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# An algorithm for a common fixed point of a family of pseudocontractive mappings

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## Abstract

We introduce an iterative process which converges strongly to a common fixed point of a family of Lipschitzian pseudocontractive mappings. Our theorems improve and unify most of the results that have been proved for this important class of nonlinear operators.

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**Keywords:** fixed points of mappings; monotone mappings; pseudocontractive mappings; strong convergence

## 1 Introduction

Let  $C$  be a nonempty subset of a real Hilbert space  $H$ . A mapping  $T : C \rightarrow H$  is called *pseudocontractive* if and only if

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + \|(I - T)x - (I - T)y\|^2, \quad \text{for all } x, y \in C, \quad (1.1)$$

and  $T$  is called  $\alpha$ -*strictly pseudocontractive* [1] if and only if there exists  $\alpha \in [0, 1)$  such that

$$\|Tx - Ty\|^2 \leq \|x - y\|^2 + \alpha \|(I - T)x - (I - T)y\|^2, \quad \text{for all } x, y \in C, \quad (1.2)$$

$T$  is called *Lipschitzian* if and only if

$$\|Tx - Ty\| \leq L\|x - y\|, \quad \text{for all } x, y \in K. \quad (1.3)$$

If in (1.3) we have  $L \leq 1$ , then  $T$  is called *nonexpansive*. We note that inequalities (1.1) and (1.2) can be equivalently written as

$$\langle x - y, Tx - Ty \rangle \leq \|x - y\|^2, \quad \text{for all } x, y \in C \quad (1.4)$$

and

$$\langle x - y, Tx - Ty \rangle \leq \|x - y\|^2 - \lambda \|(I - T)x - (I - T)y\|^2, \quad (1.5)$$

for some  $\lambda > 0$ , respectively.

We observe from (1.1), (1.2) and (1.3) that every nonexpansive mapping is an  $\alpha$ -strictly pseudocontractive mapping and every  $\alpha$ -strictly pseudocontractive mapping is a pseudocontractive mapping, and hence a class of pseudocontractive mappings is a more general class of mappings.

Furthermore, pseudocontractive mappings are related with the important class of nonlinear *monotone* mappings, where a mapping  $A$  with domain  $D(A)$  and range  $R(A)$  in  $H$  is called *monotone* if the inequality

$$\langle x - y, Ax - Ay \rangle \geq 0, \tag{1.6}$$

holds for every  $x, y \in D(A)$ . We note that  $T$  is pseudocontractive if and only if  $A := I - T$  is monotone, and hence a fixed point of  $T$ ,  $F(T) := \{x \in D(T) : Tx = x\}$ , is a zero of  $A$ ,  $N(A) := \{x \in D(A) : Ax = 0\}$ . It is now well known (see, e.g., [2]) that if  $A$  is monotone, then the solutions of the equation  $Ax = 0$  correspond to the equilibrium points of some evolution systems.

Finding a point in the intersection of fixed point sets of a family of nonexpansive mappings is a task that occurs frequently in various areas of mathematical sciences and engineering. For example, the well-known convex feasibility problem reduces to finding a point in the intersection of fixed point sets of a family of nonexpansive mappings; see, e.g., [3, 4].

Consequently, considerable research efforts have been devoted to developing iterative methods for approximating a common fixed point (when it exists) for a family of nonexpansive mappings and  $\alpha$ -strictly pseudocontractive mappings. Bauschke [5] was the first to introduce a Halpern-type iterative process (see, e.g., [6]) for approximating a common fixed point for a finite family of  $N$  nonexpansive self-mappings. He proved the following theorem.

**Theorem BSK** (Bauschke [5], Theorem 3.1) *Let  $K$  be a nonempty closed convex subset of a Hilbert space  $H$ , and let  $T_1, T_2, \dots, T_N$  be a finite family of nonexpansive mappings of  $K$  into itself with  $F := \bigcap_{i=1}^N F(T_i) \neq \emptyset$  and  $F = F(T_N T_{N-1} \cdots T_1) = F(T_1 T_N \cdots T_2) = F(T_{N-1} T_{N-2} \cdots T_1 T_N)$ . Given points  $u, x_0 \in C$ , let  $\{x_n\}$  be generated by*

$$x_{n+1} := \alpha_{n+1}u + (1 - \alpha_{n+1})T_{n+1}x_n, \quad n \geq 0, \tag{1.7}$$

where  $T_n := T_{n(\text{mod } N)}$  and  $\{\alpha_n\} \subset (0, 1)$  satisfies  $\sum_{n \geq 1} |\alpha_{n+r} - \alpha_n| < \infty$ . Then  $\{x_n\}$  converges strongly to  $P_F u$ , where  $P_F : H \rightarrow F$  is the metric projection.

Various authors have studied iterative schemes similar to that of Theorem BSK in more general Banach spaces on the one hand, and using various conditions on the sequence  $\{\alpha_n\}$  on the other hand (see, for example, Colao *et al.* [7], Yao [8], Takahashi and Takahashi [9], Plubtieng and Punpaeng [10], Ceng *et al.* [11] and the references therein). Many authors have also studied iterative methods for a family of  $\alpha$ -strictly pseudocontractive mappings (see, e.g., [12–15] and the references therein).

Our concern now is the following: *Can we construct an iterative sequence for a common fixed point of a finite family of pseudocontractive mappings?*

In 2008, Zhou [16] studied weak convergence of an implicit scheme to a common fixed point of a finite family of pseudocontractive mappings in Banach spaces more general than

Hilbert spaces. Let  $C$  be a closed convex subset of  $E$ , and let  $\{T_i\}_{i=1}^N$  be a finite family of Lipschitzian pseudocontractive self-mappings of  $C$  subset of  $E$  which is a real uniformly convex Banach space with a Fréchet differentiable norm and  $F := \bigcap_{i=1}^N F(T_i) \neq \emptyset$ . Let  $\{x_n\}$  be defined by

$$x_n = \alpha_n x_{n-1} + (1 - \alpha_n) T_n x_n, \quad n \geq 1, \tag{1.8}$$

where  $T_n = T_{n(\text{mod } N)}$ . He proved that  $\{x_n\}$  converges weakly to a common fixed point of the family  $\{T_i\}_{i=1}^N$  under certain conditions on the parameter  $\{\alpha_n\}$ .

We remark that the scheme in the theorem of Zhou [16] is *implicit* and the convergence is *weak convergence*.

Recently, Zegeye *et al.* [17] proved the following strong convergence of Ishikawa iterative process [18] for a common fixed point of a finite family of Lipschitz pseudocontractive mappings.

**Theorem ZSA** [17] *Let  $C$  be a nonempty, closed and convex subset of a real Hilbert space  $H$ . Let  $T_i : C \rightarrow C$ ,  $i = 1, 2, \dots, N$ , be a finite family of Lipschitz pseudocontractive mappings with Lipschitzian constants  $L_i$ , for  $i = 1, 2, \dots, N$ , respectively. Assume that the interior of  $F := \bigcap_{i=1}^N F(T_i)$  is nonempty. Let  $\{x_n\}$  be a sequence generated from an arbitrary  $x_0 \in E$  by*

$$\begin{cases} y_n = (1 - \beta_n)x_n + \beta_n T_n x_n, \\ x_{n+1} = (1 - \alpha_n)x_n + \alpha_n T_n y_n, \end{cases} \quad n \geq 1, \tag{1.9}$$

where  $T_n := T_{n(\text{mod } N)}$  and  $\{\alpha_n\}, \{\beta_n\} \subset (0, 1)$  satisfy certain appropriate conditions. Then  $\{x_n\}$  converges strongly to a common fixed point of  $\{T_1, T_2, \dots, T_N\}$ .

It is worth to mention that the assumption ‘interior of  $F(T)$  is nonempty’ in Theorem ZSA is *severe restriction*.

More recently, Daman and Zegeye [19] proved the following strong convergence of a Halpern-type [6] iterative process for a common fixed point of a finite family of Lipschitz pseudocontractive mappings under the assumption that the family satisfies *condition (H)*, that is, the family  $\{T_i : i = 1, 2, \dots, N\}$  of pseudocontractive mappings is said to satisfy *condition (H)* if and only if  $\langle T_i x - x, T_j x - x \rangle \geq 0$  for  $i, j \in \{1, 2, \dots, N\}$ . We can also see that condition (H) is again *severe restriction*.

It is our purpose in this paper to introduce an iterative scheme which converges strongly to a common fixed point of a family of Lipschitz pseudocontractive mappings. The assumption that *interior of  $F(T)$  is nonempty* or *condition (H)* is dispensed with. The results obtained in this paper improve and extend the results of Zhou [20], Theorem ZSA, Daman and Zegeye [19] and some other results in this direction.

## 2 Preliminaries

In what follows we shall make use of the following lemmas.

**Lemma 2.1** *Let  $H$  be a real Hilbert space. Then, for any given  $x, y \in E$ , the following inequality holds:*

$$\|x + y\|^2 \leq \|x\|^2 + 2\langle y, x + y \rangle.$$

**Lemma 2.2** [21] *Let  $C$  be a convex subset of a real Hilbert space  $H$ . Let  $x \in H$ . Then  $x_0 = P_C x$  if and only if*

$$\langle z - x_0, x - x_0 \rangle \leq 0, \quad \forall z \in C.$$

**Lemma 2.3** [22] *Let  $\{a_n\}$  be a sequence of nonnegative real numbers satisfying the following relation:*

$$a_{n+1} \leq (1 - \alpha_n)a_n + \alpha_n \delta_n, \quad n \geq n_0,$$

where  $\{\alpha_n\} \subset (0, 1)$  and  $\{\delta_n\} \subset \mathbb{R}$  satisfy the following conditions:  $\lim_{n \rightarrow \infty} \alpha_n = 0$ ,  $\sum_{n=1}^{\infty} \alpha_n = \infty$ , and  $\limsup_{n \rightarrow \infty} \delta_n \leq 0$ . Then  $\lim_{n \rightarrow \infty} a_n = 0$ .

**Lemma 2.4** [23] *Let  $H$  be a real Hilbert space,  $C$  be a closed convex subset of  $H$  and  $T : C \rightarrow C$  be a continuous pseudocontractive mapping. Then*

- (i)  $F(T)$  is a closed convex subset of  $C$ ;
- (ii)  $(I - T)$  is demiclosed at zero, i.e., if  $\{x_n\}$  is a sequence in  $C$  such that  $x_n \rightharpoonup x$  and  $Tx_n - x_n \rightarrow 0$ , as  $n \rightarrow \infty$ , then  $x = T(x)$ .

**Lemma 2.5** [24] *Let  $\{a_n\}$  be sequences of real numbers such that there exists a subsequence  $\{n_i\}$  of  $\{n\}$  such that  $a_{n_i} < a_{n_i+1}$  for all  $i \in \mathbb{N}$ . Then there exists a nondecreasing sequence  $\{m_k\} \subset \mathbb{N}$  such that  $m_k \rightarrow \infty$  and the following properties are satisfied by all (sufficiently large) numbers  $k \in \mathbb{N}$ :*

$$a_{m_k} \leq a_{m_k+1} \quad \text{and} \quad a_k \leq a_{m_k+1}.$$

In fact,  $m_k$  is the largest number  $n$  in the set  $\{1, 2, \dots, k\}$  such that the condition  $a_n \leq a_{n+1}$  holds.

**Lemma 2.6** [25] *Let  $H$  be a real Hilbert space. Then, for all  $x_i \in H$  and  $\alpha_i \in [0, 1]$  for  $i = 1, 2, \dots, n$  such that  $\alpha_1 + \alpha_2 + \dots + \alpha_n = 1$ , the following equality holds:*

$$\|\alpha_0 x_0 + \alpha_1 x_1 + \dots + \alpha_n x_n\|^2 = \sum_{i=0}^n \alpha_i \|x_i\|^2 - \sum_{0 \leq i < j \leq n} \alpha_i \alpha_j \|x_i - x_j\|^2.$$

### 3 Main result

**Theorem 3.1** *Let  $C$  be a nonempty, closed and convex subset of a real Hilbert space  $H$ . Let  $T_1, T_2 : C \rightarrow C$  be Lipschitz pseudocontractive mappings with Lipschitz constants  $L_1$  and  $L_2$ , respectively. Assume that  $\mathcal{F} = F(T_1) \cap F(T_2)$  is nonempty. Let a sequence  $\{x_n\}$  be generated from an arbitrary  $x_0, u \in C$  by*

$$\begin{cases} z_n = (1 - c_n)x_n + c_n T_2 x_n; \\ y_n = (1 - \beta_n)x_n + \beta_n T_1 x_n; \\ x_{n+1} = \alpha_n u + (1 - \alpha_n)(\theta_n x_n + \delta_n T_1 y_n + \gamma_n T_2 z_n), \quad n \geq 0, \end{cases} \quad (3.1)$$

where  $\{\delta_n\}, \{\theta_n\}, \{\gamma_n\} \subset (a, b) \subset (0, 1)$ ,  $\{\alpha_n\} \subset (0, c) \subset (0, 1)$  satisfy the following conditions: (i)  $\theta_n + \delta_n + \gamma_n = 1$ ; (ii)  $\lim_{n \rightarrow \infty} \alpha_n = 0$ ,  $\sum \alpha_n = \infty$ ; (iii)  $\delta_n + \gamma_n \leq c_n$ ,  $\beta_n \leq \beta < \frac{1}{\sqrt{1+L^2+1}}$ ,  $\forall n \geq 1$ ,

for  $L := \max\{L_1, L_2\}$ . Then  $\{x_n\}$  converges strongly to a common fixed point of  $T_1$  and  $T_2$  nearest to  $u$ .

*Proof* We make use of some ideas of the paper [19]. Let  $p \in \mathcal{F}$ . Then, from (3.1), (1.1) and Lemma 2.6, we have that

$$\begin{aligned} \|x_{n+1} - p\|^2 &= \|\alpha_n u + (1 - \alpha_n)(\theta_n x_n + \delta_n T_1 y_n + \gamma_n T_2 z_n) - p\|^2 \\ &\leq \alpha_n \|u - p\|^2 + (1 - \alpha_n) \|\delta_n (T_1 y_n - p) + \theta_n (x_n - p) + \gamma_n (T_2 z_n - p)\|^2 \\ &\leq \alpha_n \|u - p\|^2 + (1 - \alpha_n) [\delta_n \|T_1 y_n - p\|^2 + \theta_n \|x_n - p\|^2 \\ &\quad + \gamma_n \|T_2 z_n - p\|^2] - (1 - \alpha_n) \delta_n \theta_n \|T_1 y_n - x_n\|^2 \\ &\quad - (1 - \alpha_n) \theta_n \gamma_n \|T_2 z_n - x_n\|^2, \end{aligned}$$

and hence

$$\begin{aligned} \|x_{n+1} - p\|^2 &\leq \alpha_n \|u - p\|^2 + (1 - \alpha_n) \delta_n [\|y_n - p\|^2 + \|y_n - T_1 y_n\|^2] \\ &\quad + (1 - \alpha_n) \theta_n \|x_n - p\|^2 + (1 - \alpha_n) \gamma_n [\|z_n - p\|^2 + \|z_n - T_2 z_n\|^2] \\ &\quad - (1 - \alpha_n) \delta_n \theta_n \|T_1 y_n - x_n\|^2 - (1 - \alpha_n) \theta_n \gamma_n \|T_2 z_n - x_n\|^2 \\ &\leq \alpha_n \|u - p\|^2 + (1 - \alpha_n) \delta_n \|y_n - p\|^2 + (1 - \alpha_n) \delta_n \|y_n - T_1 y_n\|^2 \\ &\quad + (1 - \alpha_n) \theta_n \|x_n - p\|^2 + (1 - \alpha_n) \gamma_n \|z_n - p\|^2 \\ &\quad + (1 - \alpha_n) \gamma_n \|z_n - T_2 z_n\|^2 - (1 - \alpha_n) \delta_n \theta_n \|T_1 y_n - x_n\|^2 \\ &\quad - (1 - \alpha_n) \theta_n \gamma_n \|T_2 z_n - x_n\|^2. \end{aligned} \tag{3.2}$$

In addition, from (3.1), Lemma 2.6 and (1.1), we get that

$$\begin{aligned} \|y_n - p\|^2 &= \|(1 - \beta_n)(x_n - p) + \beta_n (T_1 x_n - p)\|^2 \\ &= (1 - \beta_n) \|x_n - p\|^2 + \beta_n \|T_1 x_n - p\|^2 \\ &\quad - \beta_n (1 - \beta_n) \|x_n - T_1 x_n\|^2 \\ &\leq (1 - \beta_n) \|x_n - p\|^2 + \beta_n [\|x_n - p\|^2 + \|x_n - T_1 x_n\|^2] \\ &\quad - \beta_n (1 - \beta_n) \|x_n - T_1 x_n\|^2 \\ &= \|x_n - p\|^2 + \beta_n^2 \|x_n - T_1 x_n\|^2. \end{aligned} \tag{3.3}$$

Similarly, we have that

$$\|z_n - p\|^2 = \|x_n - p\|^2 + c_n^2 \|x_n - T_2 x_n\|^2. \tag{3.4}$$

Furthermore, from (3.1) and Lemma 2.6, we have that

$$\begin{aligned} \|y_n - T_1 y_n\|^2 &= \|(1 - \beta_n)(x_n - T_1 y_n) + \beta_n (T_1 x_n - T_1 y_n)\|^2 \\ &= (1 - \beta_n) \|x_n - T_1 y_n\|^2 + \beta_n \|T_1 x_n - T_1 y_n\|^2 \end{aligned}$$

$$\begin{aligned}
 & -\beta_n(1-\beta_n)\|x_n - T_1x_n\|^2 \\
 \leq & (1-\beta_n)\|x_n - T_1y_n\|^2 + \beta_nL^2\|x_n - y_n\|^2 \\
 & -\beta_n(1-\beta_n)\|x_n - T_1x_n\|^2 \\
 = & (1-\beta_n)\|x_n - T_1y_n\|^2 + \beta_n^3L^2\|x_n - T_1x_n\|^2 \\
 & -\beta_n(1-\beta_n)\|x_n - T_1x_n\|^2 \\
 = & (1-\beta_n)\|x_n - T_1y_n\|^2 - \beta_n(1-L^2\beta_n^2 - \beta_n)\|x_n - T_1x_n\|^2. \tag{3.5}
 \end{aligned}$$

Similarly, we obtain that

$$\|z_n - T_2z_n\|^2 = (1-c_n)\|x_n - T_2z_n\|^2 - c_n(1-L^2c_n^2 - c_n)\|x_n - T_2x_n\|^2. \tag{3.6}$$

Substituting (3.3), (3.4), (3.5) and (3.6) into (3.2), we obtain that

$$\begin{aligned}
 \|x_{n+1} - p\|^2 \leq & \alpha_n\|u - p\|^2 + (1-\alpha_n)\delta_n[\|x_n - p\|^2 + \beta_n^2\|x_n - T_1x_n\|^2] \\
 & + (1-\alpha_n)\delta_n[(1-\beta_n)\|x_n - T_1y_n\|^2 - \beta_n(1-L^2\beta_n^2 - \beta_n) \\
 & \times \|x_n - T_1x_n\|^2] + (1-\alpha_n)\theta_n\|x_n - p\|^2 + (1-\alpha_n)\gamma_n[\|x_n - p\|^2 \\
 & + c_n^2\|x_n - T_2x_n\|^2] + (1-\alpha_n)\gamma_n[(1-c_n)\|x_n - T_2z_n\|^2 \\
 & - c_n(1-L^2c_n^2 - c_n)\|x_n - T_2x_n\|^2] - (1-\alpha_n)\delta_n\theta_n\|T_1y_n - x_n\|^2 \\
 & - (1-\alpha_n)\theta_n\gamma_n\|T_2z_n - x_n\|^2 \\
 = & \alpha_n\|u - p\|^2 + (1-\alpha_n)\|x_n - p\|^2 - (1-\alpha_n)\delta_n\beta_n \\
 & \times (1-(L^2\beta_n^2 + 2\beta_n))\|x_n - T_1x_n\|^2 \\
 & - (1-\alpha_n)\gamma_n c_n(1-(L^2c_n^2 + 2c_n))\|x_n - T_2x_n\|^2 \\
 & + (1-\alpha_n)\delta_n(1-\theta_n - \beta_n)\|T_1y_n - x_n\|^2 \\
 & + (1-\alpha_n)\gamma_n(1-\theta_n - c_n)\|T_2z_n - x_n\|^2,
 \end{aligned}$$

and hence

$$\begin{aligned}
 \|x_{n+1} - p\|^2 \leq & \alpha_n\|u - p\|^2 + (1-\alpha_n)\|x_n - p\|^2 - (1-\alpha_n)\delta_n\beta_n \\
 & \times (1-(L^2\beta_n^2 + 2\beta_n))\|x_n - T_1x_n\|^2 \\
 & - (1-\alpha_n)\gamma_n c_n(1-(L^2c_n^2 + 2c_n))\|x_n - T_2x_n\|^2 \\
 & + (1-\alpha_n)\delta_n(\delta_n + \gamma_n - \beta_n)\|T_1y_n - x_n\|^2 \\
 & + (1-\alpha_n)\gamma_n(\delta_n + \gamma_n - c_n)\|T_2z_n - x_n\|^2. \tag{3.7}
 \end{aligned}$$

Now, from (iii) of the hypothesis, we have that

$$1 - 2\beta_n - L^2\beta_n^2 \geq 1 - 2\beta - L^2\beta^2 > 0, \tag{3.8}$$

$$1 - 2c_n - L^2c_n^2 \geq 1 - 2\beta - L^2\beta^2 > 0 \tag{3.9}$$

and

$$(\delta_n + \gamma_n) - \beta_n \leq 0, \quad (\delta_n + \gamma_n) - c_n \leq 0 \quad \text{for all } n \geq 1. \tag{3.10}$$

Thus, inequality (3.7) implies that

$$\|x_{n+1} - p\|^2 \leq \alpha_n \|u - p\|^2 + (1 - \alpha_n) \|x_n - p\|^2. \tag{3.11}$$

Thus, by induction,

$$\|x_{n+1} - p\|^2 \leq \max\{\|u - p\|^2, \|x_0 - p\|^2\}, \quad \forall n \geq 0,$$

which implies that  $\{x_n\}$  and hence  $\{y_n\}$  are bounded.

Let  $x^* = P_{\mathcal{F}}(u)$ . Then, using (3.1), Lemma 2.1 and following the methods used to get (3.7), we obtain that

$$\begin{aligned} \|x_{n+1} - x^*\|^2 &= \|\alpha_n u + (1 - \alpha_n)(\theta_n x_n + \delta_n T_1 y_n + \gamma_n T_2 z_n) - x^*\|^2 \\ &= \|\alpha_n (u - x^*) + (1 - \alpha_n)[\theta_n x_n + \delta_n T_1 y_n + \gamma_n T_2 z_n - x^*]\|^2 \\ &\leq (1 - \alpha_n) \|\delta_n T_1 y_n + \theta_n x_n + \gamma_n T_2 z_n - x^*\|^2 \\ &\quad + 2\alpha_n \langle u - x^*, x_{n+1} - x^* \rangle \\ &\leq (1 - \alpha_n) \delta_n \|T_1 y_n - x^*\|^2 + (1 - \alpha_n) \theta_n \|x_n - x^*\|^2 \\ &\quad + (1 - \alpha_n) \gamma_n \|T_2 z_n - x^*\|^2 - (1 - \alpha_n) \theta_n \delta_n \|T_1 y_n - x_n\|^2 \\ &\quad - (1 - \alpha_n) \theta_n \gamma_n \|T_2 z_n - x_n\|^2 + 2\alpha_n \langle u - x^*, x_{n+1} - x^* \rangle \end{aligned}$$

and

$$\begin{aligned} \|x_{n+1} - x^*\|^2 &\leq (1 - \alpha_n) \delta_n [\|y_n - x^*\|^2 + \|y_n - T_1 y_n\|^2] \\ &\quad + (1 - \alpha_n) \theta_n \|x_n - x^*\|^2 + (1 - \alpha_n) \gamma_n [\|z_n - x^*\|^2 \\ &\quad + \|z_n - T_2 z_n\|^2] - (1 - \alpha_n) \theta_n \delta_n \|T_1 y_n - x_n\|^2 \\ &\quad - (1 - \alpha_n) \theta_n \gamma_n \|T_2 z_n - x_n\|^2 + 2\alpha_n \langle u - x^*, x_{n+1} - x^* \rangle \\ &\leq (1 - \alpha_n) \delta_n [\|x_n - x^*\|^2 + \beta_n^2 \|x_n - T_1 x_n\|^2] \\ &\quad + (1 - \alpha_n) \delta_n [(1 - \beta_n) \|x_n - T_1 y_n\|^2 - \beta_n (1 - L^2 \beta_n^2 - \beta_n) \\ &\quad \times \|x_n - T_1 x_n\|^2] + (1 - \alpha_n) \theta_n \|x_n - x^*\|^2 \\ &\quad + (1 - \alpha_n) \gamma_n [\|x_n - x^*\|^2 + c_n^2 \|x_n - T_2 x_n\|^2] \\ &\quad + (1 - \alpha_n) \gamma_n [(1 - c_n) \|x_n - T_2 z_n\|^2 - c_n (1 - L^2 c_n^2 - c_n) \\ &\quad \times \|x_n - T_2 x_n\|^2] - (1 - \alpha_n) \theta_n \delta_n \|T_1 y_n - x_n\|^2 - (1 - \alpha_n) \theta_n \gamma_n \\ &\quad \times \|T_2 z_n - x_n\|^2 + 2\alpha_n \langle u - x^*, x_{n+1} - x^* \rangle, \end{aligned}$$

which implies that

$$\begin{aligned} \|x_{n+1} - x^*\|^2 &\leq (1 - \alpha_n) \|x_n - x^*\|^2 - (1 - \alpha_n) \delta_n \beta_n [1 - L^2 \beta_n^2 - 2\beta_n] \\ &\quad \times \|x_n - T_1 x_n\|^2 + (1 - \alpha_n) \delta_n (\delta_n + \gamma_n - \beta_n) \|x_n - T_1 y_n\|^2 \\ &\quad - (1 - \alpha_n) \delta_n c_n [1 - L^2 c_n^2 - 2c_n] \|x_n - T_2 x_n\|^2 \\ &\quad + (1 - \alpha_n) \gamma_n (\delta_n + \gamma_n - c_n) \|x_n - T_2 z_n\|^2 + 2\alpha_n \langle u - x^*, x_{n+1} - x^* \rangle \quad (3.12) \\ &\leq (1 - \alpha_n) \|x_n - x^*\|^2 + 2\alpha_n \langle u - x^*, x_{n+1} - x^* \rangle. \quad (3.13) \end{aligned}$$

Now, we consider two cases.

Case 1. Suppose that there exists  $n_0 \in \mathbb{N}$  such that  $\{\|x_n - x^*\|\}$  is decreasing for all  $n \geq n_0$ . Then, we get that  $\{\|x_n - x^*\|\}$  is convergent. Thus, from (3.12), (3.8), (3.9) and (3.10), we have that

$$x_n - T_1 x_n \rightarrow 0, \quad x_n - T_2 x_n \rightarrow 0 \quad \text{as } n \rightarrow \infty. \quad (3.14)$$

Furthermore, since  $\{x_{n+1}\}$  is a bounded subset of  $H$  which is reflexive, we can choose a subsequence  $\{x_{n_i+1}\}$  of  $\{x_{n+1}\}$  such that  $x_{n_i+1} \rightharpoonup x$  and  $\limsup_{n \rightarrow \infty} \langle u - x^*, x_{n+1} - x^* \rangle = \lim_{i \rightarrow \infty} \langle u - x^*, x_{n_i+1} - x^* \rangle$ . Then, from (3.14) and Lemma 2.4, we have that  $x \in F(T_1)$  and  $x \in F(T_2)$ . Therefore, by Lemma 2.2, we immediately obtain that

$$\begin{aligned} \limsup_{n \rightarrow \infty} \langle u - x^*, x_{n+1} - x^* \rangle &= \lim_{i \rightarrow \infty} \langle u - x^*, x_{n_i+1} - x^* \rangle \\ &= \langle u - x^*, x - x^* \rangle \leq 0. \quad (3.15) \end{aligned}$$

Then it follows from (3.13), (3.15) and Lemma 2.3 that  $\|x_n - x^*\| \rightarrow 0$  as  $n \rightarrow \infty$ . Consequently,  $x_n \rightarrow x^* = P_{\mathcal{F}}(u)$ .

Case 2. Suppose that there exists a subsequence  $\{n_i\}$  of  $\{n\}$  such that

$$\|x_{n_i} - x^*\| < \|x_{n_i+1} - x^*\|$$

for all  $i \in \mathbb{N}$ . Then, by Lemma 2.5, there exists a nondecreasing sequence  $\{m_k\} \subset \mathbb{N}$  such that  $m_k \rightarrow \infty$ , and

$$\|x_{m_k} - x^*\| \leq \|x_{m_k+1} - x^*\| \quad \text{and} \quad \|x_k - x^*\| \leq \|x_{m_k+1} - x^*\| \quad (3.16)$$

for all  $k \in \mathbb{N}$ . Now, from (3.12), (3.8), (3.9) and (3.10), we get that  $x_{m_k} - T_1 x_{m_k} \rightarrow 0$  and  $x_{m_k} - T_2 x_{m_k} \rightarrow 0$  as  $k \rightarrow \infty$ . Thus, like in Case 1, we obtain that

$$\limsup_{k \rightarrow \infty} \langle u - x^*, x_{m_k+1} - x^* \rangle \leq 0. \quad (3.17)$$

Now, from (3.13) we have that

$$\|x_{m_k+1} - x^*\|^2 \leq (1 - \alpha_{m_k}) \|x_{m_k} - x^*\|^2 + 2\alpha_{m_k} \langle u - x^*, x_{m_k+1} - x^* \rangle, \quad (3.18)$$



and hence (3.16) and (3.18) imply that

$$\begin{aligned} \alpha_{m_k} \|x_{m_k} - x^*\|^2 &\leq \|x_{m_k} - x^*\|^2 - \|x_{m_{k+1}} - x^*\|^2 + 2\alpha_{m_k} \langle u - x^*, x_{m_{k+1}} - x^* \rangle \\ &\leq +2\alpha_{m_k} \langle u - x^*, x_{m_{k+1}} - x^* \rangle. \end{aligned}$$

But using the fact that  $\alpha_{m_k} > 0$  and (3.17), we obtain that

$$\limsup_{k \rightarrow \infty} \|x_{m_k} - x^*\|^2 \leq 0,$$

and hence  $\|x_{m_k} - x^*\| \rightarrow 0$  as  $k \rightarrow \infty$ . This together with (3.18) implies that  $\|x_{m_{k+1}} - x^*\| \rightarrow 0$  as  $k \rightarrow \infty$ . But  $\|x_k - x^*\| \leq \|x_{m_{k+1}} - x^*\|$  for all  $k \in N$ , thus we obtain that  $x_k \rightarrow x^*$ . Therefore, from the above two cases, we can conclude that  $\{x_n\}$  converges strongly to a common fixed point of  $T_1$  and  $T_2$  nearest to  $u$ . The proof is complete.  $\square$

We note that the method of the proof of Theorem 3.1 provides a convergence theorem for a finite family of Lipschitzian pseudocontractive mappings. In fact, we have the following theorem.

**Theorem 3.2** *Let  $C$  be a nonempty, closed and convex subset of a real Hilbert space  $H$ . Let  $T_i : C \rightarrow C, i = 1, 2, \dots, N$ , be Lipschitz pseudocontractive mappings with Lipschitz constants  $L_i, i = 1, 2, \dots, N$ , respectively. Assume that  $\mathcal{F} = \bigcap_i^N F(T_i)$  is nonempty. Let a sequence  $\{x_n\}$  be generated from an arbitrary  $x_0, u \in C$  by*

$$\begin{cases} y_{ni} = (1 - \beta_n)x_n + \beta_n T_i x_n, & i = 1, 2, \dots, N; \\ x_{n+1} = \alpha_n u + (1 - \alpha_n)(\theta_n x_n + \sum_{i=1}^N \theta_{ni} T_i y_{ni}), \end{cases} \quad (3.19)$$

where  $\{\theta_{ni} : i = 0, 1, 2, \dots, N\} \subset (a, b) \subset (0, 1), \{\alpha_n\} \subset (0, c) \subset (0, 1)$  satisfy the following conditions: (i)  $\theta_{n0} + \theta_{n1} + \dots + \theta_{nN} = 1$ ; (ii)  $\lim_{n \rightarrow \infty} \alpha_n = 0, \sum \alpha_n = \infty$ ; (iii)  $\sum_{i=1}^N \theta_{ni} \leq \beta_n \leq \beta < \frac{1}{\sqrt{1+L^2+1}}, \forall n \geq 1$ , for  $L := \max\{L_i : i = 1, 2, \dots, N\}$ . Then  $\{x_n\}$  converges strongly to a common fixed point of  $T_i (i = 1, 2, \dots, N)$  nearest to  $u$ .

If in Theorem 3.1 we assume that  $T$  is nonexpansive, then we have that  $T$  is Lipschitz pseudocontractive with  $L = 1$ , and hence we get the following corollary.

**Corollary 3.3** *Let  $C$  be a nonempty, closed and convex subset of a real Hilbert space  $H$ . Let  $T_1, T_2 : C \rightarrow C$  be nonexpansive mappings. Assume that  $\mathcal{F} = F(T_1) \cap F(T_2)$  is nonempty. Let a sequence  $\{x_n\}$  be generated from an arbitrary  $x_0, u \in C$  by*

$$\begin{cases} z_n = (1 - c_n)x_n + c_n T_2 x_n; \\ y_n = (1 - \beta_n)x_n + \beta_n T_1 x_n; \\ x_{n+1} = \alpha_n u + (1 - \alpha_n)(\theta_n x_n + \delta_n T_1 y_n + \gamma_n T_2 z_n), \end{cases} \quad (3.20)$$

where  $\{\delta_n\}, \{\theta_n\}, \{\gamma_n\} \subset (a, b) \subset (0, 1), \{\alpha_n\} \subset (0, c) \subset (0, 1)$  satisfy the following conditions: (i)  $\delta_n + \theta_n + \gamma_n = 1$ ; (ii)  $\lim_{n \rightarrow \infty} \alpha_n = 0, \sum \alpha_n = \infty$ ; (iii)  $\delta_n + \gamma_n \leq c_n, \beta_n \leq \beta < (\sqrt{2} - 1), \forall n \geq 1$ . Then  $\{x_n\}$  converges strongly to a common fixed point of  $T_1$  and  $T_2$  nearest to  $u$ .

We now state and prove the convergence theorem for a common zero of a family of monotone mappings.

**Corollary 3.4** *Let  $H$  be a real Hilbert space. Let  $A_i : H \rightarrow H, i = 1, 2$ , be Lipschitz monotone mappings with Lipschitz constants  $L_1$  and  $L_2$ , respectively. Assume that  $F := \bigcap_{i=1}^2 N(A_i)$  is nonempty. Let a sequence  $\{x_n\}$  be generated from an arbitrary  $x_1, u \in H$  by*

$$\begin{cases} z_n = x_n - c_n A_2 x_n; \\ y_n = x_n - \beta_n A_1 x_n; \\ x_{n+1} = \alpha_n u + (1 - \alpha_n)(\theta_n x_n + \delta_n(I - A_1)y_n + \gamma_n(I - A_2)z_n), \end{cases} \quad (3.21)$$

where  $\{\delta_n\}, \{\theta_n\}, \{\gamma_n\} \subset (a, b) \subset (0, 1), \{\alpha_n\} \subset (0, c) \subset (0, 1)$  satisfy the following conditions: (i)  $\delta_n + \theta_n + \gamma_n = 1$ ; (ii)  $\lim_{n \rightarrow \infty} \alpha_n = 0, \sum \alpha_n = \infty$ ; (iii)  $\delta_n + \gamma_n \leq c_n, \beta_n \leq \beta < \frac{1}{\sqrt{1+(1+L)^2+1}}, \forall n \geq 1$ , for  $L := \max\{L_1, L_2\}$ . Then  $\{x_n\}$  converges strongly to a common zero point of  $A_1$  and  $A_2$  nearest to  $u$ .

*Proof* Let  $T_i x := (I - A_i)x$  for  $i = 1, 2$ . Then we get that every  $T_i$  for all  $i \in \{1, 2\}$  is a Lipschitz pseudocontractive mapping with the Lipschitz constant  $L'_i := (1 + L_i)$  and  $\bigcap_{i=1}^2 F(T_i) = \bigcap_{i=1}^2 N(A_i) \neq \emptyset$ . Moreover, when  $A_i$  is replaced with  $(I - T_i)$ , for each  $i \in \{1, 2\}$ , then scheme (3.21) reduces to scheme (3.1), and hence the conclusion follows from Theorem 3.1.  $\square$

We may also have the following corollary for a finite family of monotone mappings.

**Corollary 3.5** *Let  $H$  be a real Hilbert space. Let  $A_i : H \rightarrow H, i = 1, 2, \dots, N$ , be Lipschitz monotone mappings with Lipschitz constants  $L_i, i = 1, 2, \dots, N$ , respectively. Assume that  $F := \bigcap_{i=1}^N N(A_i)$  is nonempty. Let a sequence  $\{x_n\}$  be generated from an arbitrary  $x_1, u \in H$  by*

$$\begin{cases} y_{ni} = x_n - \beta_n A_i x_n, \quad i = 1, 2, \dots, N; \\ x_{n+1} = \alpha_n u + (1 - \alpha_n)(\theta_{n0} x_n + \sum_{i=1}^N \theta_{ni}(I - A_i)y_{ni}), \end{cases} \quad (3.22)$$

where  $\{\theta_{ni} : i = 0, 1, 2, \dots, N\} \subset (a, b) \subset (0, 1), \{\alpha_n\} \subset (0, c) \subset (0, 1)$  satisfy the following conditions: (i)  $\theta_{n0} + \theta_{n1} + \dots + \theta_{nN} = 1$ ; (ii)  $\lim_{n \rightarrow \infty} \alpha_n = 0, \sum \alpha_n = \infty$ ; (iii)  $\sum_{i=1}^N \theta_{ni} \leq \beta_n \leq \beta < \frac{1}{\sqrt{1+(1+L)^2+1}}, \forall n \geq 1$ , for  $L := \max\{L_i : i = 1, 2, \dots, N\}$ . Then  $\{x_n\}$  converges strongly to a common zero point of  $A_i, i = 1, 2, \dots, N$ , nearest to  $u$ .

If in Corollary 3.5 we consider a single Lipschitz monotone mapping, then we obtain the following corollary.

**Corollary 3.6** *Let  $H$  be a real Hilbert space. Let  $A : H \rightarrow H$  be a Lipschitz monotone mapping with Lipschitz constant  $L$ . Assume that  $F := N(A)$  is nonempty. Let a sequence  $\{x_n\}$  be generated from an arbitrary  $x_1, u \in H$  by*

$$\begin{cases} y_n = x_n - \beta_n A x_n; \\ x_{n+1} = \alpha_n u + (1 - \alpha_n)((1 - \gamma_n)x_n + \gamma_n(I - A)y_n), \end{cases} \quad (3.23)$$

where  $\{\gamma_n\} \subset (a, b) \subset (0, 1)$ ,  $\{\alpha_n\} \subset (0, c) \subset (0, 1)$  satisfy the following conditions:  
 (i)  $\lim_{n \rightarrow \infty} \alpha_n = 0$ ,  $\sum \alpha_n = \infty$ ; (ii)  $\gamma_n \leq \beta_n \leq \beta < \frac{1}{\sqrt{1+(1+L)^2+1}}$ ,  $\forall n \geq 1$ . Then  $\{x_n\}$  converges strongly to the zero point of  $A$  nearest to  $u$ .

#### 4 Numerical example

Now, we give an example of a finite family of pseudocontractive mappings satisfying Theorem 3.1 and some numerical experiment result to explain the conclusion of the theorem as follows.

**Example 4.1** Let  $H = \mathbb{R}$  with absolute value norm. Let  $K = [-2, 1]$  and  $T_1, T_2 : K \rightarrow K$  be defined by

$$T_1x := \begin{cases} x + x^2, & x \in [-2, 0], \\ x, & x \in (0, 1] \end{cases} \tag{4.1}$$

and

$$T_2x := \begin{cases} x, & x \in [-2, \frac{1}{2}], \\ x - 2(x - \frac{1}{2})^2, & x \in (\frac{1}{2}, 1]. \end{cases} \tag{4.2}$$

Clearly,  $\mathcal{F} = F(T_1) \cap F(T_2) = [0, 1] \cap [-2, \frac{1}{2}] = [0, \frac{1}{2}]$ , and for  $x, y \in K$  we have that

$$\langle (I - T_1)x - (I - T_1)y, x - y \rangle \geq 0 \tag{4.3}$$

and

$$\langle (I - T_2)x - (I - T_2)y, x - y \rangle \geq 0, \tag{4.4}$$

which show that both mappings are pseudocontractive. Next, we show that  $T_1$  is Lipschitzian with  $L = 5$ . If  $x, y \in [-2, 0]$ , then

$$\begin{aligned} |T_1x - T_1y| &= |x + x^2 - y - y^2| \\ &= |(x + y) + 1||x - y| \leq 3|x + y|. \end{aligned} \tag{4.5}$$

If  $x, y \in (0, 1]$ , then

$$|T_1x - T_1y| = |x - y|.$$

If  $x \in [-2, 0]$  and  $y \in (0, 1]$ , then

$$\begin{aligned} |T_1x - T_1y| &= |x + x^2 - y| \\ &= |x - y + x^2| = |x - y + x^2 - y^2 + y^2| \\ &= |x - y + x^2 - y^2| + y^2 \\ &\leq |x + y + 1| \cdot |x - y| + |y - x| \\ &= |2 + x + y| \cdot |x - y| \leq 3|x - y|. \end{aligned} \tag{4.6}$$

If  $x \in (0, 1]$  and  $y \in [-2, 0]$ , then

$$\begin{aligned}
 |T_1x - T_1y| &= |x - (y + y^2)| \\
 &= |x - y - y^2 + x^2 - x^2| = |x - y + (x + y)(x - y) - x^2| \\
 &= |(1 + x + y)||x - y| + x^2 \\
 &\leq |(1 + x + y)||x - y| + (x - y)^2 \\
 &\leq (|1 + x + y| + |x - y|)|x - y| \\
 &\leq 5|x - y|.
 \end{aligned}
 \tag{4.7}$$

Thus, we get that  $T_1$  is Lipschitzian pseudocontractive with  $L = 5$ . Similarly, we can show that  $T_2$  is Lipschitzian pseudocontractive with  $L = 10$ .

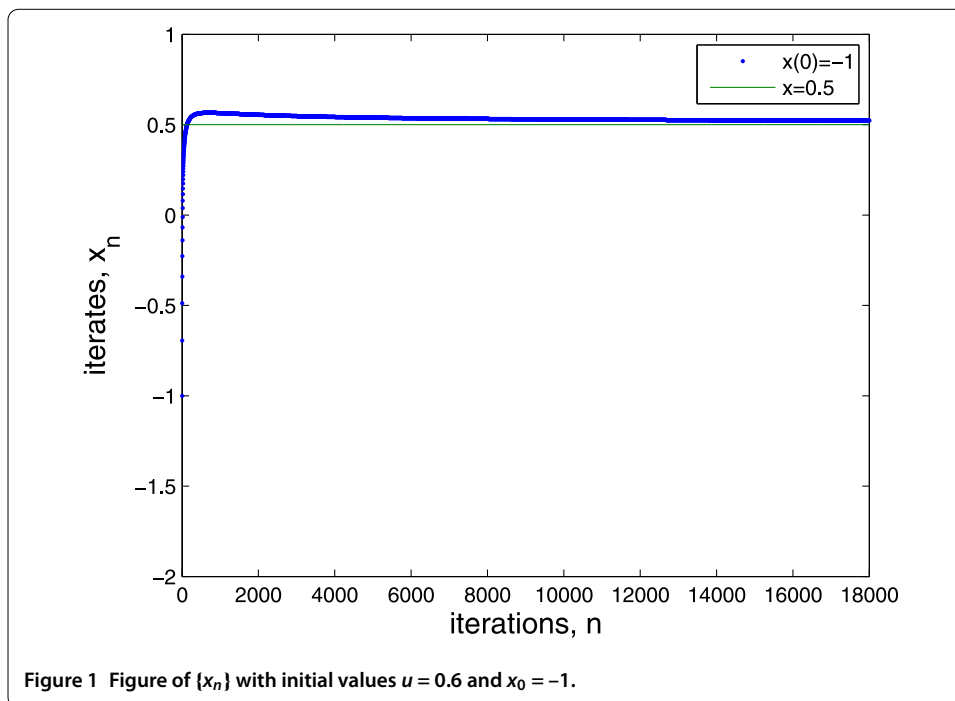
Now, taking  $\alpha_n = \frac{1}{n+10}$ ,  $c_n = \beta_n = \frac{1}{100} + \frac{1}{n+100}$ ,  $\gamma_n = \delta_n = 0.5[\frac{1}{100} + \frac{1}{n+100}]$  and  $\theta_n = 1 - [\frac{1}{100} + \frac{1}{n+100}]$ , we observe that the conditions of Theorem 3.1 are satisfied and scheme (3.1) provides the data in Tables 1 and 2 and Figures 1 and 2.

(i) When  $u = 0.6$  and  $x_0 = -1$ , we see that the sequence converges to  $x^* = 0.5$  as shown in Table 1 and Figure 1.

(ii) When  $u = -1$  and  $x_0 = 0.8$ , we see that the sequence converges to  $x^* = 0$  as shown in Table 2 and Figure 2.

**Table 1** Values of  $\{x_n\}$  with initial values  $u = 0.6$  and  $x_0 = -1$

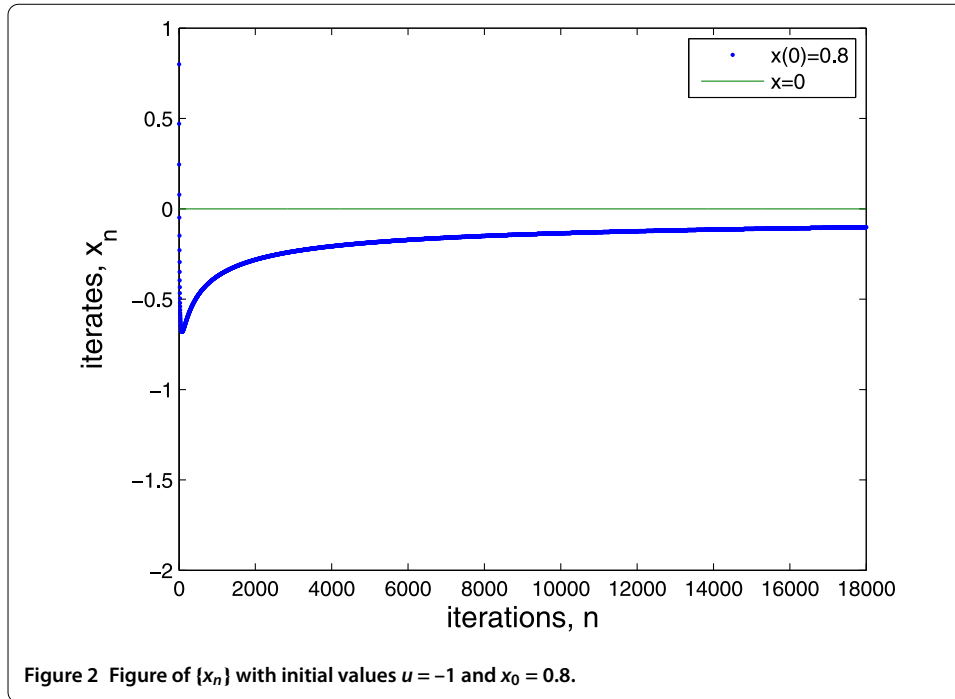
$n$	0	500	1,000	5,000	10,000	12,000	14,000	17,000
$x_n$	-1.0000	0.5639	0.5638	0.5387	0.5287	0.5264	0.5246	0.5225



**Figure 1** Figure of  $\{x_n\}$  with initial values  $u = 0.6$  and  $x_0 = -1$ .

**Table 2** Values of  $\{x_n\}$  with initial values  $u = -1$  and  $x_0 = 0.8$

$n$	0	500	1,000	5,000	10,000	12,000	14,000	17,000
$x_n$	0.8000	-0.4781	-0.3738	-0.1870	-0.1351	-0.1238	-0.1150	-0.1047



**Figure 2** Figure of  $\{x_n\}$  with initial values  $u = -1$  and  $x_0 = 0.8$ .

**Remark 4.2** Theorem 3.1 provides a convergence sequence to a common fixed point of two Lipschitzian pseudocontractive mappings, whereas Theorem 3.2 provides a convergence sequence to a common fixed point of a finite family of Lipschitzian pseudocontractive mappings. In addition, Corollary 3.4 provides a convergence sequence to a common zero of two Lipschitzian monotone mappings, whereas Theorem 3.5 provides a convergence sequence to a common zero of a finite family of Lipschitzian monotone mappings.

**Remark 4.3** Theorem 3.2 improves Theorem ZSA, Theorem 3.2 of Daman and Zegeye [19] in the sense that our convergence does not require the assumption that *interior of  $F(T)$  is nonempty* or *condition (H)*.

**Remark 4.4** Theorem 3.1 improves Theorem 3.1 of Zhou [20], Theorem 3.1 of Yao *et al.* [26] and Theorem 3.1 of Tang *et al.* [27] in the sense that our convergence requires neither compactness of  $T$  nor computation of closed and convex  $C_n$  of  $C$  for each  $n \geq 1$ .

**Competing interests**

The authors declare that they have no competing interests.

**Authors' contributions**

All the authors contributed equally. All authors read and approved the final manuscript.

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