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Ω-Distance and coupled fixed point in G-metric spaces

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Abstract

Saadati *et al.* (Math. Comput. Model. 52:797-801, 2010) introduced the concept of Ω-distance in generalized metric spaces and studied some nice fixed point theorems. Very recently, Jleli and Samet (Fixed Point Theory Appl. 2012:210, 2012) showed that some of the fixed point theorems in G-metric spaces can be obtained from quasi-metric space. In this paper, we utilize the concept of Ω-distance in the sense of Saadati *et al.* to establish some common coupled fixed point results. Also, we introduce an example to support the useability of our results. Note that the method of Jleli and Samet cannot be used in our results.

MSC: 47H10; 54H25

Keywords: coupled fixed point; Ω-distance

1 Introduction

In 2006, Mustafa and Sims [1] introduced a generalization of metric spaces, the G-metric spaces, which assigns to each triple of elements a non-negative real number. Very recently, Jleli and Samet [2] showed that some of the fixed point theorems in G-metric spaces can be obtained from quasi-metric spaces. For some works in G-metric spaces, see [3–36]. In 2010, Saadati *et al.* [26] introduced the concept of Ω-distance and studied some nice fixed point theorems (also, see [13]). Meanwhile, Bhaskar and Lakshmikantam [37] introduced the concept of coupled fixed point and proved several fixed point theorems. Lakshmikantam and Ćirić [38] generalized the concept of coupled fixed point to the the concept of coupled coincidence point of two mappings [39]. After that, many authors established coupled fixed point results (please, see [19–44]). In the present paper, we utilize the concept of Ω-distance to establish some coupled fixed point results. Also, we introduce an example to support the useability of our study.

2 Preliminaries

Definition 2.1 ([1]) Let X be an nonempty set. The mapping $G\colon X \times X \times X \to X$ is called *G-metric* if the following axioms are fulfilled:

(1) $G(x,y,z) = 0$ if $x = y = z$ (the coincidence);
(2) $G(x,x,y) > 0$ for all $x,y \in X$, $x \neq y$;
(3) $G(x,x,z) \leq G(x,y,z)$ for each triple (x,y,z) from $X \times X \times X$ with $z \neq y$;
(4) $G(x,y,z) = G(p\{x,y,z\})$ for each permutation of $\{x,y,z\}$ (the symmetry);
(5) $G(x,y,z) \leq G(x,a,a) + G(a,y,z)$ for each x, y, z and a in X (the rectangle inequality).

Definition 2.2 ([1]) Consider X a G-metric space and (x_n) a sequence in G.

(1) (x_n) is called *G-Cauchy sequence* if for each $\epsilon > 0$ there is a positive integer n_0 so that for all $m, n, l \geq n_0$, $G(x_n, x_m, x_l) < \epsilon$.

(2) (x_n) is said to be *G-convergent* to $x \in X$ if for each $\epsilon > 0$ there is a positive integer n_0 such that $G(x_m, x_n, x) < \epsilon$ for each $m, n \geq n_0$.

Definition 2.3 ([26]) Consider (X, G) a G-metric space and $\Omega\colon X \times X \times X \to [0, +\infty)$. The mapping Ω is called an *Ω-distance* on X if it satisfies the three conditions in the following:

(1) $\Omega(x, y, z) \leq \Omega(x, a, a) + \Omega(a, y, z)$ for all x, y, z, a from X.

(2) For each x, y from X, $\Omega(x, y, \cdot), \Omega(x, \cdot, y)\colon X \to [0, +\infty)$ are lower semi-continuous.

(3) for each $\epsilon > 0$ there is $\delta > 0$, so that $\Omega(x, a, a) \leq \delta$ and $\Omega(a, y, z) \leq \delta$ imply $G(x, y, z) \leq \epsilon$.

The following lemma [13, 26] is going to be very helpful in computing the limits of several sequences.

Lemma 2.1 *Let X be a metric space, endowed with metric G, and let Ω be an Ω-distance on X. (x_n), (y_n) are sequences in X, (α_n) and (β_n) are sequences in $[0, +\infty)$ with $\lim_{n\to+\infty} \alpha_n = \lim_{n\to+\infty} \beta_n = 0$. If x, y, z and $a \in X$, then*

(1) *If $\Omega(y, x_n, x_n) \leq \alpha_n$ and $\Omega(x_n, y, z) \leq \beta_n$ for $n \in \mathbb{N}$, then $G(y, y, z) < \epsilon$, and, by consequence, $y = z$.*

(2) *Inequalities $\Omega(y_n, x_n, x_n) \leq \alpha_n$ and $\Omega(x_n, y_m, z) \leq \beta_n$ for $m > n$ imply $G(y_n, y_m, z) \to 0$, hence $y_n \to z$.*

(3) *If $\Omega(x_n, x_m, x_l) \leq \alpha_n$ for $l, m, n \in \mathbb{N}$ with $n \leq m \leq l$, then (x_n) is a G-Cauchy sequence.*

(4) *If $\Omega(x_n, a, a) \leq \alpha_n$, $n \in \mathbb{N}$, then (x_n) is a G-Cauchy sequence.*

Definition 2.4 ([37]) Consider X a nonempty set. A pair $(x, y) \in X \times X$ is called *coupled fixed point* of mapping $F\colon X \times X \to X$ if

$$F(x, y) = x, \qquad F(y, x) = y.$$

Definition 2.5 ([38]) Let X be a nonempty set. The element $(x, y) \in X \times X$ is a *coupled coincidence point* of mappings $F\colon X \times X \to X$ and $g\colon X \to X$ if

$$F(x, y) = gx, \qquad F(y, x) = gy.$$

3 Main results

Theorem 3.1 *Let (X, G) be a G-metric space and Ω an Ω-distance on X such that X is Ω-bounded. $g\colon X \to X$ and $F\colon X \times X \to X$ are mappings. Suppose there exists $k \in [0, 1)$ such that for each x, y, z, x^*, y^* and z^* in X*

$$\begin{aligned}
&\Omega\big(F(x,y), F\big(x^*,y^*\big), F\big(z,z^*\big)\big) + \Omega\big(F(y,x), F\big(y^*,x^*\big), F\big(z^*,z\big)\big) \\
&\quad \leq k \max\big\{\Omega\big(gx, gx^*, gz\big) + \Omega\big(gy, gy^*, gz^*\big), \\
&\qquad \Omega\big(gx^*, gx, gz\big) + \Omega\big(gy^*, gy, gz^*\big), \\
&\qquad \Omega\big(gx, F\big(x^*,y^*\big), gz\big) + \Omega\big(gy, F\big(y^*,x^*\big), gz^*\big),
\end{aligned}$$

$$\begin{aligned}
&\Omega\big(F(x,y),gx^*,gz\big)+\Omega\big(F(y,x),gy^*,gz^*\big),\\
&\Omega\big(gx^*,F(x,y),gz\big)+\Omega\big(gy^*,F(y,x),gz^*\big),\\
&\Omega\big(F(x,y),F\big(x^*,y^*\big),gz\big)+\Omega\big(F(y,x),F\big(y^*,x^*\big),gz^*\big)\big\}.
\end{aligned}$$

Consider also that the following conditions hold true:

(1) $F(X\times X)\subseteq gX$;

(2) *gX is a complete subspace of X with respect to the topology, induced by G*;

(3) *If $F(u,v)\neq gu$ or $F(v,u)\neq gv$, then*

$$\begin{aligned}
&\inf\big\{\Omega\big(gx,F(x,y),gu\big)+\Omega\big(gy,F(y,x),gv\big)\\
&\quad+\Omega\big(gx,gu,F(x,y)\big)+\Omega\big(gy,gv,F(y,x)\big)\big\}>0.
\end{aligned}$$

Then, F and g have a unique coupled coincidence point (u,v). Moreover, $F(u,v)=gu=gv=F(v,u)$.

Proof Consider $x_0\in X$ and $y_0\in X$. Because $F(X\times X)\subseteq gX$, there exist x_1 and y_1 in X such that $gx_1=F(x_0,y_0)$ and $y_1=F(y_0,x_0)$. By continuing the process, we obtain two sequences, (x_n) and (y_n), with the properties

$$gx_{n+1}=F(x_n,y_n),\qquad gy_{n+1}=F(y_n,x_n)$$

Using the contraction condition, we obtain

$$\begin{aligned}
&\Omega(gx_n,gx_{n+1},gx_{n+s})+\Omega(gy_n,gy_{n+1},gy_{n+s})\\
&\quad=\Omega\big(F(x_{n-1},y_{n-1}),F(x_n,y_n),F(x_{n+s-1},y_{n+s-1})\big)\\
&\qquad+\Omega\big(F(y_{n-1},x_{n-1}),F(y_n,x_n),F(y_{n+s-1},x_{n+s-1})\big)\\
&\quad\le k\max\big\{\Omega(gx_{n-1},gx_n,gx_{n+s-1})+\Omega(gy_{n-1},gy_n,gy_{n+s-1}),\\
&\qquad\Omega(gx_n,gx_{n-1},gx_{n+s-1})+\Omega(gy_n,gy_{n-1},gy_{n+s-1}),\\
&\qquad\Omega(gx_{n-1},gx_{n+1},gx_{n+s-1})+\Omega(gy_{n-1},gy_{n+1},gy_{n+s-1}),\\
&\qquad\Omega(gx_n,gx_n,gx_{n+s-1})+\Omega(gy_n,gy_n,gy_{n+s-1}),\\
&\qquad\Omega(gx_n,gx_{n+1},gx_{n+s-1})+\Omega(gy_n,gy_{n+1},gy_{n+s-1})\big\}.
\end{aligned}$$

By applying the contraction inequality repeatedly, we get that

$$\begin{aligned}
&\Omega(gx_n,gx_{n+1},gx_{n+s})+\Omega(gy_n,gy_{n+1},gy_{n+s})\\
&\quad\le k^{n-1}\max_{(i,j,t)\in A}\big\{\Omega(gx_i,gx_j,gx_t)+\Omega(gy_i,gy_j,gy_t)\big\},
\end{aligned}\tag{1}$$

where $A=\{(i,j,t)|1\le i\le n,1\le j\le n+1,s+1\le t\le n+s-1\}$.

Since X is Ω-bounded, there is $M>0$ such that $\Omega(x,y,z)<M$ for each triple $(x,y,z)\in X\times X\times X$. Hence, relation (1) becomes

$$\Omega(gx_n,gx_{n+1},gx_{n+s})+\Omega(gy_n,gy_{n+1},gy_{n+s})\le 2k^{n-1}M.$$

Consider now $l > m > n > 0$, $l, m, n \in \mathbb{N}$. The following relations hold true:

$$\begin{aligned}\Omega(gx_n, gx_m, gx_l) &\leq \Omega(gx_n, gx_{n+1}, gx_{n+1}) + \Omega(gx_{n+1}, gx_m, gx_l)\\ &\leq \Omega(gx_n, gx_{n+1}, gx_{n+1})\\ &\quad + \Omega(gx_{n+1}, gx_{n+2}, gx_{n+2}) + \cdots + \Omega(gx_{m-1}, gx_m, gx_l),\end{aligned} \tag{2}$$

and, also

$$\begin{aligned}\Omega(gy_n, gy_m, gy_l) &\leq \Omega(gy_n, gy_{n+1}, gy_{n+1}) + \Omega(gy_{n+1}, gy_m, gy_l)\\ &\leq \Omega(gy_n, gy_{n+1}, gy_{n+1})\\ &\quad + \Omega(gy_{n+1}, gy_{n+2}, gy_{n+2}) + \cdots + \Omega(gy_{m-1}, gy_m, gy_l).\end{aligned} \tag{3}$$

Making the sum of relations (2) and (3), and using inequality (1), it follows that

$$\begin{aligned}&\Omega(gx_n, gx_m, gx_l) + \Omega(gy_n, gy_m, gy_l)\\ &\quad \leq 2M\bigl(k^{n-1} + k^n + \cdots + k^{m-2}\bigr)\\ &\quad \leq 2Mk^{n-1}\frac{1}{1-k}.\end{aligned}$$

Lemma 2.1, part (3), implies that (gx_n) and (gy_n) are G-Cauchy sequences. Since gX is a complete G-subspace of X, there are gu and gv in gX such that $gx_n \to gu$ and $gy_n \to gv$.

Let $\epsilon > 0$. From the lower semi-continuity of Ω, we get

$$\Omega(gx_n, gx_m, gu) \leq \liminf_{p\to+\infty} \Omega(gx_n, gx_m, gx_p) \leq \epsilon, \quad m \geq n, \tag{4}$$

$$\Omega(gy_n, gy_m, gv) \leq \liminf_{p\to+\infty} \Omega(gy_n, gy_m, gy_p) \leq \epsilon, \quad m \geq n, \tag{5}$$

$$\Omega(gx_n, gu, gx_l) \leq \liminf_{p\to+\infty} \Omega(gx_n, gx_p, gx_l) \leq \epsilon, \quad l \geq n, \tag{6}$$

$$\Omega(gy_n, gv, gy_l) \leq \liminf_{p\to+\infty} \Omega(gy_n, gy_p, gy_l) \leq \epsilon, \quad l \geq n. \tag{7}$$

Suppose that $F(u, v) \neq gu$ or $F(v, u) \neq gv$. Applying hypotheses (3) of the theorem, and using inequalities (4)-(7), we obtain

$$\begin{aligned}0 < \inf\bigl\{&\Omega\bigl(gx_n, F(x_n, y_n), gu\bigr) + \Omega\bigl(gy_n, F(y_n, x_n), gv\bigr)\\ &+ \Omega\bigl(gx_n, gu, F(x_n, y_n)\bigr) + \Omega\bigl(gy_n, gv, F(y_n, x_n)\bigr)\bigr\} \leq 4\epsilon\end{aligned}$$

for each $\epsilon > 0$, which is a contradiction.

Therefore, $F(u, v) = gu$ and $F(v, u) = gv$.

Using the contraction condition from the hypotheses, we get

$$\begin{aligned}&\Omega\bigl(F(u, v), gx_{n+1}, gx_{n+1}\bigr) + \Omega\bigl(F(v, u), gy_{n+1}, gy_{n+1}\bigr)\\ &\quad = \Omega\bigl(F(u, v), F(x_n, y_n), F(x_n, y_n)\bigr) + \Omega\bigl(F(v, u), F(y_n, x_n), F(y_n, x_n)\bigr)\\ &\quad \leq k \max\bigl\{\Omega(gu, gx_n, gx_n) + \Omega(gv, gy_n, gy_n),\end{aligned}$$

$$\begin{aligned}&\Omega(gx_n,gu,gx_n)+\Omega(gy_n,gv,gy_n),\\&\Omega(gu,gx_{n+1},gx_n)+\Omega(gv,gy_{n+1},gy_n)\\&\Omega(gx_n,gu,gx_n)+\Omega(gy_n,gv,gy_n)\bigr\}.\end{aligned}$$

We apply repeatedly the contraction inequality, and we obtain

$$\begin{aligned}&\Omega\bigl(F(u,v),gx_{n+1},gx_{n+1}\bigr)+\Omega\bigl(F(v,u),gy_{n+1},gy_{n+1}\bigr)\\&\quad\le k^n\max\bigl\{\Omega(gu,gx_i,gx_1)+\Omega(gv,gy_i,gy_1),\\&\qquad\Omega(gx_j,gu,gx_1)+\Omega(gy_j,gv,gy_1)|1\le i\le n,1\le j\le n+1\bigr\}.\end{aligned}$$

Since X is Ω-bounded, it follows that

$$\Omega\bigl(F(u,v),gx_{n+1},gx_{n+1}\bigr)+\Omega\bigl(F(v,u),gy_{n+1},gy_{n+1}\bigr)\le 2Mk^n. \tag{8}$$

In a similar manner, it can be proved that

$$\Omega\bigl(gx_{n+1},F(u,v),F(v,u)\bigr)+\Omega\bigl(gy_{n+1},F(v,u),F(u,v)\bigr)\le 2Mk^n. \tag{9}$$

Taking into account (8), (9) and the first statement of Lemma 2.1, we get $gu=gv$.

We will prove now the uniqueness of the coupled coincidence point of F and g.

Suppose (u,v) and (u^*,v^*) are coupled coincidence points of F and g. Using the contraction condition, we obtain

$$\Omega(gu,gu,gu)+\Omega(gv,gv,gv)\le k\bigl(\Omega(gu,gu,gu)+\Omega(gv,gv,gv)\bigr),$$

hence $\Omega(gu,gu,gu)=\Omega(gv,gv,gv)=0$.

On the other hand,

$$\begin{aligned}&\Omega\bigl(gu^*,gu,gu\bigr)+\Omega\bigl(gv^*,gv,gv\bigr)\\&\quad\le k\max\bigl\{\Omega\bigl(gu^*,gu,gu\bigr)+\Omega\bigl(gv^*,gv,gv\bigr),\Omega\bigl(gu,gu^*,gu\bigr)+\Omega\bigl(gv,gv^*,gv\bigr)\bigr\}\end{aligned} \tag{10}$$

and

$$\begin{aligned}&\Omega\bigl(gu,gu^*,gu\bigr)+\Omega\bigl(gv,gv^*,gv\bigr)\\&\quad\le k\max\bigl\{\Omega\bigl(gu^*,gu,gu\bigr)+\Omega\bigl(gv^*,gv,gv\bigr),\Omega\bigl(gu,gu^*,gu\bigr)+\Omega\bigl(gv,gv^*,gv\bigr)\bigr\}.\end{aligned} \tag{11}$$

Relations (10) and (11) imply that $\Omega(gu^*,gu,gu)=\Omega(gu,gu^*,gu)=0$ and also $\Omega(gv^*,gv,gv)=\Omega(gv,gv^*,gv)=0$. Lemma 2.1 imposes that $gu=gu^*$ and $gv=gv^*$, and the uniqueness is proved. □

If we take $g=Id_X$ in Theorem 3.1, we easily get the following.

Corollary 3.1 *Let (X,G) be a complete G-metric space, and let Ω be an Ω-distance on X such that X is Ω-bounded. Suppose $F\colon X\times X\to X$ is a mapping for which there exists*

$k \in [0,1)$ such that for each x, y, z, x^*, y^* and z^* in X

$$\begin{aligned}
&\Omega\bigl(F(x,y),F\bigl(x^*,y^*\bigr),F\bigl(z,z^*\bigr)\bigr) + \Omega\bigl(F(y,x),F\bigl(y^*,x^*\bigr),F\bigl(z^*,z\bigr)\bigr) \\
&\quad \le k\max\bigl\{\Omega\bigl(x,x^*,z\bigr) + \Omega\bigl(y,y^*,z^*\bigr), \Omega\bigl(x^*,x,z\bigr) + \Omega\bigl(y^*,y,z^*\bigr), \\
&\qquad \Omega\bigl(x,F\bigl(x^*,y^*\bigr),z\bigr) + \Omega\bigl(y,F\bigl(y^*,x^*\bigr),z^*\bigr), \\
&\qquad \Omega\bigl(F(x,y),x^*,z\bigr) + \Omega\bigl(F(y,x),y^*,z^*\bigr), \\
&\qquad \Omega\bigl(x^*,F(x,y),z\bigr) + \Omega\bigl(y^*,F(y,x),z^*\bigr), \\
&\qquad \Omega\bigl(F(x,y),F\bigl(x^*,y^*\bigr),z\bigr) + \Omega\bigl(F(y,x),F\bigl(y^*,x^*\bigr),z^*\bigr)\bigr\}.
\end{aligned}$$

Consider also that if $F(u,v) \neq u$ or $F(v,u) \neq v$, then

$$\begin{aligned}
&\inf\bigl\{\Omega\bigl(x,F(x,y),u\bigr) + \Omega\bigl(y,F(y,x),v\bigr) \\
&\quad + \Omega\bigl(F(x,y),u,x\bigr) + \Omega\bigl(F(y,x),v,y\bigr)\bigr\} > 0.
\end{aligned}$$

Then, F has a unique coupled fixed point (u,v). Moreover, $F(u,v) = u = v = F(v,u)$.

Corollary 3.2 *Let (X,G) be a G-metric space, and let Ω be an Ω-distance on X such that X is Ω-bounded. $g\colon X \to X$ and $F\colon X \times X \to X$ are mappings. Suppose that there exists $k_1,k_2,k_3,k_4,k_5,k_6 \in [0,1)$ with $k_1 + k_2 + k_3 + k_4 + k_5 + k_6 < 1$ such that for each x, y, z, x^*, y^* and z^* in X*

$$\begin{aligned}
&\Omega\bigl(F(x,y),F\bigl(x^*,y^*\bigr),F\bigl(z,z^*\bigr)\bigr) + \Omega\bigl(F(y,x),F\bigl(y^*,x^*\bigr),F\bigl(z^*,z\bigr)\bigr) \\
&\quad \le k_1\bigl(\Omega\bigl(gx,gx^*,gz\bigr) + \Omega\bigl(gy,gy^*,gz^*\bigr)\bigr) \\
&\qquad + k_2\bigl(\Omega\bigl(gx^*,gx,gz\bigr) + \Omega\bigl(gy^*,gy,gz^*\bigr)\bigr) \\
&\qquad + k_3\bigl(\Omega\bigl(gx,F\bigl(x^*,y^*\bigr),gz\bigr) + \Omega\bigl(gy,F\bigl(y^*,x^*\bigr),gz^*\bigr)\bigr) \\
&\qquad + k_4\bigl(\Omega\bigl(F(x,y),gx^*,gz\bigr) + \Omega\bigl(F(y,x),gy^*,gz^*\bigr)\bigr) \\
&\qquad + k_5\bigl(\Omega\bigl(gx^*,F(x,y),gz\bigr) + \Omega\bigl(gy^*,F(y,x),gz^*\bigr)\bigr) \\
&\qquad + k_6\bigl(\Omega\bigl(F(x,y),F\bigl(x^*,y^*\bigr),gz\bigr) + \Omega\bigl(F(y,x),F\bigl(y^*,x^*\bigr),gz^*\bigr)\bigr).
\end{aligned}$$

Consider also that the following conditions hold true:

(1) *$F(X \times X) \subseteq gX$;*

(2) *gX is a complete subspace of X with respect to the topology induced by G;*

(3) *If $F(u,v) \neq gu$ or $F(v,u) \neq gv$, then*

$$\begin{aligned}
&\inf\bigl\{\Omega\bigl(gx,F(x,y),gu\bigr) + \Omega\bigl(gy,F(y,x),gv\bigr) \\
&\quad + \Omega\bigl(gx,gu,F(x,y)\bigr) + \Omega\bigl(gy,gv,F(y,x)\bigr)\bigr\} > 0.
\end{aligned}$$

Then, F and g have a unique coupled coincidence point (u,v). Moreover, $F(u,v) = gu = gv = F(v,u)$.

Proof Follows from Theorem 3.1 by noting that

$$\begin{aligned}
&k_1\Omega\big(gx,gx^*,gz\big)+\Omega\big(gy,gy^*,gz^*\big)\\
&\quad+k_2\Omega\big(gx^*,gx,gz\big)+\Omega\big(gy^*,gy,gz^*\big)\\
&\quad+k_3\Omega\big(gx,F\big(x^*,y^*\big),gz\big)+\Omega\big(gy,F\big(y^*,x^*\big),gz^*\big)\\
&\quad+k_4\Omega\big(F(x,y),gx^*,gz\big)+\Omega\big(F(y,x),gy^*,gz^*\big)\\
&\quad+k_5\Omega\big(gx^*,F(x,y),gz\big)+\Omega\big(gy^*,F(y,x),gz^*\big),\\
&\quad+k_6\Omega\big(F(x,y),F\big(x^*,y^*\big),gz\big)+\Omega\big(F(y,x),F\big(y^*,x^*\big),gz^*\big)\\
&\le k\max\big\{\Omega\big(gx,gx^*,gz\big)+\Omega\big(gy,gy^*,gz^*\big),\\
&\quad\Omega\big(gx^*,gx,gz\big)+\Omega\big(gy^*,gy,gz^*\big),\\
&\quad\Omega\big(gx,F\big(x^*,y^*\big),gz\big)+\Omega\big(gy,F\big(y^*,x^*\big),gz^*\big),\\
&\quad\Omega\big(F(x,y),gx^*,gz\big)+\Omega\big(F(y,x),gy^*,gz^*\big),\\
&\quad\Omega\big(gx^*,F(x,y),gz\big)+\Omega\big(gy^*,F(y,x),gz^*\big),\\
&\quad\Omega\big(F(x,y),F\big(x^*,y^*\big),gz\big)+\Omega\big(F(y,x),F\big(y^*,x^*\big),gz^*\big)\big\}.
\end{aligned}$$

□

If we take $g = Id_X$ in Corollary 3.2, we easily get the following.

Corollary 3.3 *Let (X,G) be a complete G-metric space, and let Ω be an Ω-distance on X such that X is Ω-bounded. Suppose $F\colon X\times X\to X$ is a mapping, for which there exists $k_1,k_2,k_3,k_4,k_5,k_6\in[0,1)$ with $k_1+k_2+k_3+k_4+k_5+k_6<1$ such that for each x, y, z, x^*, y^* and z^* in X*

$$\begin{aligned}
&\Omega\big(F(x,y),F\big(x^*,y^*\big),F\big(z,z^*\big)\big)+\Omega\big(F(y,x),F\big(y^*,x^*\big),F\big(z^*,z\big)\big)\\
&\le k_1\big(\Omega\big(x,x^*,z\big)+\Omega\big(y,y^*,z^*\big)\big)\\
&\quad+k_2\big(\Omega\big(x^*,x,z\big)+\Omega\big(y^*,y,z^*\big)\big)\\
&\quad+k_3\big(\Omega\big(x,F\big(x^*,y^*\big),z\big)+\Omega\big(y,F\big(y^*,x^*\big),z^*\big)\big)\\
&\quad+k_4\big(\Omega\big(F(x,y),x^*,z\big)+\Omega\big(F(y,x),y^*,z^*\big)\big)\\
&\quad+k_5\big(\Omega\big(x^*,F(x,y),z\big)+\Omega\big(y^*,F(y,x),z^*\big)\big)\\
&\quad+k_6\big(\Omega\big(F(x,y),F\big(x^*,y^*\big),z\big)+\Omega\big(F(y,x),F\big(y^*,x^*\big),z^*\big)\big).
\end{aligned}$$

Consider also that if $F(u,v)\neq u$ or $F(v,u)\neq v$, then

$$\begin{aligned}
&\inf\big\{\Omega\big(x,F(x,y),u\big)+\Omega\big(y,F(y,x),v\big)\\
&\quad+\Omega\big(F(x,y),u,x\big)+\Omega\big(F(y,x),v,y\big)\big\}>0.
\end{aligned}$$

Then, F has a unique coupled fixed point (u,v). Moreover, $F(u,v)=u=v=F(v,u)$.

By modifying the contraction condition, we get the following theorem.

Theorem 3.2 *Let (X, G) be a G-metric space, and let Ω be an Ω-distance on X such that X is Ω-bounded. $g\colon X \to X$ and $F\colon X \times X \to X$ are mappings. Suppose that there exist $k_1, k_2 \in [0,1)$ with $k_1 + k_2 < 1$ such that for each x, y, z, x^*, y^* and z^* in X*

$$\begin{aligned}
&\Omega\bigl(F(x,y), gx^*, F\bigl(z,z^*\bigr)\bigr) + \Omega\bigl(F(y,x), gy^*, F\bigl(z^*,z\bigr)\bigr) \\
&\quad \le k_1 \max\bigl\{\Omega\bigl(gx, gx^*, gz\bigr) + \Omega\bigl(gy, gy^*, gz^*\bigr), \\
&\qquad \Omega\bigl(gx^*, gx, gz\bigr) + \Omega\bigl(gy^*, gy, gz^*\bigr), \\
&\qquad \Omega\bigl(F(x,y), gx^*, gz\bigr) + \Omega\bigl(F(y,x), gy^*, gz^*\bigr), \\
&\qquad \Omega\bigl(gx^*, F(x,y), gz\bigr) + \Omega\bigl(gy^*, F(y,x), gz^*\bigr)\bigr\} \\
&\qquad + k_2\bigl(\Omega\bigl(F(x,y), F\bigl(x^*,y^*\bigr), F\bigl(z,z^*\bigr)\bigr) + \Omega\bigl(F(y,x), F\bigl(y^*,x^*\bigr), F\bigl(z^*,z\bigr)\bigr)\bigr),
\end{aligned}$$

and the conditions (1)-(3) *from Theorem* 3.1 *hold.*

Then, F and g have a unique coupled coincidence point (u, v). Moreover, $F(u, v) = gu = gv = F(v, u)$.

Proof Let x_0 and y_0 be elements of X. Since $F(X \times X) \subseteq gX$, there exist x_1 and y_1 in X such that $gx_1 = F(x_0, y_0)$ and $y_1 = F(y_0, x_0)$. Repeating this procedure, we obtain two sequences, (x_n) and (y_n), with the properties

$$gx_{n+1} = F(x_n, y_n), \qquad gy_{n+1} = F(y_n, x_n).$$

The contraction condition implies that

$$\begin{aligned}
&\Omega(gx_n, gx_{n+1}, gx_{n+s}) + \Omega(gy_n, gy_{n+1}, gy_{n+s}) \\
&\quad = \Omega\bigl(F(x_{n-1}, y_{n-1}), gx_{n+1}, F(x_{n+s-1}, y_{n+s-1})\bigr) \\
&\qquad + \Omega\bigl(F(y_{n-1}, x_{n-1}), gy_{n+1}, F(y_{n+s-1}, x_{n+s-1})\bigr) \\
&\quad \le k_1 \max\bigl\{\Omega(gx_{n-1}, gx_{n+1}, gx_{n+s-1}) + \Omega(gy_{n-1}, gy_{n+1}, gy_{n+s-1}), \\
&\qquad \Omega(gx_{n+1}, gx_{n-1}, gx_{n+s-1}) + \Omega(gy_{n+1}, gy_{n-1}, gy_{n+s-1}), \\
&\qquad \Omega(gx_n, gx_{n+1}, gx_{n+s-1}) + \Omega(gy_n, gy_{n+1}, gy_{n+s-1}), \\
&\qquad \Omega(gx_{n+1}, gx_n, gx_{n+s-1}) + \Omega(gy_{n+1}, gy_n, gy_{n+s-1})\bigr\} \\
&\qquad + k_2\bigl(\Omega(gx_n, gx_{n+1}, gx_{n+s}) + \Omega(gy_n, gy_{n+1}, gy_{n+s})\bigr),
\end{aligned}$$

which leads us to

$$\begin{aligned}
&\Omega(gx_n, gx_{n+1}, gx_{n+s}) + \Omega(gy_n, gy_{n+1}, gy_{n+s}) \\
&\quad \le k \max\bigl\{\Omega(gx_{n-1}, gx_{n+1}, gx_{n+s-1}) + \Omega(gy_{n-1}, gy_{n+1}, gy_{n+s-1}), \\
&\qquad \Omega(gx_{n+1}, gx_{n-1}, gx_{n+s-1}) + \Omega(gy_{n+1}, gy_{n-1}, gy_{n+s-1}), \\
&\qquad \Omega(gx_n, gx_{n+1}, gx_{n+s-1}) + \Omega(gy_n, gy_{n+1}, gy_{n+s-1}), \\
&\qquad \Omega(gx_{n+1}, gx_n, gx_{n+s-1}) + \Omega(gy_{n+1}, gy_n, gy_{n+s-1})\bigr\},
\end{aligned}$$

where $k = \frac{k_1}{1-k_2} < 1$.

Following the same steps, as we did in Theorem 3.1, the conclusion is straightforward. □

Theorem 3.2 leads us to a coupled fixed point property, by considering $g = Id_X$.

Corollary 3.4 *Let (X, G) be a complete G-metric space, and let Ω be an Ω-distance on X such that X is Ω-bounded. Suppose that $F\colon X \times X \to X$ is a mapping, for which there exist $k_1, k_2 \in [0,1)$ with $k_1 + k_2 < 1$ such that for each x, y, z, x^*, y^* and z^* in X*

$$\begin{aligned}
&\Omega\bigl(F(x,y), x^*, F(z,z^*)\bigr) + \Omega\bigl(F(y,x), y^*, F(z^*,z)\bigr) \\
&\quad \le k_1 \max\bigl\{\Omega\bigl(x, x^*, z\bigr) + \Omega\bigl(y, y^*, z^*\bigr), \\
&\qquad \Omega\bigl(x^*, x, z\bigr) + \Omega\bigl(y^*, y, z^*\bigr), \\
&\qquad \Omega\bigl(F(x,y), x^*, z\bigr) + \Omega\bigl(F(y,x), y^*, z^*\bigr), \\
&\qquad \Omega\bigl(x^*, F(x,y), z\bigr) + \Omega\bigl(y^*, F(y,x), z^*\bigr)\bigr\} \\
&\qquad + k_2\bigl(\Omega\bigl(F(x,y), F\bigl(x^*, y^*\bigr), F\bigl(z, z^*\bigr)\bigr) + \Omega\bigl(F(y,x), F\bigl(y^*, x^*\bigr), F\bigl(z^*, z\bigr)\bigr)\bigr),
\end{aligned}$$

and if $F(u,v) \neq u$ or $F(v,u) \neq v$, then

$$\begin{aligned}
&\inf\bigl\{\Omega\bigl(x, F(x,y), u\bigr) + \Omega\bigl(y, F(y,x), v\bigr) \\
&\quad + \Omega\bigl(x, u, F(x,y)\bigr) + \Omega\bigl(y, v, F(y,x)\bigr)\bigr\} > 0.
\end{aligned}$$

Then, F has a coupled fixed point (u,v). Moreover, $F(u,v) = u = v = F(v,u)$.

Theorem 3.3 *Let (X, G) be a G-metric space, and let Ω be an Ω-distance on X. Consider $F\colon X \times X \to X$, $g\colon X \to X$ and $\phi\colon gX \to \mathbb{R}_+$ such that*

$$\begin{aligned}
&\Omega\bigl(gx, F(x,y), F(z,z^*)\bigr) + \Omega\bigl(gy, F(y,x), F(z^*,z)\bigr) \\
&\quad \le \phi(gx) + \phi(gy) + \phi(gz) + \phi\bigl(gz^*\bigr) \\
&\qquad - \phi\bigl(F(x,y)\bigr) - \phi\bigl(F(y,x)\bigr) - \phi\bigl(F\bigl(z,z^*\bigr)\bigr) - \phi\bigl(F\bigl(z^*,z\bigr)\bigr)
\end{aligned}$$

for all $x, y, z, z^* \in X$. Suppose that the following conditions are fulfilled:

(1) *$F(X \times X) \subseteq gX$.*
(2) *gX is a complete subspace of X with respect to the topology, induced by G.*
(3) *There exists $k > 0$ such that $\Omega(x,x,y) \le k\Omega(x,y,y)$ holds for all $x, y \in X$.*
(4) *If $F(u,v) \neq gu$ or $F(v,u) \neq gv$, then*

$$\begin{aligned}
&\inf\bigl\{\Omega\bigl(gx, F(x,y), gu\bigr) + \Omega\bigl(gy, F(y,x), gv\bigr) \\
&\quad + \Omega\bigl(gx, gu, F(x,y)\bigr) + \Omega\bigl(gy, gv, F(y,x)\bigr)\bigr\} > 0.
\end{aligned}$$

Then F and g have a coupled coincidence point (u,v).

Proof Consider (x_0, y_0) a pair from $X \times X$. As $F(X \times X) \subseteq gX$, there exist $(x_1, y_1) \in X \times X$ so that $gx_1 = F(x_0, y_0)$, $gy_1 = F(y_0, x_0)$.

We continue the process, and we obtain two sequences (x_n), (y_n) from X, having the properties that

$$gx_{n+1} = F(x_n, y_n), \qquad gy_{n+1} = F(y_n, x_n).$$

Using the contraction condition, we get

$$\begin{aligned}
&\Omega(gx_n, gx_{n+1}, gx_{n+1}) + \Omega(gy_n, gy_{n+1}, gy_{n+1}) \\
&\quad = \Omega\bigl(gx_n, F(x_n, y_n), F(x_n, y_n)\bigr) + \Omega(gy_n, F(y_n, x_n), F(y_n, x_n) \\
&\quad \leq 2\Omega(gx_n) + 2\Omega(gy_n) - 2\Omega(gx_{n+1}) - 2\Omega(gy_{n+1}).
\end{aligned} \tag{12}$$

For $m > n$, the first part of the definition Ω-distance and (12) yields

$$\begin{aligned}
&\Omega(gx_n, gx_m, gx_m) + \Omega(gy_n, gy_m, gy_m) \\
&\quad \leq \sum_{k=n}^{m-1} \bigl[\Omega(gx_k, gx_{k+1}, gx_{k+1}) + \Omega(gy_k, gy_{k+1}, gy_{k+1})\bigr].
\end{aligned} \tag{13}$$

Let

$$S_n = \sum_{k=0}^{n} \bigl[\Omega(gx_k, gx_{k+1}, gx_{k+1}) + \Omega(gy_k, gy_{k+1}, gy_{k+1})\bigr].$$

According to (12),

$$S_n \leq 2\phi(gx_0) + 2\phi(gy_0) - 2\phi(gx_{n+1}) - 2\phi(gy_{n+1}) \leq 2\phi(gx_0) + 2\phi(gy_0).$$

Thus, (S_n) is an increasing bounded sequence, so

$$\lim_{n\to+\infty} s_n = \sum_{n=0}^{\infty} \bigl[\Omega(gx_k, gx_{k+1}, gx_{k+1}) + \Omega(gy_k, gy_{k+1}, gy_{k+1})\bigr]$$

exists.

Now, we shall show that (gx_n) and (gy_n) are G-Cauchy sequences in gX. Consider $\epsilon > 0$. By part (3) of the definition of an Ω-distance, we choose $\delta > 0$ such that if $\Omega(x, a, a) < \delta$ and $\Omega(x, y, z) < \delta$, then $G(x, y, z) < \epsilon$. Let $\eta = \min\{\delta, \frac{\delta}{k}\}$.

Using the fact that

$$\sum_{n=0}^{+\infty} \bigl[\Omega(gx_k, gx_{k+1}, gx_{k+1}) + \Omega(gy_k, gy_{k+1}, gy_{k+1})\bigr] < +\infty$$

and letting $n \to +\infty$ in (13), we choose $n_0 \in \mathbb{N}$ such that

$$\Omega(gx_n, gx_m, gx_m) + \Omega(gy_n, gy_m, gy_m) < \eta \leq \delta$$

for all $m > n \geq n_0$.

Thus,

$$\Omega(gx_n, gx_m, gx_m) < \delta$$

and

$$\Omega(gy_n, gy_m, gy_m) < \delta$$

for all $m > n \geq n_0$. Also we have

$$\Omega(gx_m, gx_m, gx_l) + \Omega(gy_m, gy_m, gy_l) \leq k\big[\Omega(gx_m, gx_m, gx_l) + \Omega(gy_m, gy_m, gy_l)\big] < k\eta \leq \delta$$

for all $l > m \geq n_0$. Thus,

$$\Omega(gx_m, gx_m, gx_l) < \delta$$

and

$$\Omega(gy_m, gy_m, gy_l) < \delta$$

for all $m > n \geq n_0$. Thus, by part (3) of the definition of Ω-distance, we have

$$G(gx_n, gx_m, gx_l) < \epsilon$$

and

$$G(gy_n, gy_m, gy_l) < \epsilon$$

for $l > m > n \geq n_0$.

Therefore, (gx_n) and (gy_n) are G-Cauchy sequences. As gX is G-complete, it follows that there are $u, v \in X$ so that $\lim_{n\to+\infty} gx_n = gu$ and $\lim_{n\to+\infty} gv_n = gv$.

Since Ω is lower semi-continuous in its second and third variable, we obtain, for $\epsilon > 0$

$$\Omega(gx_n, gx_m, gu) \leq \liminf_{p\to+\infty} \Omega(gx_n, gx_m, gx_p) \leq \epsilon, \quad m \geq n, \tag{14}$$

$$\Omega(gy_n, gy_m, gv) \leq \liminf_{p\to+\infty} \Omega(gy_n, gy_m, gy_p) \leq \epsilon, \quad m \geq n, \tag{15}$$

$$\Omega(gx_n, gu, gx_l) \leq \liminf_{p\to+\infty} \Omega(gx_n, gx_p, gx_l) \leq \epsilon, \quad l \geq n, \tag{16}$$

$$\Omega(gy_n, gv, gy_l) \leq \liminf_{p\to+\infty} \Omega(gy_n, gy_p, gy_l) \leq \epsilon, \quad l \geq n. \tag{17}$$

We make the sum of inequalities (14), (15), (16) and (17). It follows that

$$\begin{aligned} 0 < \inf\big\{&\Omega\big(gx_n, F(x_n, y_n), gu\big) + \Omega\big(gy_n, F(y_n, x_n), gv\big) \\ &+ \Omega\big(gx_n, gu, F(x_n, y_n)\big) + \Omega\big(gy_n, gv, F(y_n, x_n)\big)\big\} \leq 4\epsilon, \end{aligned}$$

for each $\epsilon > 0$, which contradicts the hypothesis.

Hence, $F(u,v) = gu$ and $F(v,u) = gv$, that is, (u,v) is a coupled coincidence point of F and g. □

By considering $g = Id_X$, we get the following corrolary.

Corollary 3.5 *Let (X,G) be a complete G-metric space, and let Ω be an Ω-distance on X. Consider $F\colon X \times X \to X$ and $\phi\colon X \to \mathbb{R}_+$ such that*

$$\begin{aligned}&\Omega\bigl(x,F(x,y),F\bigl(z,z^*\bigr)\bigr) + \Omega\bigl(y,F(y,x),F\bigl(z^*,z\bigr)\bigr)\\ &\quad\le \phi(x) + \phi(y) + \phi(z) + \phi\bigl(z^*\bigr)\\ &\qquad - \phi\bigl(F(x,y)\bigr) - \phi\bigl(F(y,x)\bigr) - \phi\bigl(F\bigl(z,z^*\bigr)\bigr) - \phi\bigl(F\bigl(z^*,z\bigr)\bigr)\end{aligned}$$

for all $x,y,z,z^* \in X$. Suppose that the following conditions are fulfilled:

(1) *There exists $k > 0$ such that $\Omega(x,x,y) \le k\Omega(x,y,y)$ holds for all $x,y \in X$.*
(2) *If $F(u,v) \ne u$ or $F(v,u) \ne v$, then*

$$\begin{aligned}&\inf\bigl\{\Omega\bigl(x,F(x,y),u\bigr) + \Omega\bigl(y,F(y,x),v\bigr)\\ &\quad + \Omega\bigl(x,u,F(x,y)\bigr) + \Omega\bigl(y,v,F(y,x)\bigr)\bigr\} > 0.\end{aligned}$$

Then F has coupled fixed point (u,v).

Now, we introduce the following example to support the useability of our result.

Example 3.1 Let $X = [0,1]$. Define

$$G\colon X \times X \times X \to \mathbb{R}^+, \qquad G(x,y,z) = |x-y| + |x-z| + |y-z|$$

and

$$\Omega\colon X \times X \times X \to \mathbb{R}^+, \qquad \Omega(x,y,z) = |x-y| + |x-z|.$$

Also define

$$F\colon X\times X \to X, \quad F(x,y) = \frac{1}{2}x; \qquad g\colon X \to X, \quad gx = x; \qquad \phi\colon X \to \mathbb{R}^+, \quad \phi(x) = 4x.$$

Then,

(1) (X,G) is a complete G-metric space.
(2) Ω is Ω-distance.
(3) $\Omega(x,x,y) \le 2\Omega(x,y,y)$ for all $x,y \in X$.
(4) $F(X \times X) \subseteq gX$.
(5) For $x,y,z,z^* \in X$ we have

$$\begin{aligned}&\Omega\bigl(x,F(x,y),F\bigl(z,z^*\bigr)\bigr) + \Omega\bigl(y,F(y,x),F\bigl(z^*,z\bigr)\bigr)\\ &\quad\le \phi(x) + \phi(y) + \phi(z) + \phi\bigl(z^*\bigr)\\ &\qquad - \phi\bigl(F(x,y)\bigr) - \phi\bigl(F(y,x)\bigr) - \phi\bigl(F\bigl(z,z^*\bigr)\bigr) - \phi\bigl(F\bigl(z^*,z\bigr)\bigr).\end{aligned}$$

(6) If $F(u,v) \neq u$ or $F(v,u) \neq v$, then

$$\begin{aligned}&\inf\bigl\{\Omega\bigl(x,F(x,y),u\bigr)+\Omega\bigl(y,F(y,x),v\bigr)\\&\quad+\Omega\bigl(x,u,F(x,y)\bigr)+\Omega\bigl(y,v,F(y,x)\bigr)\bigr\}>0.\end{aligned}$$

Proof The proof of (1), (2), (3) and (4) is clear. To prove (5) given $x,y,z,z^* \in X$.

$$\begin{aligned}&\Omega\bigl(x,F(x,y),F\bigl(z,z^*\bigr)\bigr)+\Omega\bigl(y,F(y,x),F\bigl(z^*,z\bigr)\bigr)\\&\quad=\Omega\left(x,\frac{1}{2}x,\frac{1}{2}z\right)+\Omega\left(y,\frac{1}{2}y,\frac{1}{2}z^*\right)\\&\quad=\frac{1}{2}x+\left|x-\frac{1}{2}z\right|+\frac{1}{2}y+\left|y-\frac{1}{2}z^*\right|\\&\quad\leq\frac{3}{2}x+\frac{1}{2}z+\frac{3}{2}y+\frac{1}{2}z^*\\&\quad\leq 2x+2y+2z+2z^*\\&\quad=\phi(x)+\phi(y)+\phi(z)+\phi\bigl(z^*\bigr)\\&\qquad-\phi\bigl(F(x,y)\bigr)-\phi\bigl(F(y,x)\bigr)-\phi\bigl(F\bigl(z,z^*\bigr)\bigr)-\phi\bigl(F\bigl(z^*,z\bigr)\bigr).\end{aligned}$$

To prove (6), let $F(u,v) \neq u$ or $F(v,u) \neq v$. Then $u \neq 0$ or $v \neq 0$. Thus,

$$\begin{aligned}&\inf\bigl\{\Omega\bigl(x,F(x,y),u\bigr)+\Omega\bigl(y,F(y,x),v\bigr)\\&\qquad+\Omega\bigl(x,u,F(x,y)\bigr)+\Omega\bigl(y,v,F(y,x)\bigr):x,y\in X\bigr\}\\&\quad=\inf\left\{\Omega\left(x,\frac{1}{2}x,u\right)+\Omega\left(y,\frac{1}{2}y,v\right)\right.\\&\qquad\left.+\Omega\left(x,u,\frac{1}{2}x\right)+\Omega\left(y,v,\frac{1}{2}y\right):x,y\in X\right\}\\&\quad=\inf\bigl\{x+2|x-u|+y+2|y-v|:x,y\in X\bigr\}\\&\quad=\inf\bigl\{x+2|x-u|:x\in X\bigr\}+\inf\bigl\{y+2|y-v|:y\in X\bigr\}\\&\quad\geq u+v>0.\end{aligned}$$

So, F and g satisfy all the hypotheses of Corollary 3.5. Hence the mappings F and g have a coupled coincidence point, Here $(0,0)$ is the coupled coincidence point of F and g. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors contributed equally and significantly in writing this article. Both authors read and approved the final manuscript.

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