(f^m u) = f^{m+1} u = f^m(fu)$, and Proposition 1.4(iii), we obtain

$$\begin{aligned} G(fu, u, u) &\le k \max \left\{\begin{matrix} G(fu, u, u), G(fu, fu, fu), G(u, u, u), G(u, u, u), G(fu, u, u), \\ G(fu, u, u), G(u, fu, fu), G(u, u, u), G(u, fu, fu), G(u, u, u) \end{matrix}\right\} \\ &= k \max\big\{G(fu, u, u), G(u, fu, fu)\big\} \\ &\le 2kG(fu, u, u), \end{aligned}$$

which implies that $G(fu, u, u) = 0$, and so $fu = u$.

By the same argument, we can prove $gu = u$ and $hu = u$. Thus we have $u = fu = gu = hu = Au = Bu = Cu$, so that f, g, h, A, B and C have a common fixed point u in X. Let v be any other common fixed point of f, g, h, A, B and C, then by use of condition (2.13) and Proposition 1.4(iii), we have

$$\begin{aligned} G(u,u,v) &= G(f^m u, g^m u, h^m v) \\ &\le k \max \left\{ \begin{array}{c} G(Au,Bu,Cv), G(Au,f^m u,f^m u), \\ G(Bu,g^m u,g^m u), G(Cv,h^m v,h^m v), \\ G(Au,g^m u,g^m u), G(Au,h^m v,h^m v), \\ G(Bu,f^m u,f^m u), G(Bu,h^m v,h^m v), \\ G(Cv,f^m u,f^m u), G(Cv,g^m u,g^m u) \end{array} \right\} \\ &= k \max \left\{ \begin{array}{c} G(u,u,v), G(u,u,u), G(u,u,u), G(v,v,v), G(u,u,u), \\ G(u,v,v), G(u,u,u), G(u,v,v), G(v,u,u), G(v,u,u) \end{array} \right\} \\ &= k \max\{G(u,u,v), G(u,v,v)\} \\ &\le 2kG(u,u,v). \end{aligned}$$

This implies that $G(u,u,v) = 0$, and so $u = v$. Thus common fixed point is unique.

The proof using (2.14) is similar. This completes the proof. □

In Theorem 2.9, if we take $A = B = C = I$, then we have the following corollary.

Corollary 2.10 *Let* (X, G) *be a complete G-metric space and let* f, g *and* h *be three mappings of* X *into itself satisfying the following conditions*:

$$G(f^m x, g^m y, h^m z) \le k \max \left\{ \begin{array}{c} G(x,y,z), G(x,f^m x,f^m x), \\ G(y,g^m y,g^m y), G(z,h^m z,h^m z), \\ G(x,g^m y,g^m y), G(x,h^m z,h^m z), \\ G(y,f^m x,f^m x), G(y,h^m z,h^m z), \\ G(z,f^m x,f^m x), G(z,g^m y,g^m y) \end{array} \right\} \tag{2.15}$$

or

$$G(f^m x, g^m y, h^m z) \le k \max \left\{ \begin{array}{c} G(x,y,z), G(x,x,f^m x), \\ G(y,y,g^m y), G(z,z,h^m z), \\ G(x,x,g^m y), G(x,x,h^m z), \\ G(y,y,f^m x), G(y,y,h^m z), \\ G(z,z,f^m x), G(z,z,g^m y) \end{array} \right\} \tag{2.16}$$

$\forall x,y,z \in X$, *where* $k \in [0, \frac{1}{2})$, $m \in \mathbb{N}$, *then* f, g *and* h *have a unique common fixed point in* X.

Remark 2.11 Corollary 2.10 generalizes and extends the corresponding results in Abbas *et al.* [7, Corollary 2.3].

Also, if we take $f = g = h$ and $A = B = C = I$ in Theorem 2.9, then we get the following.

Corollary 2.12 *Let (X, G) be a complete G-metric space and let f be a mapping of X into itself satisfying the following conditions*:

$$G(f^m x, f^m y, f^m z) \le k \max \left\{ \begin{array}{c} G(x,y,z), G(x, f^m x, f^m x), \\ G(y, f^m y, f^m y), G(z, f^m z, f^m z), \\ G(x, f^m y, f^m y), G(x, f^m z, f^m z), \\ G(y, f^m x, f^m x), G(y, f^m z, f^m z), \\ G(z, f^m x, f^m x), G(z, f^m y, f^m y) \end{array} \right\} \tag{2.17}$$

or

$$G(f^m x, f^m y, f^m z) \le k \max \left\{ \begin{array}{c} G(x,y,z), G(x,x,f^m x), \\ G(y,y,f^m y), G(z,z,f^m z), \\ G(x,x,f^m y), G(x,x,f^m z), \\ G(y,y,f^m x), G(y,y,f^m z), \\ G(z,z,f^m x), G(z,z,f^m y) \end{array} \right\} \tag{2.18}$$

$\forall x, y, z \in X$, *where* $k \in [0, \frac{1}{2})$, $m \in \mathbb{N}$, *then f has a unique fixed point in X*.

Remark 2.13 Corollary 2.12 generalizes and extends the corresponding results in Mustafa and Sims [8, Corollary 2.3].

Remark 2.14 Theorem 2.9, Corollaries 2.10 and 2.12 generalize and extend the corresponding results in Mustafa *et al.* [10, Corollaries 2.4 and 2.7].

Remark 2.15 In Theorem 2.9, we have taken: (1) $f = g = h$; (2) $A = B = C$; (3) $g = h$ and $B = C$; (4) $g = h$, $B = C = I$, several new results can be obtained.

Theorem 2.16 *Let (X, G) be a complete G-metric space and let f, g, h, A, B and C be six mappings of X into itself satisfying the following conditions*:

(i) $f(X) \subset B(X)$, $g(X) \subset C(X)$, $h(X) \subset A(X)$;
(ii) $\forall x, y, z \in X$,

$$\begin{aligned} G(fx, gy, hz) \le{} & a_1 G(Ax, By, Cz) + a_2 G(Ax, fx, fx) \\ & + a_3 G(By, gy, gy) + a_4 G(Cz, hz, hz) \\ & + a_5 G(Ax, gy, gy) + a_6 G(Ax, hz, hz) + a_7 G(By, fx, fx) \\ & + a_8 G(By, hz, hz) + a_9 G(Cz, fx, fx) + a_{10} G(Cz, gy, gy) \end{aligned} \tag{2.19}$$

or

$$\begin{aligned} G(fx, gy, hz) \le{} & a_1 G(Ax, By, Cz) + a_2 G(Ax, Ax, fx) \\ & + a_3 G(By, By, gy) + a_4 G(Cz, Cz, hz) \\ & + a_5 G(Ax, Ax, gy) + a_6 G(Ax, Ax, hz) + a_7 G(By, By, fx) \\ & + a_8 G(By, By, hz) + a_9 G(Cz, Cz, fx) + a_{10} G(Cz, Cz, gy), \end{aligned} \tag{2.20}$$

where $a_i \geq 0$ $(i = 1, 2, 3, \ldots, 10)$ *and* $0 \leq \sum_{i=1}^{10} a_i < \frac{1}{2}$. *If one of the following conditions are satisfied*:

(a) *Either* f *or* A *is G-continuous, the pair* (f, A) *is weakly commuting, the pairs* (g, B) *and* (h, C) *are weakly compatible*;

(b) *Either* g *or* B *is G-continuous, the pair* (g, B) *is weakly commuting, the pairs* (f, A) *and* (h, C) *are weakly compatible*;

(c) *Either* h *or* C *is G-continuous, the pair* (h, C) *is weakly commuting, the pairs* (f, A) *and* (g, B) *are weakly compatible.*

Then

(I) *one of the pairs* (f, A), (g, B) *and* (h, C) *has a coincidence point in* X;

(II) *the mappings* f, g, h, A, B *and* C *have a unique common fixed point in* X.

Proof Suppose that mappings f, g, h, A, B and C satisfy condition (2.19). For $x, y, z \in X$, let

$$M(x,y,z) = \max \left\{ \begin{array}{c} G(Ax,By,Cz), G(Ax,fx,fx), \\ G(By,gy,gy), G(Cz,hz,hz), \\ G(Ax,gy,gy), G(Ax,hz,hz), \\ G(By,fx,fx), G(By,hz,hz), \\ G(Cz,fx,fx), G(Cz,gy,gy) \end{array} \right\}.$$

Then

$$\begin{aligned} &a_1 G(Ax,By,Cz) + a_2 G(Ax,fx,fx) + a_3 G(By,gy,gy) + a_4 G(Cz,hz,hz) \\ &\quad + a_5 G(Ax,gy,gy) + a_6 G(Ax,hz,hz) + a_7 G(By,fx,fx) \\ &\quad + a_8 G(By,hz,hz) + a_9 G(Cz,fx,fx) + a_{10} G(Cz,gy,gy) \\ &\leq \left(\sum_{i=1}^{10} a_i \right) M(x,y,z). \end{aligned}$$

Therefore, it follows from (2.19) that

$$G(fx,gy,hz) \leq \left(\sum_{i=1}^{10} a_i \right) M(x,y,z).$$

Taking $k = \sum_{i=1}^{10} a_i$ in Theorem 2.1, the conclusion of Theorem 2.16 can be obtained from Theorem 2.1 immediately.

The proof using (2.20) is similar. This completes the proof. □

Remark 2.17 Theorem 2.16 generalizes and extends the corresponding results in Mustafa *et al.* [10, Theorem 2.1], Mustafa *et al.* [12, Theorem 2.5].

Remark 2.18 In Theorem 2.16, we have taken: (1) $A = B = C = I$; (2) $f = g = h$; (3) $A = B = C$; (4) $g = h$ and $B = C$; (5) $g = h$, $B = C = I$, several new results can be obtained.

Corollary 2.19 *Let* (X, G) *be a complete G-metric space and let* f, g, h, A, B *and* C *be six mappings of* X *into itself satisfying the following conditions*:

(i) $f(X) \subset B(X)$, $g(X) \subset C(X)$, $h(X) \subset A(X)$;
(ii) *The pairs* (f, A), (g, B) *and* (h, C) *are commuting mappings*;
(iii) $\forall x, y, z \in X$,

$$\begin{aligned} G\bigl(f^m x, g^m y, h^m z\bigr) \leq{}& a_1 G(Ax, By, Cz) + a_2 G\bigl(Ax, f^m x, f^m x\bigr) + a_3 G\bigl(By, g^m y, g^m y\bigr) \\ &+ a_4 G\bigl(Cz, h^m z, h^m z\bigr) + a_5 G\bigl(Ax, g^m y, g^m y\bigr) \\ &+ a_6 G\bigl(Ax, h^m z, h^m z\bigr) + a_7 G\bigl(By, f^m x, f^m x\bigr) \\ &+ a_8 G\bigl(By, h^m z, h^m z\bigr) + a_9 G\bigl(Cz, f^m x, f^m x\bigr) \\ &+ a_{10} G\bigl(Cz, g^m y, g^m y\bigr) \end{aligned} \tag{2.21}$$

or

$$\begin{aligned} G\bigl(f^m x, g^m y, h^m z\bigr) \leq{}& a_1 G(Ax, By, Cz) + a_2 G\bigl(Ax, Ax, f^m x\bigr) + a_3 G\bigl(By, By, g^m y\bigr) \\ &+ a_4 dG\bigl(Cz, Bz, h^m z\bigr) + a_5 G\bigl(Ax, Ax, g^m y\bigr) \\ &+ a_6 G\bigl(Ax, Ax, h^m z\bigr) + a_7 G\bigl(By, By, f^m x\bigr) \\ &+ a_8 G\bigl(By, By, h^m z\bigr) + a_9 G\bigl(Cz, Cz, f^m x\bigr) \\ &+ a_{10} G\bigl(Cz, Cz, g^m y\bigr), \end{aligned} \tag{2.22}$$

where $m \in \mathbb{N}$, $a_i \geq 0$ $(i = 1, 2, 3, \ldots, 10)$ *and* $0 \leq \sum_{i=1}^{10} a_i < \frac{1}{2}$. *Then* f, g, h, A, B *and* C *have a unique common fixed point in* X.

Proof The proof follows from Theorem 2.9, and from an argument similar to that used in Theorem 2.16 □

Remark 2.20 In Theorem 2.1, we have taken: (1) $A = B = C = I$; (2) $f = g = h$; (3) $A = B = C$; (4) $g = h$ and $B = C$; (5) $g = h$, $B = C = I$, several new results can be obtained.

Remark 2.21 Theorems 2.1, 2.9 and 2.16 in this paper also improve and generalize the corresponding results of Manro *et al.* [14], Vats *et al.* [15].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors contributed equally to this work. Both authors read and approved the final manuscript.

Author details
[1]Institute of Applied Mathematics and Department of Mathematics, Hangzhou Normal University, Hangzhou, Zhejiang 310036, China. [2]Accounting School, Zhejiang University of Finance and Economics, Hangzhou, Zhejiang 310018, China.

Acknowledgements
The authors are grateful to the editor and the reviewer for suggestions which improved the contents of the article. This work is supported by the National Natural Science Foundation of China (11271105) and the Natural Science Foundation of Zhejiang Province (Y6110287, LY12A01030).

Received: 1 March 2013 Accepted: 8 June 2013 Published: 3 July 2013

References

1. Gähler, S: 2-Metrische räume und ihre topologische struktur. Math. Nachr. **26**, 115-148 (1963)
2. Gähler, S: Zur geometric 2-metrische räume. Rev. Roum. Math. Pures Appl. **11**, 665-667 (1966)
3. Dhage, BC: Generalized metric space and mapping with fixed point. Bull. Calcutta Math. Soc. **84**, 329-336 (1992)
4. Dhage, BC: Generalized metric space and topological structure *I*. An. Stiint. Univ. Al. I. Cuza Iasi. Mat. **46**(1), 3-24 (2000)
5. Mustafa, Z, Sims, B: Some remarks concerning *D*-metric spaces. In: Proceedings of the International Conference on Fixed Point Theory and Applications, Valencia (Spain), July, pp. 189-198 (2003)
6. Mustafa, Z, Sims, B: A new approach to a generalized metric spaces. J. Nonlinear Convex Anal. **7**(2), 289-297 (2006)
7. Abbas, M, Nazir, T, Saadati, R: Common fixed point results for three maps in generalized metric space. Adv. Differ. Equ. **2011**, 49 (2011)
8. Mustafa, Z, Sims, B: Fixed point theorems for contractive mappings in complete *G*-metric spaces. Fixed Point Theory Appl. **2009**, Article ID 917175 (2009)
9. Abbas, M, Rhoades, BE: Common fixed point results for noncommuting mappings without continuity in generalized metric spaces. Appl. Math. Comput. **215**, 262-269 (2009)
10. Mustafa, Z, Obiedat, H, Awawdeh, F: Some fixed point theorems for mappings on complete *G*-metric space. Fixed Point Theory Appl. **2008**, Article ID 189870 (2008)
11. Mustafa, Z, Shatanawi, W, Bataineh, M: Existence of fixed point results in *G*-metric spaces. Int. J. Math. Math. Sci. **2009**, Article ID 283028 (2009)
12. Abbas, M, Khan, SH, Nazir, T: Common fixed points of *R*-weakly commuting maps in generalized metric spaces. Fixed Point Theory Appl. **2011**, Article ID 784595 (2011)
13. Chugh, R, Kadian, T, Rani, A, Rhoades, BE: Property *P* in *G*-metric spaces. Fixed Point Theory Appl. **2010**, Article ID 401684 (2010)
14. Manro, S, Kumar, S, Bhatia, SS: *R*-Weakly commuting maps in *G*-metric spaces. Fasc. Math. **47**, 11-18 (2011)
15. Vats, RK, Kumar, S, Sihag, V: Fixed point theorems in complete *G*-metric spaces. Fasc. Math. **47**, 127-139 (2011)
16. Abbas, M, Nazir, T, Vetro, P: Common fixed point results for three maps in *G*-metric spaces. Filomat **25**(4), 1-17 (2011)
17. Mustafa, Z, Awawdeh, F, Shatanawi, W: Fixed point theorem for expansive mappings in *G*-metric spaces. Int. J. Contemp. Math. Sci. **5**(50), 2463-2472 (2010)
18. Rao, KPR, Sombabu, A, Prasad, JR: A common fixed point theorem for six expansive mappings in *G*-metric spaces. J. Sci. Eng. Technol. **7**(1), 113-120 (2011)
19. Shatanawi, W: Fixed point theory for contractive mappings satisfying Φ-maps in *G*-metric spaces. Fixed Point Theory Appl. **2010**, Article ID 181650 (2010)
20. Gajić, L, Lozanov-Crvenković, Z: A fixed point results for mappings with contractive iterate at a point in *G*-metric spaces. Filomat **25**(2), 53-58 (2011)
21. Vats, RK, Kumar, S, Sihag, V: Some common fixed point theorem for compatible mappings of type (*A*) in complete *G*-metric space. Adv. Fuzzy Math. **6**(1), 27-38 (2011)
22. Aydi, H: A fixed point result involving a generalized weakly contractive condition in *G*-metric spaces. Bull. Math. Anal. Appl. **3**(4), 180-188 (2011)
23. Gajić, L, Stojaković, M: On Ćirić generalization of mappings with a contractive iterate at a point in *G*-metric spaces. Appl. Math. Comput. **219**(1), 435-441 (2012)
24. Kaewcharoen, A: Some common fixed point theorems for contractive mappings satisfying Φ-maps in *G*-metric spaces. Banach J. Math. Anal. **6**(1), 101-111 (2012)
25. Mustafa, Z, Khandagji, M, Shatanawi, W: Fixed point results on complete *G*-metric spaces. Studia Sci. Math. Hung. **48**(3), 304-319 (2011)
26. Aydi, H, Shatanawi, W, Vetro, C: On generalized weakly *G*-contraction mapping in *G*-metric spaces. Comput. Math. Appl. **62**, 4222-4229 (2011)
27. Abbas, M, Nazir, T, Radenović, S: Some periodic point results in generalized metric spaces. Appl. Math. Comput. **217**, 4094-4099 (2010)
28. Obiedat, H, Mustafa, Z: Fixed point results on a nonsymmetric *G*-metric spaces. Jordan J. Math. Stat. **3**(2), 65-79 (2010)
29. Shatanawi, W: Coupled fixed point theorems in generalized metric spaces. Hacet. J. Math. Stat. **40**(3), 441-447 (2011)
30. Mustafa, Z, Obiedat, H: A fixed point theorem of Reich in complete *G*-metric spaces. CUBO **12**(1), 83-93 (2010)
31. Gu, F: Common fixed point theorems for six mappings in generalized metric spaces. Abstr. Appl. Anal. **2012**, Article ID 379212 (2012)
32. Gu, F, Ye, H: Common fixed point theorems of Altman integral type mappings in *G*-metric spaces. Abstr. Appl. Anal. **2012**, Article ID 630457 (2012)
33. Ye, H, Gu, F: Common fixed point theorems for a class of twice power type contraction maps in *G*-metric spaces. Abstr. Appl. Anal. **2012**, Article ID 736214 (2012)
34. Yin, Y, Gu, F: Common fixed point theorem about four mappings in *G*-metric spaces. J. Hangzhou Norm. Univ., Nat. Sci. **11**(6), 511-515 (2012)
35. Gu, F, Yin, Y: Common fixed point for three pairs of self-maps satisfying common (*E.A*) property in generalized metric spaces. Abstr. Appl. Anal. **2013**, Article ID 808092 (2013)
36. Gu, F, Zhou, S: Coupled common fixed point theorems for a pair of commuting mappings in partially ordered *G*-metric spaces. Fixed Point Theory Appl. **2013**, 64 (2013). doi:10.1186/1687-1812-2013-64
37. Ye, H, Lu, J, Gu, F: A new common fixed point theorem for noncompatible mappings of type (A_f) in *G*-metric spaces. J. Hangzhou Norm. Univ., Nat. Sci. **12**(1), 50-56 (2013)
38. Zhou, S, Gu, F: Some new fixed point theorems in *G*-metric spaces. J. Hangzhou Norm. Univ., Nat. Sci. **11**(1), 47-50 (2012)
39. Mustafa, Z: Common fixed points of weakly compatible mappings in *G*-metric spaces. Appl. Math. Sci. **6**(92), 4589-4600 (2012)
40. Mustafa, Z: Some new common fixed point theorems under strict contractive conditions in *G*-metric spaces. J. Appl. Math. **2012**, Article ID 248937 (2012)
41. Tahat, N, Aydi, H, Karapinar, E, Shatanawi, W: Common fixed points for single-valued and multi-valued maps satisfying a generalized contraction in *G*-metric spaces. Fixed Point Theory Appl. **2012**, 48 (2012)

42. Aydi, H: A common fixed point of integral type contraction in generalized metric spaces. J. Adv. Math. Stud. **5**(1), 111-117 (2012)
43. Abbas, M, Nazir, T, Doric, D: Common fixed point of mappings satisfying (*E.A*) property in generalized metric spaces. Appl. Math. Comput. **218**(14), 7665-7670 (2012)
44. Mustafa, Z, Aydi, H, Karapinar, E: On common fixed points in *G*-metric spaces using (*E.A*) property. Comput. Math. Appl. **64**(6), 1944-1956 (2012)
45. Long, W, Abbas, M, Nazir, T, Radenovic, S: Common fixed point for two pairs of mappings satisfying (*E.A*) property in generalized metric spaces. Abstr. Appl. Anal. **2012**, Article ID 394830 (2012)
46. Gugnani, M, Aggarwal, M, Chugh, R: Common fixed point results in *G*-metric spaces and applications. Int. J. Comput. Appl. **43**(11), 38-42 (2012)
47. Popa, V, Patriciu, A-M: A general fixed point theorem for pairs of weakly compatible mappings in *G*-metric spaces. J. Nonlinear Sci. Appl. **5**, 151-160 (2012)
48. Shatanawi, W, Postolache, M: Some fixed point results for a *G*-weak contraction in *G*-metric spaces. Abstr. Appl. Anal. **2012**, Article ID 815870 (2012). doi:10.1155/2012/815870
49. Aydi, H, Karapinar, E, Shatanawi, W: Tripled common fixed point results for generalized contractions in ordered generalized metric spaces. Fixed Point Theory Appl. **2012**, 101 (2012)
50. Aydi, H, Karapinar, E, Shatanawi, W: Tripled fixed point results in generalized metric spaces. J. Appl. Math. **2012**, Article ID 314279 (2012)
51. Gholizadeh, L, Saadati, R, Shatanawi, W, Vaezpour, SM: Contractive mapping in generalized, ordered metric spaces with application in integral equations. Math. Probl. Eng. **2011**, Article ID 380784 (2011). doi:10.1155/2011/380784

doi:10.1186/1687-1812-2013-174
Cite this article as: Gu and Yang: Some new common fixed point results for three pairs of mappings in generalized metric spaces. *Fixed Point Theory and Applications* 2013 **2013**:174.