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Coupled coincidence point results for (ψ,φ)-weakly contractive mappings in partially ordered G_b-metric spaces

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Abstract

In this paper we present some coupled coincidence point results for (ψ,φ)-weakly contractive mappings in the setup of partially ordered G_b-metric spaces. Our results extend the results of Cho *et al.* (Fixed Point Theory Appl. 2012:8, 2012) and the results of Choudhury and Maity (Math. Comput. Model. 54:73-79, 2011). Moreover, examples of the main results are given.

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1 Introduction

Existence of coupled fixed points in partially ordered metric spaces was first investigated in 1987 by Guo and Lakshmikantham [1]. Also, Bhaskar and Lakshmikantham [2] established some coupled fixed point theorems for a mixed monotone mapping in partially ordered metric spaces.

Recently, Lakshmikantham and Ćirić [3] introduced the notions of mixed g-monotone mapping and coupled coincidence point and proved some coupled coincidence point and common coupled fixed point theorems in partially ordered complete metric spaces.

Definition 1.1 [3] Let $(X,\preceq)$ be a partially ordered set, and let $F: X\times X\to X$ and $g: X\to X$ be two mappings. F has the mixed g-monotone property if F is monotone g-non-decreasing in its first argument and is monotone g-non-increasing in its second argument, that is, for all $x_1,x_2\in X$, $gx_1\preceq gx_2$ implies $F(x_1,y)\preceq F(x_2,y)$ for any $y\in X$ and for all $y_1,y_2\in X$, $gy_1\preceq gy_2$ implies $F(x,y_1)\succeq F(x,y_2)$ for any $x\in X$.

Definition 1.2 [2] An element $(x,y)\in X\times X$ is called a coupled fixed point of the mapping $F: X\times X\to X$ if $x=F(x,y)$ and $y=F(y,x)$.

Definition 1.3 [3] An element $(x,y)\in X\times X$ is called

(1) a coupled coincidence point of mappings $F: X\times X\to X$ and $g: X\to X$ if $g(x)=F(x,y)$ and $g(y)=F(y,x)$;

(2) a common coupled fixed point of mappings $F: X\times X\to X$ and $g: X\to X$ if $x=g(x)=F(x,y)$ and $y=g(y)=F(y,x)$.

Recently, Abbas *et al.* [4] introduced the concept of w-compatible mappings to obtain some coupled coincidence point results in a cone metric space.

Definition 1.4 [4] Two mappings $F : X \times X \to X$ and $g : X \to X$ are called w-compatible if $g(F(x,y)) = F(gx,gy)$, whenever $g(x) = F(x,y)$ and $g(y) = F(y,x)$.

The concept of generalized metric space, or a G-metric space, was introduced by Mustafa and Sims [5]. Mustafa and others studied fixed point theorems for mappings satisfying different contractive conditions (see [5–22]).

Definition 1.5 (G-metric space [5]) Let X be a nonempty set, and let $G : X \times X \times X \to R^+$ be a function satisfying the following properties:
(G1) $G(x,y,z) = 0$ iff $x = y = z$;
(G2) $0 < G(x,x,y)$ for all $x,y \in X$ with $x \neq y$;
(G3) $G(x,x,y) \leq G(x,y,z)$ for all $x,y,z \in X$ with $z \neq y$;
(G4) $G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots$ (symmetry in all three variables);
(G5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for all $x,y,z,a \in X$ (rectangle inequality).
Then the function G is called a G-metric on X and the pair (X,G) is called a G-metric space.

Definition 1.6 [5] Let (X,G) be a G-metric space, and let $\{x_n\}$ be a sequence of points of X. A point $x \in X$ is said to be the limit of the sequence $\{x_n\}$ if $\lim_{n,m\to\infty} G(x,x_n,x_m) = 0$ and one says that the sequence $\{x_n\}$ is G-convergent to x. Thus, if $x_n \to x$ in a G-metric space (X,G), then for any $\varepsilon > 0$, there exists a positive integer N such that $G(x,x_n,x_m) < \varepsilon$ for all $n,m \geq N$.

Definition 1.7 [5] Let (X,G) be a G-metric space. A sequence $\{x_n\}$ is called G-Cauchy if for every $\varepsilon > 0$, there is a positive integer N such that $G(x_n,x_m,x_l) < \varepsilon$ for all $n,m,l \geq N$, that is, if $G(x_n,x_m,x_l) \to 0$ as $n,m,l \to \infty$.

Lemma 1.8 [5] *Let (X,G) be a G-metric space. Then the following are equivalent*:
(1) $\{x_n\}$ *is G-convergent to x.*
(2) $G(x_n,x_n,x) \to 0$ *as* $n \to \infty$.
(3) $G(x_n,x,x) \to 0$ *as* $n \to \infty$.

Lemma 1.9 [23] *If (X,G) is a G-metric space, then $\{x_n\}$ is a G-Cauchy sequence if and only if for every $\varepsilon > 0$, there exists a positive integer N such that $G(x_n,x_m,x_m) < \varepsilon$ for all $m > n \geq N$.*

Definition 1.10 [5] A G-metric space (X,G) is said to be G-complete if every G-Cauchy sequence in (X,G) is convergent in X.

Definition 1.11 [5] Let (X,G) and (X',G') be two G-metric spaces. Then a function $f : X \to X'$ is G-continuous at a point $x \in X$ if and only if it is G-sequentially continuous at x, that is, whenever $\{x_n\}$ is G-convergent to x, $\{f(x_n)\}$ is G'-convergent to $f(x)$.

Definition 1.12 [23] Let (X,G) be a G-metric space. A mapping $F : X \times X \to X$ is said to be continuous at (x,y) if for any two G-convergent sequences $\{x_n\}$ and $\{y_n\}$ converging to x and y, respectively, $\{F(x_n,y_n)\}$ is G-convergent to $F(x,y)$.

Definition 1.13 [3] Let X be a nonempty set. We say that the mappings $F : X \times X \to X$ and $g : X \to X$ are commutative if $g(F(x,y)) = F(gx,gy)$ for all $x, y \in X$.

Choudhury and Maity [23] established some coupled fixed point results for mappings with mixed monotone property in partially ordered G-metric spaces. They obtained the following results.

Theorem 1.14 ([23], Theorem 3.1) *Let $(X, \preceq)$ be a partially ordered set, and let G be a G-metric on X such that (X, G) is a complete G-metric space. Let $F : X \times X \to X$ be a continuous mapping having the mixed monotone property on X. Assume that there exists $k \in [0,1)$ such that*

$$G\bigl(F(x,y), F(u,v), F(w,z)\bigr) \leq \frac{k}{2}\bigl[G(x,u,w) + G(y,v,z)\bigr] \tag{1.1}$$

for all $x \preceq u \preceq w$ and $y \succeq v \succeq z$, where either $u \neq w$ or $v \neq z$.

If there exist $x_0, y_0 \in X$ such that $x_0 \preceq F(x_0, y_0)$ and $y_0 \succeq F(y_0, x_0)$, then F has a coupled fixed point in X, that is, there exist $x, y \in X$ such that $x = F(x,y)$ and $y = F(y,x)$.

Theorem 1.15 ([23], Theorem 3.2) *If in the above theorem, in place of the continuity of F, we assume the following conditions, namely,*

(i) *if a non-decreasing sequence $\{x_n\} \to x$, then $x_n \preceq x$ for all n, and*

(ii) *if a non-increasing sequence $\{y_n\} \to y$, then $y_n \succeq y$ for all n,*

then F has a coupled fixed point.

The concept of an altering distance function was introduced by Khan *et al.* [24] as follows.

Definition 1.16 The function $\psi : [0,\infty) \to [0,\infty)$ is called an altering distance function if the following properties are satisfied:

1. ψ is continuous and non-decreasing.
2. $\psi(t) = 0$ if and only if $t = 0$.

In [25], Cho *et al.* studied coupled coincidence and coupled common fixed point theorems in ordered generalized metric spaces for a nonlinear contractive condition related to a pair of altering distance functions.

Theorem 1.17 ([25], Theorem 3.1) *Let $(X, \preceq)$ be a partially ordered set, and let (X, G) be a complete G-metric space. Let $F : X^2 \to X$ and $g : X \to X$ be continuous mappings such that F has the mixed g-monotone property and g commutes with F. Assume that there are altering distance functions ψ and φ such that*

$$\begin{aligned} &\psi\bigl(G\bigl(F(x,y), F(u,v), F(s,t)\bigr)\bigr) \\ &\quad \leq \psi\bigl(\max\bigl\{G(gx,gu,gs), G(gy,gv,gt)\bigr\}\bigr) - \varphi\bigl(\max\bigl\{G(gx,gu,gs), G(gy,gv,gt)\bigr\}\bigr) \end{aligned} \tag{1.2}$$

for all $x, y, u, v, w, z \in X$ with $gw \preceq gu \preceq gx$ and $gy \preceq gv \preceq gz$. Also, suppose that $F(X \times X) \subseteq g(X)$. If there exist $x_0, y_0 \in X$ such that $gx_0 \preceq F(x_0, y_0)$ and $F(y_0, x_0) \preceq gy_0$, then F and g have a coupled coincidence point.

Definition 1.18 [25] Let $(X, \preceq)$ be a partially ordered set, and let G be a G-metric on X. We say that $(X, G, \preceq)$ is regular if the following conditions hold:

(i) If $\{x_n\}$ is a non-decreasing sequence with $x_n \to x$, then $x_n \preceq x$ for all $n \in \mathbb{N}$.

(ii) If $\{x_n\}$ is a non-increasing sequence with $x_n \to x$, then $x_n \succeq x$ for all $n \in \mathbb{N}$.

Theorem 1.19 ([25], Theorem 3.2) *Let $(X, \preceq)$ be a partially ordered set, and let G be a G-metric on X such that $(X, G, \preceq)$ is regular. Assume that there exist altering distance functions ψ, φ and mappings $F : X \times X \to X$ and $g : X \to X$ satisfying* (1.2) *for all $x, y, u, v, w, z \in X$ with $gw \preceq gu \preceq gx$ and $gy \preceq gv \preceq gz$. Suppose also that $(g(X), G)$ is G-complete, F has the mixed g-monotone property and $F(X \times X) \subseteq g(X)$. If there exist $x_0, y_0 \in X$ such that $gx_0 \preceq F(x_0, y_0)$ and $F(y_0, x_0) \preceq gy_0$, then F and g have a coupled coincidence point.*

So far, many authors have discussed fixed point results, periodic point results, coupled and tripled fixed point results and many other related topics in fixed point theory in different extensions of the concept of metric spaces such as b-metric spaces, partial metric spaces, cone metric spaces, G-metric spaces, *etc.* (see, *e.g.*, [6, 14, 20, 26–33]). Motivated by the work in [34], Aghajani *et al.*, in a submitted paper [35], extended the notion of G-metric space to the concept of G_b-metric space (see Section 2). In this paper, we obtain some coupled coincidence point theorems for nonlinear (ψ, φ)-weakly contractive mappings in partially ordered G_b-metric spaces. These results generalize and modify several comparable results in the literature.

2 Mathematical preliminaries

Aghajani *et al.* in [35] introduced the concept of generalized b-metric spaces (G_b-metric spaces) and then they presented some basic properties of G_b-metric spaces.

The following is their definition of G_b-metric spaces.

Definition 2.1 [35] Let X be a nonempty set, and let $s \geq 1$ be a given real number. Suppose that a mapping $G : X \times X \times X \to \mathbb{R}^+$ satisfies:

(G_b1) $G(x, y, z) = 0$ if $x = y = z$,
(G_b2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$,
(G_b3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$,
(G_b4) $G(x, y, z) = G(p\{x, y, z\})$, where p is a permutation of x, y, z (symmetry),
(G_b5) $G(x, y, z) \leq s[G(x, a, a) + G(a, y, z)]$ for all $x, y, z, a \in X$ (rectangle inequality).

Then G is called a generalized b-metric and the pair (X, G) is called a generalized b-metric space or a G_b-metric space.

It should be noted that the class of G_b-metric spaces is effectively larger than that of G-metric spaces given in [5]. Indeed, each G-metric space is a G_b-metric space with $s = 1$ (see also [36]).

The following example shows that a G_b-metric on X need not be a G-metric on X.

Example 2.2 [35] Let (X, G) be a G-metric space, and let $G_*(x, y, z) = G(x, y, z)^p$, where $p > 1$ is a real number.

Note that G_* is a G_b-metric with $s = 2^{p-1}$. Obviously, G_* satisfies conditions (G_b1)-(G_b4) of Definition 2.1, so it suffices to show that (G_b5) holds. If $1 < p < \infty$, then the convexity of the function $f(x) = x^p$ ($x > 0$) implies that $(a+b)^p \leq 2^{p-1}(a^p + b^p)$. Thus, for each $x, y, z, a \in X$, we obtain

$$\begin{aligned} G_*(x,y,z) &= G(x,y,z)^p \\ &\leq \bigl(G(x,a,a) + G(a,y,z)\bigr)^p \\ &\leq 2^{p-1}\bigl(G(x,a,a)^p + G(a,y,z)^p\bigr) \\ &= 2^{p-1}\bigl(G_*(x,a,a) + G_*(a,y,z)\bigr). \end{aligned}$$

So, G_* is a G_b-metric with $s = 2^{p-1}$.

Also, in the above example, (X, G_*) is not necessarily a G-metric space. For example, let $X = \mathbb{R}$ and G-metric G be defined by

$$G(x,y,z) = \frac{1}{3}\bigl(|x-y| + |y-z| + |x-z|\bigr)$$

for all $x, y, z \in \mathbb{R}$ (see [5]). Then $G_*(x,y,z) = G(x,y,z)^2 = \frac{1}{9}(|x-y| + |y-z| + |x-z|)^2$ is a G_b-metric on $\mathbb{R}$ with $s = 2^{2-1} = 2$, but it is not a G-metric on $\mathbb{R}$. To see this, let $x = 3$, $y = 5$, $z = 7$ and $a = \frac{7}{2}$. Hence, we get $G_*(3,5,7) = \frac{64}{9}$, $G_*(3, \frac{7}{2}, \frac{7}{2}) = \frac{1}{9}$, $G_*(\frac{7}{2}, 5, 7) = \frac{49}{9}$. Therefore, $G_*(3,5,7) = \frac{64}{9} \nleq \frac{50}{9} = G_*(3, \frac{7}{2}, \frac{7}{2}) + G_*(\frac{7}{2}, 5, 7)$.

Example 2.3 Let $X = \mathbb{R}$ and $d(x,y) = |x-y|^2$. We know that (X, d) is a b-metric space with $s = 2$. Let $G(x,y,z) = d(x,y) + d(y,z) + d(z,x)$, we show that (X, G) is not a G_b-metric space. Indeed, (G_b3) is not true for $x = 0$, $y = 2$ and $z = 1$. To see this, we have

$$G(0,0,2) = d(0,0) + d(0,2) + d(2,0) = 2d(0,2) = 8$$

and

$$G(0,2,1) = d(0,2) + d(2,1) + d(1,0) = 4 + 1 + 1 = 6.$$

So, $G(0,0,2) > G(0,2,1)$.

However, $G(x,y,z) = \max\{d(x,y), d(y,z), d(z,x)\}$ is a G_b-metric on $\mathbb{R}$ with $s = 2$.

Now we present some definitions and propositions in a G_b-metric space.

Definition 2.4 [35] Let (X, G) be a G_b-metric space. Then, for any $x_0 \in X$ and any $r > 0$, the G_b-ball with center x_0 and radius r is

$$B_G(x_0, r) = \bigl\{y \in X \mid G(x_0, y, y) < r\bigr\}.$$

For example, let $X = \mathbb{R}$ and consider the G_b-metric G defined by

$$G(x,y,z) = \frac{1}{9}\bigl(|x-y| + |y-z| + |x-z|\bigr)^2$$

for all $x, y, z \in \mathbb{R}$. Then

$$\begin{aligned} B_G(3,4) &= \{y \in X \mid G(3,y,y) < 4\} \\ &= \left\{y \in X \,\middle|\, \frac{1}{9}\big(|y-3| + |y-3|\big)^2 < 4\right\} \\ &= \{y \in X \mid |y-3|^2 < 9\} \\ &= (0,6). \end{aligned}$$

By some straightforward calculations, we can establish the following.

Proposition 2.5 [35] *Let X be a G_b-metric space. Then, for each $x, y, z, a \in X$, it follows that*:

(1) *if $G(x,y,z) = 0$, then $x = y = z$,*
(2) $G(x,y,z) \leq s(G(x,x,y) + G(x,x,z))$,
(3) $G(x,y,y) \leq 2sG(y,x,x)$,
(4) $G(x,y,z) \leq s(G(x,a,z) + G(a,y,z))$.

Definition 2.6 [35] Let X be a G_b-metric space. We define $d_G(x,y) = G(x,y,y) + G(x,x,y)$ for all $x, y \in X$. It is easy to see that d_G defines a b-metric d on X, which we call the b-metric associated with G.

Proposition 2.7 [35] *Let X be a G_b-metric space. Then, for any $x_0 \in X$ and any $r > 0$, if $y \in B_G(x_0, r)$, then there exists $\delta > 0$ such that $B_G(y,\delta) \subseteq B_G(x_0, r)$.*

Proof For $s = 1$, see Proposition 4 in [5]. Suppose that $s > 1$ and let $y \in B_G(x_0, r)$. If $y = x_0$, then we choose $\delta = r$. If $y \neq x_0$, then $0 < G(x_0, y, y) < r$. Let $A = \{n \in \mathbb{N} \mid \frac{r}{4s^{n+2}} < G(x_0,y,y)\}$. Since $\lim_{n\to\infty} \frac{1}{4s^{n+2}} = 0$, hence, for $0 < \epsilon = \frac{G(x_0,y,y)}{r} < 1$, there exists $n_0 \in \mathbb{N}$ such that $\frac{1}{4s^{n_0+2}} < \frac{G(x_0,y,y)}{r}$ or $\frac{r}{4s^{n_0+2}} < G(x_0,y,y)$. Hence, $n_0 \in A$ and A is a nonempty set, then by the well-ordering principle, A has a least element m. Since $m - 1 \notin A$, we have $G(x_0,y,y) \leq \frac{r}{4s^{m+1}}$. Now, if $G(x_0,y,y) = \frac{r}{4s^{m+1}}$, then we choose $\delta = \frac{r}{4s^{m+1}}$ and if $G(x_0,y,y) < \frac{r}{4s^{m+1}}$, we choose $\delta = \frac{r}{4s^{m+1}} - G(x_0,y,y)$. □

From the above proposition, the family of all G_b-balls

$$\digamma = \{B_G(x,r) \mid x \in X, r > 0\}$$

is a base of a topology $\tau(G)$ on X, which we call G_b-metric topology.

Now, we generalize Proposition 5 in [5] for a G_b-metric space as follows.

Proposition 2.8 [35] *Let X be a G_b-metric space. Then, for any $x_0 \in X$ and $r > 0$, we have*

$$B_G\left(x_0, \frac{r}{2s+1}\right) \subseteq B_{d_G}(x_0, r) \subseteq B_G(x_0, r).$$

Thus, every G_b-metric space is topologically equivalent to a b-metric space. This allows us to readily transport many concepts and results from b-metric spaces into the G_b-metric space setting.

Definition 2.9 [35] Let X be a G_b-metric space. A sequence $\{x_n\}$ in X is said to be:

(1) G_b-Cauchy if for each $\varepsilon > 0$ there exists a positive integer n_0 such that for all $m, n, l \geq n_0$, $G(x_n, x_m, x_l) < \varepsilon$;

(2) G_b-convergent to a point $x \in X$ if for each $\varepsilon > 0$ there exists a positive integer n_0 such that for all $m, n \geq n_0$, $G(x_n, x_m, x) < \varepsilon$.

Using the above definitions, we can easily prove the following two propositions.

Proposition 2.10 [35] *Let X be a G_b-metric space. Then the following are equivalent*:

(1) *The sequence $\{x_n\}$ is G_b-Cauchy.*

(2) *For any $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$ for all $m, n \geq n_0$.*

Proposition 2.11 [35] *Let X be a G_b-metric space. The following are equivalent*:

(1) *$\{x_n\}$ is G_b-convergent to x.*

(2) *$G(x_n, x_n, x) \to 0$ as $n \to +\infty$.*

(3) *$G(x_n, x, x) \to 0$ as $n \to +\infty$.*

Definition 2.12 [35] A G_b-metric space X is called G_b-complete if every G_b-Cauchy sequence is G_b-convergent in X.

Definition 2.13 Let (X, G) and (X', G') be two G_b-metric spaces. Then a function $f : X \to X'$ is G_b-continuous at a point $x \in X$ if and only if it is G_b-sequentially continuous at x, that is, whenever $\{x_n\}$ is G_b-convergent to x, $\{f(x_n)\}$ is G'_b-convergent to $f(x)$.

Definition 2.14 Let (X, G) be a G_b-metric space. A mapping $F : X \times X \to X$ is said to be continuous if for any two G_b-convergent sequences $\{x_n\}$ and $\{y_n\}$ converging to x and y, respectively, $\{F(x_n, y_n)\}$ is G_b-convergent to $F(x, y)$.

Mustafa and Sims proved that each G-metric function $G(x, y, z)$ is jointly continuous in all three of its variables (see Proposition 8 in [5]). But, in general, a G_b-metric function $G(x, y, z)$ for $s > 1$ is not jointly continuous in all its variables. Now, we present an example of a discontinuous G_b-metric.

Example 2.15 Let $X = \mathbb{N} \cup \{\infty\}$ and let $D : X \times X \to \mathbb{R}$ be defined by

$$D(m,n) = \begin{cases} 0 & \text{if } m = n, \\ |\frac{1}{m} - \frac{1}{n}| & \text{if one of } m, n \text{ is even and the other is even or } \infty, \\ 5 & \text{if one of } m, n \text{ is odd and the other is odd (and } m \neq n\text{) or } \infty, \\ 2 & \text{otherwise.} \end{cases}$$

Then it is easy to see that for all $m, n, p \in X$, we have

$$D(m,p) \leq \frac{5}{2}\bigl(D(m,n) + D(n,p)\bigr).$$

Thus, (X, D) is a b-metric space with $s = \frac{5}{2}$ (see corrected Example 3 from [37]).

Let $G(x,y,z) = \max\{D(x,y), D(y,z), D(z,x)\}$. It is easy to see that G is a G_b-metric with $s = \frac{5}{2}$. Now, we show that $G(x,y,z)$ is not a continuous function. Take $x_n = 2n$ and $y_n = z_n = 1$. Then we have $x_n \to \infty$, $y_n \to 1$ and $z_n \to 1$. Also,

$$\begin{aligned} G(2n,\infty,\infty) &= \max\bigl\{D(2n,\infty), D(\infty,\infty), D(\infty,2n)\bigr\} \\ &= \max\bigl\{D(2n,\infty), D(\infty,\infty)\bigr\} = \frac{1}{2n} \to 0 \end{aligned}$$

and

$$G(y_n,1,1) = G(z_n,1,1) = 0 \to 0.$$

On the other hand,

$$G(x_n,y_n,z_n) = \max\bigl\{D(x_n,1), D(1,1), D(1,x_n)\bigr\} = D(x_n,1) = 2$$

and

$$G(\infty,1,1) = \max\bigl\{D(\infty,1), D(1,1), D(1,\infty)\bigr\} = 5.$$

Hence, $\lim_{n\to\infty} G(x_n,y_n,z_n) \neq G(x,y,z)$.

So, from the above discussion, we need the following simple lemma about the G_b-convergent sequences in the proof of our main result.

Lemma 2.16 *Let (X,G) be a G_b-metric space with $s > 1$, and suppose that $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ are G_b-convergent to x, y and z, respectively. Then we have*

$$\frac{1}{s^3} G(x,y,z) \le \liminf_{n\to\infty} G(x_n,y_n,z_n) \le \limsup_{n\to\infty} G(x_n,y_n,z_n) \le s^3 G(x,y,z).$$

In particular, if $x = y = z$, then we have $\lim_{n\to\infty} G(x_n,y_n,z_n) = 0$.

Proof Using the triangle inequality in a G_b-metric space, it is easy to see that

$$G(x,y,z) \le sG(x,x_n,x_n) + s^2 G(y,y_n,y_n) + s^3 G(z,z_n,z_n) + s^3 G(x_n,y_n,z_n)$$

and

$$G(x_n,y_n,z_n) \le sG(x_n,x,x) + s^2 G(y_n,y,y) + s^3 G(z_n,z,z) + s^3 G(x,y,z).$$

Taking the lower limit as $n \to \infty$ in the first inequality and the upper limit as $n \to \infty$ in the second inequality, we obtain the desired result. □

3 Main results

Our first result is the following.

Theorem 3.1 *Let $(X, \preceq)$ be a partially ordered set, and let G be a G_b-metric on X such that (X, G) is a complete G_b-metric space. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings such that*

$$\begin{aligned} &\psi\bigl(s \max\bigl\{G\bigl(F(x,y), F(u,v), F(w,t)\bigr), G\bigl(F(y,x), F(v,u), F(t,w)\bigr)\bigr\}\bigr) \\ &\quad \le \psi\bigl(\max\bigl\{G(gx,gu,gw), G(gy,gv,gt)\bigr\}\bigr) - \varphi\bigl(\max\bigl\{G(gx,gu,gw), G(gy,gv,gt)\bigr\}\bigr) \end{aligned} \tag{3.1}$$

for every pair $(x,y), (u,v), (w,t) \in X \times X$ such that $gx \preceq gu \preceq gw$ and $gy \succeq gv \succeq gt$, or $gw \preceq gu \preceq gx$ and $gt \succeq gv \succeq gy$, where $\psi, \varphi : [0,\infty) \to [0,\infty)$ are altering distance functions.

Also, suppose that:

1. *$F(X \times X) \subseteq g(X)$.*
2. *F has the mixed g-monotone property.*
3. *F is continuous.*
4. *g is continuous and commutes with F.*

If there exist $x_0, y_0 \in X$ such that $gx_0 \preceq F(x_0,y_0)$ and $gy_0 \succeq F(y_0,x_0)$, then F and g have a coupled coincidence point in X.

Proof Let $x_0, y_0 \in X$ be such that $gx_0 \preceq F(x_0,y_0)$ and $gy_0 \succeq F(y_0,x_0)$. Since $F(X \times X) \subseteq g(X)$, we can choose $x_1, y_1 \in X$ such that $gx_1 = F(x_0,y_0)$ and $gy_1 = F(y_0,x_0)$. Then, $gx_0 \preceq F(x_0,y_0) = gx_1$ and $gy_0 \succeq F(y_0,x_0) = gy_1$. Since F has the mixed g-monotone property, we have $F(x_0,y_0) \preceq F(x_1,y_0) \preceq F(x_1,y_1)$ and $F(y_0,x_0) \succeq F(y_1,x_0) \succeq F(y_1,x_1)$, that is, $gx_0 \preceq gx_1$ and $gy_0 \succeq gy_1$. In this way, we construct the sequences $\{z_n\}$ and $\{t_n\}$ as $z_n = gx_n = F(x_{n-1}, y_{n-1})$ and $t_n = gy_n = F(y_{n-1}, x_{n-1})$ for all $n \ge 1$, inductively.

One can easily show that for all $n \in \mathbb{N}$, $z_{n-1} \preceq z_n$ and $t_{n-1} \succeq t_n$.

We complete the proof in three steps.

Step I. Let

$$\delta_n = \max\bigl\{G(z_{n+1}, z_{n+1}, z_n), G(t_{n+1}, t_{n+1}, t_n)\bigr\},$$

we shall prove that $\lim_{n\to\infty} \delta_n = 0$.

Since $gx_{n-1} \preceq gx_n \preceq gx_n$ and $gy_{n-1} \succeq gy_n \succeq gy_n$, using (3.1) we obtain that

$$\begin{aligned} &\psi\bigl(s \max\bigl\{G(z_{n+1}, z_{n+1}, z_n), G(t_{n+1}, t_{n+1}, t_n)\bigr\}\bigr) \\ &\quad = \psi\bigl(s \max\bigl\{G\bigl(F(x_n,y_n), F(x_n,y_n), F(x_{n-1},y_{n-1})\bigr), \\ &\qquad G\bigl(F(y_n,x_n), F(y_n,x_n), F(y_{n-1},x_{n-1})\bigr)\bigr\}\bigr) \\ &\quad \le \psi\bigl(\max\bigl\{G(z_n,z_n,z_{n-1}), G(t_n,t_n,t_{n-1})\bigr\}\bigr) \\ &\qquad - \varphi\bigl(\max\bigl\{G(z_n,z_n,z_{n-1}), G(t_n,t_n,t_{n-1})\bigr\}\bigr). \end{aligned} \tag{3.2}$$

If for an $n \ge 1$, $\delta_n = 0$, then the conclusion of the theorem follows. So, we assume that

$$\delta_n \ne 0 \quad \text{for all } n \ge 1. \tag{3.3}$$

Let, for some n, $\delta_{n-1} < \delta_n$. So, from (3.2) as ψ is non-decreasing, we have

$$\psi(s\delta_{n-1}) \le \psi(s\delta_n) \le \psi(\delta_{n-1}) - \varphi(\delta_{n-1}) \le \psi(s\delta_{n-1}) - \varphi(\delta_{n-1}),$$

that is, $\varphi(\delta_{n-1}) \leq 0$. By our assumptions, we have $\delta_{n-1} = 0$, which contradicts (3.3). Therefore, for all $n \geq 1$, we deduce that $\delta_{n+1} \leq \delta_n$, that is, $\{\delta_n\}$ is a non-increasing sequence of nonnegative real numbers. Thus, there exists $r \geq 0$ such that $\lim_{n\to\infty} \delta_n = r$.

Letting $n \to \infty$ in (3.2), we get that

$$\psi(sr) \leq \psi(r) - \varphi(r) \leq \psi(sr) - \varphi(r).$$

So, $\varphi(r) = 0$. Thus,

$$\lim_{n\to\infty} \delta_n = 0. \tag{3.4}$$

Step II. We shall show that $\{z_n\}$ and $\{t_n\}$ are G_b-Cauchy sequences in X. So, we shall show that for every $\varepsilon > 0$, there exists $k \in \mathbb{N}$ such that for all $m, n \geq k$,

$$\max\bigl\{G(z_m, z_n, z_n), G(t_m, t_n, t_n)\bigr\} < \varepsilon. \tag{3.5}$$

Suppose that the above statement is false. Then there exists $\varepsilon > 0$, for which we can find subsequences $\{z_{m(k)}\}$ and $\{z_{n(k)}\}$ of $\{z_n\}$ and $\{t_{m(k)}\}$ and $\{t_{n(k)}\}$ of $\{t_n\}$ such that

$$\max\bigl\{G(z_{m(k)}, z_{n(k)}, z_{n(k)}), G(t_{m(k)}, t_{n(k)}, t_{n(k)})\bigr\} \geq \varepsilon. \tag{3.6}$$

Further, corresponding to $m(k)$ we can choose $n(k)$ in such a way that it is the smallest integer with $n(k) > m(k)$ satisfying (3.6). So,

$$\max\bigl\{G(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}), G(t_{m(k)}, t_{n(k)-1}, t_{n(k)-1})\bigr\} < \varepsilon. \tag{3.7}$$

From the rectangle inequality we have

$$G(z_{m(k)}, z_{n(k)}, z_{n(k)}) \leq s\bigl[G(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}) + G(z_{n(k)-1}, z_{n(k)}, z_{n(k)})\bigr]$$

and

$$G(t_{m(k)}, t_{n(k)}, t_{n(k)}) \leq s\bigl[G(t_{m(k)}, t_{n(k)-1}, t_{n(k)-1}) + G(t_{n(k)-1}, t_{n(k)}, t_{n(k)})\bigr].$$

So,

$$\begin{aligned}
&\max\bigl\{G(z_{m(k)}, z_{n(k)}, z_{n(k)}), G(t_{m(k)}, t_{n(k)}, t_{n(k)})\bigr\} \\
&\quad \leq s\bigl[\max\bigl\{G(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}), G(t_{m(k)}, t_{n(k)-1}, t_{n(k)-1})\bigr\} \\
&\qquad + \max\bigl\{G(z_{n(k)-1}, z_{n(k)}, z_{n(k)}), G(t_{n(k)-1}, t_{n(k)}, t_{n(k)})\bigr\}\bigr] \\
&\quad = s\max\bigl\{G(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}), G(t_{m(k)}, t_{n(k)-1}, t_{n(k)-1})\bigr\} + s\delta_{n(k)-1}.
\end{aligned}$$

If $k \to \infty$, as $\lim_{n\to\infty} \delta_n = 0$, from (3.6) and (3.7) we conclude that

$$\frac{\varepsilon}{s} \leq \liminf_{k\to\infty} \max\bigl\{G(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}), G(t_{m(k)}, t_{n(k)-1}, t_{n(k)-1})\bigr\}. \tag{3.8}$$

On the other hand, we have

$$G(z_{m(k)}, z_{n(k)}, z_{n(k)}) \leq s\big[G(z_{m(k)}, z_{m(k)+1}, z_{m(k)+1}) + G(z_{m(k)+1}, z_{n(k)}, z_{n(k)})\big].$$

Similarly,

$$G(t_{m(k)}, t_{n(k)}, t_{n(k)}) \leq sG(t_{m(k)}, t_{m(k)+1}, t_{m(k)+1}) + sG(t_{m(k)+1}, t_{n(k)}, t_{n(k)}).$$

So, we have

$$\begin{aligned}
&\max\big\{G(z_{m(k)}, z_{n(k)}, z_{n(k)}), G(t_{m(k)}, t_{n(k)}, t_{n(k)})\big\} \\
&\quad\leq s\max\big\{G(z_{m(k)}, z_{m(k)+1}, z_{m(k)+1}), G(t_{m(k)}, t_{m(k)+1}, t_{m(k)+1})\big\} \\
&\qquad+ s\max\big\{G(z_{m(k)+1}, z_{n(k)}, z_{n(k)}), G(t_{m(k)+1}, t_{n(k)}, t_{n(k)})\big\}.
\end{aligned}$$

Taking the upper limit as $k \to \infty$ in the above inequality and using (3.6), we obtain

$$\begin{aligned}
\varepsilon &\leq \limsup_{k\to\infty} \max\big\{G(z_{m(k)}, z_{n(k)}, z_{n(k)}), G(t_{m(k)}, t_{n(k)}, t_{n(k)})\big\} \\
&\leq s\limsup_{k\to\infty} \delta_{m(k)} + s\limsup_{k\to\infty} \max\big\{G(z_{m(k)+1}, z_{n(k)}, z_{n(k)}), G(t_{m(k)+1}, t_{n(k)}, t_{n(k)})\big\}.
\end{aligned}$$

Consequently, from (3.4) we get

$$\frac{\varepsilon}{s} \leq \limsup_{k\to\infty} \max\big\{G(z_{m(k)+1}, z_{n(k)}, z_{n(k)}), G(t_{m(k)+1}, t_{n(k)}, t_{n(k)})\big\}. \tag{3.9}$$

As $gx_{m(k)} \preceq gx_{n(k)-1} \preceq gx_{n(k)-1}$ and $gy_{m(k)} \succeq gy_{n(k)-1} \succeq gy_{n(k)-1}$, putting $x = x_{m(k)}$, $y = y_{m(k)}$, $u = x_{n(k)-1}$, $v = y_{n(k)-1}$, $w = x_{n(k)-1}$ and $t = y_{n(k)-1}$ in (3.1), for all $k \geq 0$, we have

$$\begin{aligned}
&\psi\big(s\max\big\{G(z_{m(k)+1}, z_{n(k)}, z_{n(k)}), G(t_{m(k)+1}, t_{n(k)}, t_{n(k)})\big\}\big) \\
&\quad= \psi\big(s\max\big\{G\big(F(x_{m(k)}, y_{m(k)}), F(x_{n(k)-1}, y_{n(k)-1}), F(x_{n(k)-1}, y_{n(k)-1})\big), \\
&\qquad G\big(F(y_{m(k)}, x_{m(k)}), F(y_{n(k)-1}, x_{n(k)-1}), F(y_{n(k)-1}, x_{n(k)-1})\big)\big\}\big) \\
&\quad\leq \psi\big(\max\big\{G(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}), G(t_{m(k)}, t_{n(k)-1}, t_{n(k)-1})\big\}\big) \\
&\qquad- \varphi\big(\max\big\{G(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}), G(t_{m(k)}, t_{n(k)-1}, t_{n(k)-1})\big\}\big).
\end{aligned}$$

Taking the upper limit as $k \to \infty$ in the above inequality and using (3.7) and (3.9), we have

$$\begin{aligned}
\psi(\varepsilon) &= \psi\left(s\frac{\varepsilon}{s}\right) \\
&\leq \psi\Big(s\limsup_{k\to\infty} \max\big\{G(z_{m(k)+1}, z_{n(k)}, z_{n(k)}), G(t_{m(k)+1}, t_{n(k)}, t_{n(k)})\big\}\Big) \\
&\leq \psi\Big(\limsup_{k\to\infty} \max\big\{G(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}), G(t_{m(k)}, t_{n(k)-1}, t_{n(k)-1})\big\}\Big) \\
&\quad- \varphi\Big(\liminf_{k\to\infty} \max\big\{G(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}), G(t_{m(k)}, t_{n(k)-1}, t_{n(k)-1})\big\}\Big) \\
&\leq \psi(\varepsilon) - \varphi\Big(\liminf_{k\to\infty} \max\big\{G(z_{m(k)}, z_{n(k)-1}, z_{n(k)-1}), G(t_{m(k)}, t_{n(k)-1}, t_{n(k)-1})\big\}\Big),
\end{aligned}$$

which implies that

$$\varphi\Big(\liminf_{k\to\infty}\max\big\{G(z_{m(k)},z_{n(k)-1},z_{n(k)-1}),G(t_{m(k)},t_{n(k)-1},t_{n(k)-1})\big\}\Big)\le 0,$$

or, equivalently, $\liminf_{k\to\infty}\max\{G(z_{m(k)},z_{n(k)-1},z_{n(k)-1}),G(t_{m(k)},t_{n(k)-1},t_{n(k)-1})\}=0$, which is a contradiction to (3.8). Consequently, $\{z_n\}$ and $\{t_n\}$ are G_b-Cauchy.

Step III. We shall show that F and g have a coupled coincidence point.

Since X is G_b-complete and $\{z_n\}\subseteq X$ is G_b-Cauchy, there exists $z\in X$ such that

$$\lim_{n\to\infty}G(z_n,z_n,z)=\lim_{n\to\infty}G(gx_n,gx_n,z)=0.$$

Similarly, there exists $t\in X$ such that

$$\lim_{n\to\infty}G(t_n,t_n,t)=\lim_{n\to\infty}G(gy_n,gy_n,t)=0.$$

Now, we prove that (z,t) is a coupled coincidence point of F and g.

The continuity of g and Lemma 2.16 yield that

$$\begin{aligned}0&=\frac{1}{s^3}G(gz,gz,gz)\le\liminf_{n\to\infty}G\big(g(gx_n),g(gx_n),gz\big)\\&\le\limsup_{n\to\infty}G\big(g(gx_n),g(gx_n),gz\big)\le s^3G(gz,gz,gz)=0.\end{aligned}$$

Hence,

$$\lim_{n\to\infty}G\big(g(gx_n),g(gx_n),gz\big)=0,$$

and, similarly, we get

$$\lim_{n\to\infty}G\big(g(gy_n),g(gy_n),gt\big)=0.$$

Since $gx_{n+1}=F(x_n,y_n)$ and $gy_{n+1}=F(y_n,x_n)$, the commutativity of F and g yields that

$$g(gx_{n+1})=g\big(F(x_n,y_n)\big)=F(gx_n,gy_n)$$

and

$$g(gy_{n+1})=g\big(F(y_n,x_n)\big)=F(gy_n,gx_n).$$

From the continuity of F, $\{g(gx_{n+1})\}$ is G_b-convergent to $F(z,t)$ and $\{g(gy_{n+1})\}$ is G_b-convergent to $F(t,z)$. By uniqueness of the limit, we have $F(z,t)=gz$ and $F(t,z)=gt$. That is, g and F have a coupled coincidence point. □

In the following theorem, we omit the continuity and commutativity assumptions of g and F.

Theorem 3.2 *Let $(X, \preceq)$ be a partially ordered set, and let G be a G_b-metric on X such that $(X, G, \preceq)$ is a regular G_b-metric space. Suppose that $F : X \times X \to X$ and $g : X \to X$ are two mappings satisfying* (3.1) *for every pair $(x,y), (u,v), (w,t) \in X \times X$ such that $gx \preceq gu \preceq gw$ and $gy \succeq gv \succeq gt$, or $gw \preceq gu \preceq gx$ and $gt \succeq gv \succeq gy$, where ψ and φ are the same as in Theorem* 3.1.

Let $F(X \times X) \subseteq g(X)$, $g(X)$ is a G_b-complete subset of X and F has the mixed g-monotone property.

If there exist $x_0, y_0 \in X$ such that $gx_0 \preceq F(x_0, y_0)$ and $gy_0 \succeq F(y_0, x_0)$, then F and g have a coupled coincidence point in X.

Moreover, if gy_0 and gx_0 are comparable, then $gu = F(u,v) = F(v,u) = gv$, and if F and g are w-compatible, then F and g have a coupled coincidence point of the form (t,t).

Proof Following the proof of the previous theorem, as $g(X)$ is a G_b-complete subset of X and $\{z_n\}, \{t_n\} \subseteq g(X)$, there exist $u, v \in X$ such that

$$\lim_{n\to\infty} G(gx_n, gx_n, gu) = \lim_{n\to\infty} G(gy_n, gy_n, gv) = 0.$$

Now, we prove that $F(u,v) = gu$ and $F(v,u) = gv$.

Since $\{gx_n\}$ is non-decreasing and $\{gy_n\}$ is non-increasing, from regularity of X we have $gx_n \preceq gu$ and $gy_n \succeq gv$ for all $n \geq 0$.

Using (3.1), we have

$$\begin{aligned}
&\psi\bigl(s\max\bigl\{G\bigl(z_{n+1}, z_{n+1}, F(u,v)\bigr), G\bigl(t_{n+1}, t_{n+1}, F(v,u)\bigr)\bigr\}\bigr) \\
&\quad = \psi\bigl(s\max\bigl\{G\bigl(F(x_n,y_n), F(x_n,y_n), F(u,v)\bigr), G\bigl(F(y_n,x_n), F(y_n,x_n), F(v,u)\bigr)\bigr\}\bigr) \\
&\quad \leq \psi\bigl(\max\bigl\{G(gx_n, gx_n, gu), G(gy_n, gy_n, gv)\bigr\}\bigr) \\
&\qquad - \varphi\bigl(\max\bigl\{G(gx_n, gx_n, gu), G(gy_n, gy_n, gv)\bigr\}\bigr).
\end{aligned}$$

In the above inequality, by using Lemma 2.16, if $n \to \infty$, we have

$$\begin{aligned}
&\psi\left(\frac{1}{s^2}\max\bigl\{G\bigl(gu, gu, F(u,v)\bigr), G\bigl(gv, gv, F(v,u)\bigr)\bigr\}\right) \\
&\quad = \psi\left(s\max\left\{\frac{1}{s^3}G\bigl(gu, gu, F(u,v)\bigr), \frac{1}{s^3}G\bigl(gv, gv, F(v,u)\bigr)\right\}\right) \\
&\quad \leq \psi\Bigl(s\max\Bigl\{\limsup_{n\to\infty} G\bigl(z_{n+1}, z_{n+1}, F(u,v)\bigr), \limsup_{n\to\infty} G\bigl(t_{n+1}, t_{n+1}, F(v,u)\bigr)\Bigr\}\Bigr) \\
&\quad \leq \psi(0) - \varphi(0) = 0,
\end{aligned}$$

and hence, $gu = F(u,v)$ and $F(v,u) = gv$.

Now, let $gy_0 \preceq gx_0$. Then $gv \preceq gy_n \preceq gy_0 \preceq gx_0 \preceq gx_n \preceq gu$ for all $n \in \mathbb{N}$. We shall show that $gu = gv$.

From (3.1), we have

$$\begin{aligned}
&\psi\bigl(s\max\bigl\{G(gu, gu, gv), G(gv, gv, gu)\bigr\}\bigr) \\
&\quad \leq \psi\bigl(\max\bigl\{G(gu, gu, gv), G(gv, gv, gu)\bigr\}\bigr) - \varphi\bigl(\max\bigl\{G(gu, gu, gv), G(gv, gv, gu)\bigr\}\bigr) \\
&\quad \leq \psi\bigl(s\max\bigl\{G(gu, gu, gv), G(gv, gv, gu)\bigr\}\bigr) - \varphi\bigl(\max\bigl\{G(gu, gu, gv), G(gv, gv, gu)\bigr\}\bigr).
\end{aligned}$$

Therefore, $\max\{G(gu,gu,gv),G(gv,gv,gu)\} = 0$. Hence, we get that $G(gv,gv,gu) = G(gu,gu,gv) = 0$, and this means that $gu = gv$.

Now, let $t = gu = gv$. Since F and g are w-compatible, then $gt = g(gu) = g(F(u,v)) = F(gu,gv) = F(t,t)$. Thus, F and g have a coupled coincidence point of the form (t,t). □

Remark 3.3 In Theorems 3.1 and 3.2, we have extended the results of Cho *et al.* [25] (Theorems 1.17 and 1.19).

Note that if $(X,\preceq)$ is a partially ordered set, then we can endow $X \times X$ with the following partial order relation:

$$(x,y) \preceq (u,v) \quad \Longleftrightarrow \quad x \preceq u, y \succeq v$$

for all $(x,y),(u,v) \in X \times X$ [3].

In the following theorem, we give a sufficient condition for the uniqueness of the common coupled fixed point. Similar conditions were introduced by many authors (see, *e.g.*, [2, 3, 9, 20, 38–45]).

Theorem 3.4 *Let all the conditions of Theorem* 3.1 *be fulfilled, and let the following condition hold*:

For arbitrary two points (x,y), (z,t), *there exists* (u,v) *such that* $(F(u,v),F(v,u))$ *is comparable with* $(F(x,y),F(y,x))$ *and* $(F(z,t),F(t,z))$.

Then F and g have a unique common coupled fixed point.

Proof Let (x,y) and (z,t) be two coupled coincidence points of F and g, *i.e.*,

$$g(x) = F(x,y), \qquad g(y) = F(y,x)$$

and

$$g(z) = F(z,t), \qquad g(t) = F(t,z).$$

We shall show that $g(x) = g(z)$ and $g(y) = g(t)$.

Suppose that (x,y) and (z,t) are not comparable. Choose an element $(u,v) \in X \times X$ such that $(F(u,v),F(v,u))$ is comparable with $(F(x,y),F(y,x))$ and $(F(z,t),F(t,z))$.

Let $u_0 = u$, $v_0 = v$ and choose $u_1, v_1 \in X$ so that $gu_1 = F(u_0,v_0)$ and $gv_1 = F(v_0,u_0)$. Then, similarly as in the proof of Theorem 3.1, we can inductively define sequences $\{gu_n\}$ and $\{gv_n\}$ such that $gu_{n+1} = F(u_n,v_n)$ and $gv_{n+1} = F(v_n,u_n)$. Since $(gx,gy) = (F(x,y),F(y,x))$ and $(F(u,v),F(v,u)) = (gu_1,gv_1)$ are comparable, we may assume that $(gx,gy) \preceq (gu_1,gv_1)$. Then $gx \preceq gu_1$ and $gy \succeq gv_1$. Using the mathematical induction, it is easy to prove that $gx \preceq gu_n$ and $gy \succeq gv_n$ for all $n \in \mathbb{N}$.

Let $\gamma_n = \max\{G(gx,gx,gu_n),G(gy,gy,gv_n)\}$. We shall show that $\lim_{n\to\infty} \gamma_n = 0$. First, assume that $\gamma_n = 0$ for an $n \geq 1$.

Applying (3.1), as $gx \preceq gu_n$ and $gy \succeq gv_n$, one obtains that

$$\begin{aligned}
&\psi\left(s\max\left\{G(gx,gx,gu_{n+1}),G(gy,gy,gv_{n+1})\right\}\right)\\
&\quad = \psi\left(s\max\left\{G\big(F(x,y),F(x,y),F(u_n,v_n)\big),G\big(F(y,x),F(y,x),F(v_n,u_n)\big)\right\}\right)
\end{aligned}$$

$$\begin{aligned}
&\le \psi\left(\max\left\{G(gx,gx,gu_n),G(gy,gy,gv_n)\right\}\right)\\
&\quad - \varphi\left(\max\left\{G(gx,gx,gu_n),G(gy,gy,gv_n)\right\}\right)\\
&= \psi(\gamma_n) - \varphi(\gamma_n)\\
&= \psi(0) - \varphi(0) = 0.
\end{aligned} \tag{3.10}$$

So, from the properties of ψ and φ, we deduce that $\gamma_{n+1} = 0$. Repeating this process, we can show that $\gamma_m = 0$ for all $m \ge n$. So, $\lim_{n\to\infty} \gamma_n = 0$.

Now, let $\gamma_n \neq 0$ for all n, and let $\gamma_n < \gamma_{n+1}$ for some n.

As ψ is an altering distance function, from (3.10)

$$\begin{aligned}
\psi(s\gamma_n) &= \psi\left(s\max\left\{G(gx,gx,gu_n),G(gy,gy,gv_n)\right\}\right)\\
&\le \psi(s\gamma_{n+1})\\
&= \psi\left(s\max\left\{G(gx,gx,gu_{n+1}),G(gy,gy,gv_{n+1})\right\}\right)\\
&\le \psi\left(\max\left\{G(gx,gx,gu_n),G(gy,gy,gv_n)\right\}\right)\\
&\quad - \varphi\left(\max\left\{G(gx,gx,gu_n),G(gy,gy,gv_n)\right\}\right)\\
&= \psi(\gamma_n) - \varphi(\gamma_n)\\
&\le \psi(s\gamma_n) - \varphi(\gamma_n).
\end{aligned}$$

This implies that $\gamma_n = 0$, which is a contradiction.

Hence, $\gamma_{n+1} \le \gamma_n$ for all $n \ge 1$. Now, if we proceed as in Theorem 3.1, we can show that

$$\lim_{n\to\infty} \max\left\{G(gx,gx,gu_n),G(gy,gy,gv_n)\right\} = 0.$$

So, $\{gu_n\} \to gx$ and $\{gv_n\} \to gy$.

Similarly, we can show that

$$\lim_{n\to\infty} \left\{G(gz,gz,gu_n),G(gt,gt,gv_n)\right\} = 0,$$

that is, $\{gu_n\} \to gz$ and $\{gv_n\} \to gt$. Finally, since the limit is unique, $gx = gz$ and $gy = gt$.

Since $gx = F(x,y)$ and $gy = F(y,x)$, by the commutativity of F and g, we have $g(gx) = g(F(x,y)) = F(gx,gy)$ and $g(gy) = g(F(y,x)) = F(gy,gx)$. Let $gx = a$ and $gy = b$. Then $ga = F(a,b)$ and $gb = F(b,a)$. Thus, (a,b) is another coupled coincidence point of F and g. Then $a = gx = ga$ and $b = gy = gb$. Therefore, (a,b) is a coupled common fixed point of F and g.

To prove the uniqueness of a coupled common fixed point, assume that (p,q) is another coupled common fixed point of F and g. Then $p = gp = F(p,q)$ and $q = gq = F(q,p)$. Since (p,q) is a coupled coincidence point of F and g, we have $gp = ga$ and $gq = gb$. Thus, $p = gp = ga = a$ and $q = gq = gb = b$. Hence, the coupled common fixed point is unique. □

The following corollary can be deduced from our previous obtained results.

Theorem 3.5 *Let $(X, \preceq)$ be a partially ordered set, and let (X, G) be a complete G_b-metric space. Let $F : X \times X \to X$ be a mapping with the mixed monotone property such that*

$$\begin{aligned} &\psi\bigl(s \max\bigl\{G\bigl(F(x,y), F(u,v), F(w,t)\bigr), G\bigl(F(y,x), F(v,u), F(t,w)\bigr)\bigr\}\bigr) \\ &\quad \leq \psi\left(\frac{G(x,u,w) + G(y,v,t)}{2}\right) - \varphi\bigl(\max\bigl\{G(x,u,w), G(y,v,t)\bigr\}\bigr) \end{aligned} \tag{3.11}$$

for every pair $(x,y), (u,v), (w,t) \in X \times X$ such that $x \preceq u \preceq w$ and $y \succeq v \succeq t$, or $w \preceq u \preceq x$ and $t \succeq v \succeq y$, where $\psi, \varphi : [0,\infty) \to [0,\infty)$ are altering distance functions.

Also, suppose that either

(a) *F is continuous, or*

(b) *X is regular.*

If there exist $x_0, y_0 \in X$ such that $x_0 \preceq F(x_0, y_0)$ and $y_0 \succeq F(y_0, x_0)$, then F has a coupled fixed point in X.

Proof If F satisfies (3.11), then F satisfies (3.1). So, the result follows from Theorems 3.1 and 3.2. □

In Theorems 3.1 and 3.2, if we take $\psi(t) = t$ and $\varphi(t) = (1-k)t$ for all $t \in [0,\infty)$, where $k \in [0,1)$, we obtain the following result.

Theorem 3.6 *Let $(X, \preceq)$ be a partially ordered set, and let (X, G) be a complete G_b-metric space. Let $F : X \times X \to X$ be a mapping having the mixed monotone property such that*

$$\begin{aligned} &\max\bigl\{G\bigl(F(x,y), F(u,v), F(w,t)\bigr), G\bigl(F(y,x), F(v,u), F(t,w)\bigr)\bigr\} \\ &\quad \leq \frac{k}{s} \max\bigl\{G(x,u,w), G(y,v,t)\bigr\} \end{aligned}$$

for every pair $(x,y), (u,v), (w,t) \in X \times X$ such that $x \preceq u \preceq w$ and $y \succeq v \succeq t$, or $w \preceq u \preceq x$ and $t \succeq v \succeq y$, where $k \in [0,1)$.

Also, suppose that either

(a) *F is continuous, or*

(b) *X is regular.*

If there exist $x_0, y_0 \in X$ such that $x_0 \preceq F(x_0, y_0)$ and $y_0 \succeq F(y_0, x_0)$, then F has a coupled fixed point in X.

The following corollary is an extension of the results by Choudhury and Maity (Theorems 1.14 and 1.15).

Theorem 3.7 *Let $(X, \preceq)$ be a partially ordered set, and let (X, G) be a complete G_b-metric space. Let $F : X \times X \to X$ be a mapping with the mixed monotone property such that*

$$\begin{aligned} &\max\bigl\{G\bigl(F(x,y), F(u,v), F(w,t)\bigr), G\bigl(F(y,x), F(v,u), F(t,w)\bigr)\bigr\} \\ &\quad \leq \frac{k}{2s}\bigl[G(x,u,w) + G(y,v,t)\bigr] \end{aligned} \tag{3.12}$$

for every pair $(x,y), (u,v), (w,t) \in X \times X$ such that $x \preceq u \preceq w$ and $y \succeq v \succeq t$, or $w \preceq u \preceq x$ and $t \succeq v \succeq y$.

Also, suppose that either

(a) *F is continuous, or*

(b) *X is regular.*

If there exist $x_0, y_0 \in X$ such that $x_0 \preceq F(x_0, y_0)$ and $y_0 \succeq F(y_0, x_0)$, then F has a coupled fixed point in X.

Proof If F satisfies (3.12), then F satisfies (3.11). □

Now, we present an example to illustrate Theorem 3.1.

Example 3.8 Let $X = \mathbb{R}$ be endowed with the usual ordering, and let G_b-metric on X be given by $G(x,y,z) = (|x-y| + |y-z| + |x-z|)^2$, where $s = 2$.

Define $F : X \times X \to X$ and $g : X \to X$ as

$$F(x,y) = \frac{x-y}{12},$$

and $g(x) = x$ for all $x, y \in X$.

Define $\psi, \varphi : [0,\infty) \to [0,\infty)$ by $\psi(t) = bt$, $\varphi(t) = (b-1)t$, where $1 \leq b \leq \frac{72}{4} = 18$.

Let $x, y, u, v, w, t \in X$ be such that $x \leq u \leq w$ and $y \geq v \geq t$. Now, we have

$$\begin{aligned}
&\psi\bigl(s \max\bigl\{G\bigl(F(x,y), F(u,v), F(w,t)\bigr), G\bigl(F(y,x), F(v,u), F(t,w)\bigr)\bigr\}\bigr) \\
&\quad = 2b \frac{(|(x-y)-(u-v)| + |(u-v)-(w-t)| + |(w-t)-(x-y)|)^2}{144} \\
&\quad \leq 2b \frac{(|x-u| + |y-v| + |u-w| + |v-t| + |w-x| + |t-y|)^2}{144} \\
&\quad \leq \frac{4b}{72} \frac{(|x-u| + |u-w| + |w-x|)^2 + (|y-v| + |v-t| + |t-y|)^2}{2} \\
&\quad \leq \frac{(|x-u| + |u-w| + |w-x|)^2 + (|y-v| + |v-t| + |t-y|)^2}{2} \\
&\quad \leq \max\bigl\{\bigl(|x-u| + |u-w| + |w-x|\bigr)^2, \bigl(|y-v| + |v-t| + |t-y|\bigr)^2\bigr\} \\
&\quad = \max\bigl\{G(gx, gu, gw), G(gy, gv, gt)\bigr\} \\
&\quad = \psi\bigl(\max\bigl\{G(gx, gu, gw), G(gy, gv, gt)\bigr\}\bigr) - \varphi\bigl(\max\bigl\{G(gx, gu, gw), G(gy, gv, gt)\bigr\}\bigr).
\end{aligned}$$

Obviously, all the conditions of Theorem 3.1 are satisfied. Moreover, $(0,0)$ is a coupled coincidence point of F and g.

4 Applications

In this section, we obtain some coupled coincidence point theorems for mappings satisfying some contractive conditions of integral type in an ordered complete G_b-metric space.

Denote by Λ the set of all functions $\mu : [0,+\infty) \to [0,+\infty)$ verifying the following conditions:

(I) μ is a positive Lebesgue integrable mapping on each compact subset of $[0,+\infty)$.

(II) For all $\varepsilon > 0$, $\int_0^\varepsilon \mu(t)\,dt > 0$.

Corollary 4.1 *Replace the contractive condition* (3.1) *of Theorem* 3.1 *by the following condition*:

There exists $\mu \in \Lambda$ *such that*

$$\begin{aligned} &\int_0^{\psi(s\max\{G(F(x,y),F(u,v),F(w,t)),G(F(y,x),F(v,u),F(t,w))\})} \mu(t)\,dt \\ &\quad \leq \int_0^{\psi(\max\{G(gx,gu,gw),G(gy,gv,gt)\})} \mu(t)\,dt - \int_0^{\varphi(\max\{G(gx,gu,gw),G(gy,gv,gt)\})} \mu(t)\,dt. \end{aligned} \tag{4.1}$$

If the other conditions of Theorem 3.1 *hold, then F and g have a coupled coincidence point.*

Proof Consider the function $\Gamma(x) = \int_0^x \mu(t)\,dt$. Then (4.1) becomes

$$\begin{aligned} &\Gamma\bigl(\psi\bigl(s\max\bigl\{G\bigl(F(x,y),F(u,v),F(w,t)\bigr),G\bigl(F(y,x),F(v,u),F(t,w)\bigr)\bigr\}\bigr)\bigr) \\ &\quad \leq \Gamma\bigl(\psi\bigl(\max\bigl\{G(gx,gu,gw),G(gy,gv,gt)\bigr\}\bigr)\bigr) \\ &\qquad - \Gamma\bigl(\varphi\bigl(\max\bigl\{G(gx,gu,gw),G(gy,gv,gt)\bigr\}\bigr)\bigr). \end{aligned}$$

Taking $\psi_1 = \Gamma o\psi$ and $\varphi_1 = \Gamma o\varphi$ and applying Theorem 3.1, we obtain the proof. □

Corollary 4.2 *Substitute the contractive condition* (3.1) *of Theorem* 3.1 *by the following condition*:

There exists $\mu \in \Lambda$ *such that*

$$\begin{aligned} &\psi\biggl(\int_0^{s\max\{G(F(x,y),F(u,v),F(w,t)),G(F(y,x),F(v,u),F(t,w))\}} \mu(t)\,dt\biggr) \\ &\quad \leq \psi\biggl(\int_0^{\max\{G(gx,gu,gw),G(gy,gv,gt)\}} \mu(t)\,dt\biggr) - \varphi\biggl(\int_0^{\max\{G(gx,gu,gw),G(gy,gv,gt)\}} \mu(t)\,dt\biggr). \end{aligned} \tag{4.2}$$

Then F and g have a coupled coincidence point if the other conditions of Theorem 3.1 *hold.*

Proof Again, as in Corollary 4.1, define the function $\Gamma(x) = \int_0^x \phi(t)\,dt$. Then (4.2) changes to

$$\begin{aligned} &\psi\bigl(\Gamma\bigl(s\max\bigl\{G\bigl(F(x,y),F(u,v),F(w,t)\bigr),G\bigl(F(y,x),F(v,u),F(t,w)\bigr)\bigr\}\bigr)\bigr) \\ &\quad \leq \psi\bigl(\Gamma\bigl(\max\bigl\{G(gx,gu,gw),G(gy,gv,gt)\bigr\}\bigr)\bigr) \\ &\qquad - \varphi\bigl(\Gamma\bigl(\max\bigl\{G(gx,gu,gw),G(gy,gv,gt)\bigr\}\bigr)\bigr). \end{aligned}$$

Now, if we define $\psi_1 = \psi o\Gamma$ and $\varphi_1 = \varphi o\Gamma$ and apply Theorem 3.1, then the proof is obtained. □

As in [46], let $n \in \mathbb{N}$ be fixed. Let $\{\mu_i\}_{1\leq i\leq N}$ be a family of N functions which belong to Λ. For all $t \geq 0$, we define

$$\begin{aligned} I_1(t) &= \int_0^t \mu_1(l)\,dl, \\ I_2(t) &= \int_0^{I_1 t} \mu_2(l)\,dl = \int_0^{\int_0^t \mu_1(l)\,dl} \mu_2(l)\,dl, \end{aligned}$$

$$I_3(t) = \int_0^{I_2 t} \mu_3(l)\,dl = \int_0^{\int_0^{\int_0^t \mu_1(l)\,dl} \mu_2(l)\,dl} \mu_3(l)\,dl,$$

$$\ldots,$$

$$I_N(t) = \int_0^{I_{(N-1)}t} \mu_N(l)\,dl.$$

We have the following result.

Corollary 4.3 *Replace the inequality* (3.1) *of Theorem* 3.1 *by the following condition*:

$$\begin{aligned}&\psi\bigl(I_N\bigl(s\max\bigl\{G\bigl(F(x,y),F(u,v),F(w,t)\bigr),G\bigl(F(y,x),F(v,u),F(t,w)\bigr)\bigr\}\bigr)\bigr)\\&\quad\le \psi\bigl(I_N\bigl(\max\bigl\{G(gx,gu,gw),G(gy,gv,gt)\bigr\}\bigr)\bigr)\\&\qquad-\varphi\bigl(I_N\bigl(\max\bigl\{G(gx,gu,gw),G(gy,gv,gt)\bigr\}\bigr)\bigr).\end{aligned} \tag{4.3}$$

Assume further that the other conditions of Theorem 3.1 *are also satisfied. Then F and g have a coupled coincidence point.*

Proof Consider $\hat{\psi} = \psi o I_N$ and $\hat{\varphi} = \varphi o I_N$. Then the above inequality becomes

$$\begin{aligned}&\hat{\psi}\bigl(s\max\bigl\{G\bigl(F(x,y),F(u,v),F(w,t)\bigr),G\bigl(F(y,x),F(v,u),F(t,w)\bigr)\bigr\}\bigr)\\&\quad\le \hat{\psi}\bigl(\max\bigl\{G(gx,gu,gw),G(gy,gv,gt)\bigr\}\bigr)-\hat{\varphi}\bigl(\max\bigl\{G(gx,gu,gw),G(gy,gv,gt)\bigr\}\bigr).\end{aligned}$$

Now, applying Theorem 3.1, we obtain the desired result. □

Another consequence of our theorems is the following result.

Corollary 4.4 *Replace the contractive condition* (3.1) *of Theorem* 3.1 *by the following condition*:

There exist $\mu_1, \mu_2, \mu_3 \in \Lambda$ *such that*

$$\begin{aligned}&\int_0^{s\max\{G(F(x,y),F(u,v),F(w,t)),G(F(y,x),F(v,u),F(t,w))\}} \mu_1(t)\,dt\\&\quad\le \int_0^{\max\{G(gx,gu,gw),G(gy,gv,gt)\}} \mu_2(t)\,dt - \int_0^{\max\{G(gx,gu,gw),G(gy,gv,gt)\}} \mu_3(t)\,dt.\end{aligned}$$

Let the other conditions of Theorem 3.1 *be satisfied. Then F and g have a coupled coincidence point.*

5 Conclusions

We saw that the results of Cho *et al.* [25] and the results of Choudhury and Maity [23] also hold in the context of G_b-metric spaces with some simple changes in the contractive conditions. The most difference between the concepts of G-metric and G_b-metric is that the G_b-metric function is not necessarily continuous in all its three variables (see, Example 2.15). On the other hand, by a simple but essential lemma (Lemma 2.16), we can prove many fixed point results in this new structure.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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References
1. Guo, D, Lakshmikantham, V: Coupled fixed points of nonlinear operators with applications. Nonlinear Anal. **11**, 623-632 (1987)
2. Gnana Bhaskar, T, Lakshmikantham, V: Fixed point theorems in partially ordered metric spaces and applications. Nonlinear Anal. TMA **65**, 1379-1393 (2006)
3. Lakshmikantham, V, Ćirić, L: Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces. Nonlinear Anal. **70**(12), 4341-4349 (2009)
4. Abbas, M, Ali Khan, M, Radenović, S: Common coupled fixed point theorems in cone metric spaces for *w*-compatible mappings. Appl. Math. Comput. **217**, 195-202 (2010)
5. Mustafa, Z, Sims, B: A new approach to generalized metric spaces. J. Nonlinear Convex Anal. **7**(2), 289-297 (2006)
6. Aydi, H, Karapınar, E, Shatanawi, W: Tripled common fixed point results for generalized contractions in ordered generalized metric spaces. Fixed Point Theory Appl. **2012**, 101 (2012)
7. Aydi, H, Shatanawi, W, Vetro, C: On generalized weakly *G*-contraction mapping in *G*-metric spaces. Comput. Math. Appl. **62**, 4222-4229 (2011)
8. Gholizadeh, L, Saadati, R, Shatanawi, W, Vaezpour, SM: Contractive mapping in generalized ordered metric spaces with application in integral equations. Math. Probl. Eng. **2011**, Article ID 380784 (2011). doi:10.1155/2011/380784
9. Luong, NV, Thuan, NX: Coupled fixed point theorems in partially ordered *G*-metric spaces. Math. Comput. Model. **55**, 1601-1609 (2012)
10. Mustafa, Z: Common fixed points of weakly compatible mappings in *G*-metric spaces. Appl. Math. Sci. **6**(92), 4589-4600 (2012)
11. Mustafa, Z: Some new common fixed point theorems under strict contractive conditions in *G*-metric spaces. J. Appl. Math. **2012**, Article ID 248937 (2012). doi:10.1155/2012/248937
12. Mustafa, Z: Mixed *g*-monotone property and quadruple fixed point theorems in partially ordered *G*-metric spaces using (ϕ,ψ) contractions. Fixed Point Theory Appl. **2012**, 199 (2012). doi:10.1186/1687-1812-2012-199
13. Mustafa, Z, Awawdeh, F, Shatanawi, W: Fixed point theorem for expansive mappings in *G*-metric spaces. Int. J. Contemp. Math. Sci. **5**, 49-52 (2010)
14. Mustafa, Z, Aydi, H, Karapınar, E: Mixed *g*-monotone property and quadruple fixed point theorems in partially ordered metric space. Fixed Point Theory Appl. **2012**, 71 (2012). doi:10.1186/1687-1812-2012-71
15. Mustafa, Z, Aydi, H, Karapınar, E: On common fixed points in *G*-metric spaces using (E.A) property. Comput. Math. Appl. **64**, 1944-1956 (2012). doi:10.1016/j.camwa.2012.03.051
16. Mustafa, Z, Khandagjy, M, Shatanawi, W: Fixed point results on complete *G*-metric spaces. Studia Sci. Math. Hung. **48**(3), 304-319 (2011). doi:10.1556/SScMath.48.2011.3.1170
17. Mustafa, Z, Obiedat, H, Awawdeh, F: Some of fixed point theorem for mapping on complete *G*-metric spaces. Fixed Point Theory Appl. **2008**, Article ID 189870 (2008)
18. Mustafa, Z, Shatanawi, W, Bataineh, M: Existence of fixed point result in *G*-metric spaces. Int. J. Math. Math. Sci. **2009**, Article ID 283028 (2009)
19. Mustafa, Z, Sims, B: Fixed point theorems for contractive mappings in complete *G*-metric space. Fixed Point Theory Appl. **2009**, Article ID 917175 (2009)
20. Shatanawi, W: Coupled fixed point theorems in generalized metric spaces. Hacet. J. Math. Stat. **40**, 441-447 (2011)
21. Shatanawi, W, Postolache, M: Some fixed point results for a *G*-weak contraction in *G*-metric spaces. Abstr. Appl. Anal. **2012**, Article ID 815870 (2012)
22. Tahat, N, Aydi, H, Karapınar, E, Shatanawi, W: Common fixed points for single-valued and multi-valued maps satisfying a generalized contraction in *G*-metric spaces. Fixed Point Theory Appl. **2012**, 48 (2012). doi:10.1186/1687-1812-2012-48
23. Choudhury, BS, Maity, P: Coupled fixed point results in generalized metric spaces. Math. Comput. Model. **54**, 73-79 (2011)
24. Khan, MS, Swaleh, M, Sessa, S: Fixed point theorems by altering distances between the points. Bull. Aust. Math. Soc. **30**, 1-9 (1984)
25. Cho, YJ, Rhoades, BE, Saadati, R, Samet, B, Shatanawi, W: Nonlinear coupled fixed point theorems in ordered generalized metric spaces with integral type. Fixed Point Theory Appl. **2012**, 8 (2012)
26. Aydi, H, Karapınar, E, Postolache, M: Tripled coincidence point theorems for weak φ-contractions in partially ordered metric spaces. Fixed Point Theory Appl. **2012**, 44 (2012)
27. Aydi, H, Shatanawi, W, Postolache, M, Mustafa, Z, Tahat, N: Theorems for Boyd-Wong type contractions in ordered metric spaces. Abstr. Appl. Anal. **2012**, Article ID 359054 (2012)
28. Parvaneh, V, Roshan, JR, Radenović, S: Existence of tripled coincidence points in ordered *b*-metric spaces and an application to a system of integral equations. Fixed Point Theory Appl. **2013**, 130 (2013)

29. Razani, A, Parvaneh, V: Coupled coincidence point results for (ψ, α, β)-weak contractions in partially ordered metric spaces. J. Appl. Math. **2012**, Article ID 496103 (2012). doi:10.1155/2012/496103
30. Roshan, JR, Parvaneh, V, Sedghi, S, Shobkolaei, N, Shatanawi, W: Common fixed points of almost generalized $(\psi, \varphi)_s$-contractive mappings in ordered *b*-metric spaces. Fixed Point Theory Appl. **2013**, 159 (2013). doi:10.1186/1687-1812-2013-159
31. Shatanawi, W, Mustafa, Z: On coupled random fixed point results in partially ordered metric spaces. Mat. Vesn. **64**, 139-146 (2012)
32. Shatanawi, W, Mustafa, Z, Tahat, N: Some coincidence point theorems for nonlinear contractions in ordered metric spaces. Fixed Point Theory Appl. **2011**, 68 (2011). doi:10.1186/1687-1812-2011-68
33. Shatanawi, W, Postolache, M: Common fixed point results of mappings for nonlinear contractions of cyclic form in ordered metric spaces. Fixed Point Theory Appl. **2013**, 60 (2013)
34. Czerwik, S: Contraction mappings in *b*-metric spaces. Acta Math. Inf. Univ. Ostrav. **1**, 5-11 (1993)
35. Aghajani, A, Abbas, M, Roshan, JR: Common fixed point of generalized weak contractive mappings in partially ordered G_b-metric spaces. Filomat (2013, in press)
36. Aghajani, A, Abbas, M, Roshan, JR: Common fixed point of generalized weak contractive mappings in partially ordered *b*-metric spaces. Math. Slovaca (2012, in press)
37. Hussain, N, Dorić, D, Kadelburg, Z, Radenović, S: Suzuki-type fixed point results in metric type spaces. Fixed Point Theory Appl. (2012). doi:10.1186/1687-1812-2012-126
38. Ćirić, L, Lakshmikantham, V: Coupled random fixed point theorems for nonlinear contractions in partially ordered metric spaces. Stoch. Anal. Appl. **27**, 1246-1259 (2009)
39. Aydi, H, Karapınar, E, Shatanawi, W: Coupled fixed point results for (ψ, φ)-weakly contractive condition in ordered partial metric spaces. Comput. Math. Appl. **62**, 4449-4460 (2011)
40. Aydi, H, Postolache, M, Shatanawi, W: Coupled fixed point results for (ψ, φ)-weakly contractive mappings in ordered *G*-metric spaces. Comput. Math. Appl. **63**, 298-309 (2012)
41. Berinde, V: Coupled fixed point theorems for contractive mixed monotone mappings in partially ordered metric spaces. Nonlinear Anal. **75**, 3218-3228 (2012)
42. Choudhury, BS, Kundu, A: A coupled coincidence point result in partially ordered metric spaces for compatible mappings. Nonlinear Anal. **73**, 2524-2531 (2010)
43. Ding, HS, Li, L, Radenović, S: Coupled coincidence point theorems for generalized nonlinear contraction in partially ordered metric spaces. Fixed Point Theory Appl. **2012**, 96 (2012). doi:10.1186/1687-1812-2012-96
44. Ćirić, L, Damjanović, B, Jleli, M, Samet, B: Coupled fixed point theorems for generalized Mizoguchi-Takahashi contraction and applications to ordinary differential equations. Fixed Point Theory Appl. **2012**, 51 (2012)
45. Ćirić, L, Cakić, N, Rajović, M, Ume, JS: Monotone generalized nonlinear contractions in partially ordered metric spaces. Fixed Point Theory Appl. **2008**, Article ID 131294 (2008)
46. Nashine, HK, Samet, B: Fixed point results for mappings satisfying (ψ, φ)-weakly contractive condition in partially ordered metric spaces. Nonlinear Anal. **74**, 2201-2209 (2011)

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