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Coupled coincidence point theorems for α-ψ-contractive type mappings in partially ordered metric spaces

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Abstract

In this paper, we introduce the notion of **α-ψ**-contractions and (**α**)-admissibility for a pair of mappings. In fact, our theorem is a generalization of the result of Mursaleen *et al.* (Fixed Point Theory Appl. 2012, doi:10.1186/1687-1812-2012-228). At the end, we shall provide an example in support of our main result.
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1 Introduction

Fixed point problems of contractive mappings in metric spaces endowed with a partial order have been studied in a number of works. Bhaskar and Lakshmikantham [1] established coupled fixed point results for mixed monotone operators in partially ordered metric spaces. Afterwards, Lakshmikantham and Ćirić [2] proved coupled coincidence and coupled common fixed point theorems for nonlinear contractive mappings in partially ordered complete metric spaces. Choudhury and Kundu [3], Ćirić *et al.* [2], Luong and Thuan [4], Nieto and López [5, 6], Ran and Reurings [7] and Samet [8] presented some new results for contractions in partially ordered metric spaces. Ilić and Rakočević [9] determined some common fixed point theorems by considering the maps on cone metric spaces. For more details on fixed point theory and related concepts, we refer to [4, 8–18] and the references therein.

Most recently, Samet *et al.* [12, 15] defined an α-ψ-contractive and α-admissible mapping and proved fixed point theorems for such mappings in complete metric spaces.

The aim of this paper is to determine some coupled coincidence point theorems for generalized contractive mappings in the framework of partially ordered metric spaces.

2 Preliminaries

Before proceeding to our main result, we give some preliminaries.

Definition 2.1 [1] Let $(X, \leq)$ be a partially ordered set, and let $F : X \times X \to X$ be a mapping. Then F is said to have the mixed monotone property if $F(x, y)$ is monotone non-

decreasing in x and is monotone non-increasing in y; that is, for any $x, y \in X$,

$$x_1, x_2 \in X, \qquad x_1 \le x_2 \quad \text{implies} \quad F(x_1, y) \le F(x_2, y) \quad \text{and}$$

$$y_1, y_2 \in X, \qquad y_1 \le y_2 \quad \text{implies} \quad F(x, y_1) \ge F(x, y_2).$$

Definition 2.2 [1] An element $(x, y) \in X \times X$ is said to be a coupled fixed point of the mapping $F : X \times X \to X$ if

$$F(x, y) = x \quad \text{and} \quad F(y, x) = y.$$

Definition 2.3 [2] Let (X, d) be a partially ordered set, and let $F : X \times X \to X$ and $g : X \to X$ be two mappings. We say F has the mixed g-monotone property if F is monotone g-non-decreasing in its first argument and is monotone g-non-increasing in its second argument, that is, for any $x, y \in X$

$$x_1, x_2 \in X, \qquad g(x_1) \le g(x_2) \quad \text{implies} \quad F(x_1, y) \le F(x_2, y) \quad \text{and}$$

$$y_1, y_2 \in X, \qquad g(y_1) \le g(y_2) \quad \text{implies} \quad F(x, y_1) \ge F(x, y_2).$$

Definition 2.4 [2] An element $(x, y) \in X \times X$ is called a coupled coincidence point of mappings $F : X \times X \to X$ and $g : X \to X$ if

$$F(x, y) = g(x) \quad \text{and} \quad F(y, x) = g(y).$$

Definition 2.5 [2] Let X be a non-empty set and $F : X \times X \to X$ and $g : X \to X$. We say F and g are commutative if $g(F(x, y)) = F(g(x), g(y))$.

Definition 2.6 [19] Denote by Ψ the family of non-decreasing functions $\psi : [0, +\infty) \to [0, +\infty)$ such that $\sum_{n=0}^{\infty} \psi_n(t) < t$ for all $t > 0$, where ψ_n is the nth iterate of ψ satisfying
  (i) $\psi^{-1}(\{0\}) = \{0\}$,
  (ii) $\psi(t) < t$ for all $t > 0$, and
  (iii) $\lim_{r \to t+} \psi(r) < t$ for all $t > 0$.

Lemma 2.7 [19] *If* $\psi : [0, \infty] \to [0, \infty]$ *is non-decreasing and right continuous, then* $\psi_n(t) \to 0$ *as* $n \to \infty$ *for all* $t \ge 0$ *if and only if* $\psi(t) < t$ *for all* $t > 0$.

Definition 2.8 [19] Let (X, d) be a partially ordered metric space and $F : X \times X \to X$, then F is said to be (α, ψ)-*contractive* if there exist two functions $\alpha : X^2 \times X^2 \to [0, +\infty)$ and $\psi \in \Psi$ such that

$$\alpha\big((x, y), (u, v)\big) d\big(F(x, y), F(u, v)\big) \le \psi\left(\frac{d(x, u) + d(y, v)}{2}\right),$$

$x, y, u, v \in X$ with $x \ge u$ and $y \le v$.

Definition 2.9 [19] Let $F : X \times X \to X$ and $\alpha : X^2 \times X^2 \to [0, +\infty)$ be two mappings. Then F is said to be (α)-*admissible* if

$$\alpha\big((x,y),(u,v)\big) \geq 1 \quad \text{implies} \quad \alpha\big((F(x,y),F(y,x)),(F(u,v),F(v,u))\big) \geq 1$$

for all $x, y, u, v \in X$.

Now, we will introduce our notions.

Definition 2.10 Let (X, d) be a partially ordered metric space, and let $F : X \times X \to X$ and $g : X \to X$ be two mappings. Then the maps F and g are said to be (α, ψ)-*contractive* if there exist two functions $\alpha : X^2 \times X^2 \to [0, +\infty)$ and $\psi \in \Psi$ such that

$$\alpha\big((g(x),g(y)),(g(u),g(v))\big)d\big(F(x,y),F(u,v)\big) \leq \psi\left(\frac{d(g(x),g(u)) + d(g(y),g(v))}{2}\right)$$

for $x, y, u, v \in X$ with $g(x) \geq g(u)$ and $g(y) \leq g(v)$.

Definition 2.11 Let $F : X \times X \to X$, $g : X \to X$ and $\alpha : X^2 \times X^2 \to [0, +\infty)$ be mappings. Then F and g are said to be (α)-*admissible* if

$$\alpha\big((g(x),g(y)),(g(u),g(v))\big) \geq 1 \quad \text{implies} \quad \alpha\big((F(x,y),F(y,x)),(F(u,v),F(v,u))\big) \geq 1$$

for all $x, y, u, v \in X$.

3 Main results

Recently, Mursaleen *et al.* [19] proved the coupled fixed point theorem with α-ψ-contractive conditions in a partially ordered metric space as follows.

Theorem 3.1 [19] *Let $(X, \leq)$ be a partially ordered set, and let there exist a metric d on X such that (X, d) is a complete metric space. Let $F : X \times X \to X$ be a mapping, and suppose that F has the mixed monotone property. Suppose that there exist $\psi \in \Psi$ and $\alpha : X^2 \times X^2 \to [0, +\infty)$ such that for $x, y, u, v \in X$, the following holds:*

$$\alpha\big((x,y),(u,v)\big)d\big(F(x,y),F(u,v)\big) \leq \psi\left(\frac{d(x,u) + d(y,v)}{2}\right)$$

for $x, y, u, v \in X$ with $x \geq u$ and $y \leq v$.

Suppose also that
  (i) *F is (α)-admissible.*
 (ii) *There exist $x_0, y_0 \in X$ such that*

$$\alpha\big((x_0,y_0),(F(x_0,y_0),F(y_0,x_0))\big) \geq 1 \quad \text{and} \quad \alpha\big((y_0,x_0),(F(y_0,x_0),F(x_0,y_0))\big) \geq 1.$$

(iii) *F is continuous.*
  If there exist $x_0, y_0 \in X$ such that $x_0 \leq F(x_0, y_0)$ and $y_0 \geq F(y_0, x_0)$, then F has a coupled fixed point, that is, there exist $x, y \in X$ such that

$$F(x,y) = x \quad \text{and} \quad F(y,x) = y.$$

Theorem 3.2 *Let $(X, \leq)$ be a partially ordered set, and let there exist a metric d on X such that (X, d) is a complete metric space. Let $F : X \times X \to X$ and $g : X \to X$ be maps, and let F have the g-mixed monotone property. Suppose that there exist $\psi \in \Psi$ and $\alpha : X^2 \times X^2 \to [0, +\infty)$ such that for $x, y, u, v \in X$, the following holds:*

$$\alpha\big((g(x), g(y)), (g(u), g(v))\big) d\big(F(x, y), F(u, v)\big)$$

$$\leq \psi\left(\frac{d(g(x), g(u)) + d(g(y), g(v))}{2}\right) \tag{3.1}$$

for all $x, y, u, v \in X$ with $g(x) \geq g(u)$ and $g(y) \leq g(v)$.

Suppose also that

(i) *F and g are (α)-admissible.*

(ii) *There exist $x_0, y_0 \in X$ such that*

$$\alpha\big((g(x_0), g(y_0)), (F(x_0, y_0), F(y_0, x_0))\big) \geq 1 \quad and$$

$$\alpha\big((g(y_0), g(x_0)), (F(y_0, x_0), F(x_0, y_0))\big) \geq 1.$$

(iii) *$F(X \times X) \subseteq g(X)$, g is continuous and commutes with F.*

(iv) *F is continuous.*

If there exist $x_0, y_0 \in X$ such that $g(x_0) \leq F(x_0, y_0)$ and $g(y_0) \geq F(y_0, x_0)$, then F and g have a coupled coincidence point, that is, there exist $x, y \in X$ such that

$$F(x, y) = g(x) \quad and \quad F(y, x) = g(y).$$

Proof Let $x_0, y_0 \in X$ be such that

$$\alpha\big(g(x_0), g(y_0)), (F(x_0, y_0), F(y_0, x_0)\big) \geq 1 \quad \text{and}$$

$$\alpha\big(g(y_0), g(x_0)), (F(y_0, x_0), F(x_0, y_0)\big) \geq 1$$

and

$$g(x_0) \leq F(x_0, y_0) = g(x_1) \quad \text{and} \quad g(y_0) \geq F(y_0, x_0) = g(y_1).$$

Let $x_2, y_2 \in X$ be such that $F(x_1, y_1) = g(x_2)$ and $F(y_1, x_1) = g(y_2)$.

Continuing this process, we can construct two sequences $\{x_n\}$ and $\{y_n\}$ in X as follows:

$$g(x_{n+1}) = F(x_n, y_n) \quad \text{and} \quad g(y_{n+1}) = F(y_n, x_n) \quad \text{for all } n \geq 0.$$

Now we will show that

$$g(x_n) \leq g(x_{n+1}) \quad \text{and} \quad g(y_n) \geq g(y_{n+1}) \quad \text{for all } n \geq 0. \tag{3.2}$$

For $n = 0$, since $g(x_0) \leq F(x_0, y_0)$ and $g(y_0) \geq F(y_0, x_0)$ and as $g(x_1) = F(x_0, y_0)$ and $g(y_1) = F(y_0, x_0)$, we have

$$g(x_0) \leq g(x_1) \quad \text{and} \quad g(y_0) \geq g(y_1).$$

Thus (3.2) holds for $n = 0$. Now suppose that (3.2) holds for some fixed $n \geq 0$.

Then since $g(x_n) \leq g(x_{n+1})$ and $g(y_n) \geq g(y_{n+1})$, therefore, by the g-mixed monotone property of F, we have

$$g(x_{n+2}) = F(x_{n+1}, y_{n+1}) \geq F(x_n, y_{n+1}) \geq F(x_n, y_n) = g(x_{n+1}) \quad \text{and}$$

$$g(y_{n+2}) = F(y_{n+1}, x_{n+1}) \leq F(y_n, x_{n+1}) \leq F(y_n, x_n) = g(y_{n+1}).$$

From above, we conclude that

$$g(x_{n+1}) \leq g(x_{n+2}) \quad \text{and} \quad g(y_{n+1}) \geq g(y_{n+2}).$$

Thus, by mathematical induction, we conclude that (3.2) holds for all $n \geq 0$.

If the following holds for some n

$$\big(g(x_{n+1}), g(y_{n+1})\big) = \big(g(x_n), g(y_n)\big),$$

then, obviously, $F(x_n, y_n) = g(x_n)$ and $F(y_n, x_n) = g(y_n)$, i.e., F has a coupled coincidence point.

Now, we assume that $(x_{n+1}, y_{n+1}) \neq (x_n, y_n)$ for all $n \geq n(\varepsilon)$.

Since F and g are α-*admissible*, we have that

$$\alpha\big(\big(g(x_0), g(y_0)\big), \big(g(x_1), g(y_1)\big)\big) = \alpha\big(\big(g(x_0), g(y_0)\big), \big(F(x_0, y_0), F(y_0, x_0)\big)\big) \geq 1 \quad \text{implies}$$

$$\alpha\big(\big(F(x_0, y_0), F(y_0, x_0)\big), \big(F(x_1, y_1), F(y_1, x_1)\big)\big) = \alpha\big(\big(g(x_1), g(y_1)\big), \big(g(x_2), g(y_2)\big)\big) \geq 1.$$

Thus, by mathematical induction, we have

$$\alpha\big(\big(g(x_n), g(y_n)\big), \big(g(x_{n+1}), g(y_{n+1})\big)\big) \geq 1. \tag{3.3}$$

Similarly,

$$\alpha\big(\big(g(y_n), g(x_n)\big), \big(g(y_{n+1}), g(x_{n+1})\big)\big) \geq 1 \quad \text{for } n \in N. \tag{3.4}$$

From (3.1) and conditions (i) and (ii) of the hypothesis, we get

$$
\begin{aligned}
d\big(g(x_n), g(x_{n+1})\big) &= d\big(F(x_{n-1}, y_{n-1}), F(x_n, y_n)\big) \\
&\leq \alpha\big(\big(g(x_{n-1}), g(y_{n-1})\big), \big(g(x_n), g(y_n)\big)\big) d\big(F(x_{n-1}, y_{n-1}), F(x_n, y_n)\big) \\
&\leq \psi\left(\frac{d(g(x_{n-1}), g(x_n)) + d(g(y_{n-1}), g(y_n))}{2}\right).
\end{aligned} \tag{3.5}
$$

Similarly, we have

$$
\begin{aligned}
d\big(g(y_n), g(y_{n+1})\big) &= d\big(F(y_{n-1}, x_{n-1}), F(y_n, x_n)\big) \\
&\leq \alpha\big(\big(g(y_{n-1}), g(x_{n-1})\big), \big(g(y_n), g(x_n)\big)\big) d\big(F(y_{n-1}, x_{n-1}), F(y_n, x_n)\big) \\
&\leq \psi\left(\frac{d(g(y_{n-1}), g(y_n)) + d(g(x_{n-1}), g(x_n))}{2}\right).
\end{aligned} \tag{3.6}
$$

Adding (3.5) and (3.6), we get

$$\frac{d(g(x_n), g(x_{n+1})) + d(g(y_n), g(y_{n+1}))}{2} \leq \psi\left(\frac{d(g(x_{n-1})g(x_n)) + d(g(y_{n-1}), g(y_n))}{2}\right).$$

Repeating the above process, we get

$$\frac{d(g(x_n), g(x_{n+1})) + d(g(y_n), g(y_{n+1}))}{2} \leq \psi^n\left(\frac{d(g(x_0)g(x_1)) + d(g(y_0), g(y_1))}{2}\right), \quad n \in N.$$

For $\varepsilon > 0$, there exists $n(\varepsilon) \in N$ such that

$$\sum_{n \geq n(\epsilon)} \psi^n\left(\frac{d(g(x_0), g(x_1)) + d(g(y_0), g(y_1))}{2}\right) < \frac{\varepsilon}{2}.$$

Let $n, m \in N$ be such that $m > n > n(\varepsilon)$, then by using the triangle inequality, we have

$$\frac{d(g(x_n), g(x_m)) + d(g(y_n), g(y_m))}{2} \leq \sum_{k=n}^{m-1} \frac{d(g(x_k), g(x_{k+1})) + d(g(y_n), g(y_{n+1}))}{2}$$

$$\leq \sum_{n \geq n(\epsilon)} \psi^n\left(\frac{d(g(x_0), g(x_1)) + d(g(y_0), g(y_1))}{2}\right)$$

$$< \frac{\varepsilon}{2},$$

$$d(g(x_n), g(x_m)) + d(g(y_n), g(y_m)) < \varepsilon.$$

Since

$$d(g(x_n), g(x_m)) \leq d(g(x_n), g(x_m)) + d(g(y_n), g(y_m)) < \varepsilon \quad \text{and}$$

$$d(g(y_n), g(y_m)) \leq d(g(x_n), g(x_m)) + d(g(y_n), g(y_m)) < \varepsilon,$$

hence $g(x_n)$ and $g(y_n)$ are Cauchy sequences in (X, d).

Since (X, d) is complete, therefore, $g(x_n)$ and $g(y_n)$ are convergent in (X, d).

There exist $x, y \in X$ such that $\lim_{n \to \infty} g(x_n) = x$ and $\lim_{n \to \infty} g(y_n) = y$.

By the continuity of g, we have

$$\lim_{n \to \infty} g(g(x_n)) = g(x) \quad \text{and} \quad \lim_{n \to \infty} g(g(y_n)) = g(y).$$

By commutativity of F and g, we get

$$g(g(x_{n+1})) = g(F(x_n, y_n)) = F(g(x_n), g(y_n)), \tag{3.7}$$

$$g(g(y_{n+1})) = g(F(y_n, x_n)) = F(g(y_n), g(x_n)). \tag{3.8}$$

We now show that

$$g(x) = F\left(\lim_{n \to \infty} g(x_n), \lim_{n \to \infty} g(y_n)\right)$$

$$= F(x, y),$$

$$g(y) = \lim_{n \to \infty} g(g(y_{n+1})) = \lim_{n \to \infty} F\big(g(y_n), g(x_n)\big)$$

$$= F(x, y) \quad \text{and} \quad g(y) = F(y, x).$$

Proceeding with limit $n \to \infty$ and using the continuity of F in (3.7) and (3.8), we get

$$g(x) = \lim_{n \to \infty} g\big(g(x_{n+1})\big) = \lim_{n \to \infty} F\big(g(x_n), g(y_n)\big)$$

$$= F\Big(\lim_{n \leftarrow \infty} g(y_n), \lim_{n \to \infty} g(x_n)\Big)$$

$$= F(y, x).$$

Thus,

$$g(x) = F(x, y) \quad \text{and} \quad g(y) = F(y, x).$$

Hence, we have proved that F and g have a coupled coincidence point. $\qquad\square$

Now, we will replace the continuity of F in Theorem 3.2 by a condition on sequences.

Theorem 3.3 *Let $(X, \le)$ be a partially ordered set, and let there exist a metric d on X such that (X, d) is a complete metric space. Let $F : X \times X \to X$ and $g : X \to X$ be maps, and let F have the g-mixed monotone property. Suppose that there exist $\psi \in \Psi$ and $\alpha : X^2 \times X^2 \to [0, +\infty)$ such that for $x, y, u, v \in X$ and the following*:

  (i) *conditions* (i), (ii) *and* (iii) *of Theorem 3.2 and* (3.1),
 (ii) *if $\{x_n\}$ and $\{y_n\}$ are sequences in X such that*

$$\alpha\big((g(x_n), g(y_n)), (g(x_{n+1}), g(y_{n+1}))\big) \ge 1 \quad and$$
$$\alpha\big((g(y_n), g(x_n)), (g(y_{n+1}), g(x_{n+1}))\big) \ge 1 \quad for\ all\ n$$

  and

$$\lim_{n \to \infty} g(x_n) = x \quad and \quad \lim_{n \to \infty} g(y_n) = y \quad for\ all\ x, y \in X,$$

  then

$$\alpha\big((g(x_n), g(y_n)), (g(x), g(y))\big) \ge 1 \quad and \quad \alpha\big((g(y_n), g(x_n)), (g(y), g(x))\big) \ge 1.$$

If there exist $x_0, y_0 \in X$ such that $g(x_0) \le F(x_0, y_0)$ and $g(y_0) \ge F(y_0, x_0)$, then F and g have a coupled coincidence point, that is, there exist $x, y \in X$ such that

$$F(x, y) = g(x) \quad and \quad F(y, x) = g(y).$$

Proof Proceeding along the same lines as in the proof of Theorem 3.2, we know that $\{g(x_n)\}$ and $\{g(y_n)\}$ are Cauchy sequences in the complete metric space (X, d). Then there exist $x, y \in X$ such that $\lim_{n \to \infty} g(x_n) = x$ and $\lim_{n \to \infty} g(y_n) = y$.

On the other hand, from the hypothesis, we obtain

$$\alpha\big((g(x_n),g(y_n)),(g(x),g(y))\big) \geq 1, \tag{3.9}$$

and similarly,

$$\alpha\big((g(y_n),g(x_n)),(g(y),g(x))\big) \geq 1. \tag{3.10}$$

Using the triangle inequality, (3.9) and the property of $\psi(t) < t$ for all $t > 0$, we get

$$
\begin{aligned}
d\big(F(x,y),g(x)\big) &\leq d\big(F(x,y),F(x_n,y_n)\big) + d\big(g(x_{n+1}),g(x)\big) \\
&\leq \alpha\big((g(x_n),g(y_n)),(g(x),g(y))\big) d\big(F(x_n,y_n),F(x,y)\big) + d\big(g(x_{n+1}),g(x)\big) \\
&\leq \psi\left(\frac{d(g(x_n),g(x)) + d(g(y_n),g(y))}{2}\right) + d\big(g(x_{n+1}),g(x)\big) \\
&< \frac{d(g(x_n),g(x)) + d(g(y_n),g(y))}{2} + d\big(g(x_{n+1}),g(x)\big).
\end{aligned}
$$

Similarly, using (3.10), we have

$$
\begin{aligned}
d\big(F(y,x),g(y)\big) &\leq d\big(F(y,x),F(y_n,x_n)\big) + d\big(g(y_{n+1}),g(y)\big) \\
&\leq \alpha\big((g(y_n),g(x_n)),(g(y),g(x))\big) d\big(F(y_n,x_n),F(y,x)\big) + d\big(g(y_{n+1}),g(y)\big) \\
&\leq \psi\left(\frac{d(g(y_n),g(y)) + d(g(x_n),g(x))}{2}\right) + d\big(g(y_{n+1}),g(y)\big) \\
&< \frac{d(g(y_n),g(y)) + d(g(x_n),g(x))}{2} + d\big(g(y_{n+1}),g(y)\big).
\end{aligned}
$$

Proceeding with limit $n \to \infty$ in above two inequalities, we get

$$d\big(F(x,y),g(x)\big) = 0 \quad \text{and} \quad d\big(F(y,x),g(y)\big) = 0.$$

Thus, $F(x,y) = g(x)$ and $F(y,x) = g(y)$. $\qquad\square$

Example 3.4 Let $X = [0,1]$ and $d : X \times X \to R$ be a standard metric.

Define a mapping $F : X \times X \to X$ by $F(x,y) = \frac{1}{4}(x-y)$ and $g : X \to X$ by $g(x) = \frac{x}{2}$ for all $x,y \in X$.

Consider a mapping $\alpha : X^2 \times X^2 \to [0,+\infty)$ to be such that

$$\alpha\big((g(x),g(y)),(g(u),g(v))\big) = \begin{cases} 1 & \text{if } g(x) \geq g(y) \text{ and } g(u) \geq g(v), \\ 0 & \text{otherwise,} \end{cases}$$

$$d\big(F(x,y),F(u,v)\big) = \left|\frac{(x-y)}{4} - \frac{(u-v)}{4}\right| \leq \frac{1}{4}\big(|x-u| + |v-y|\big) = \frac{1}{4}\big(d(x,u) + d(y,v)\big).$$

It follows that

$$\alpha\big(\big(g(x), g(y)\big), g\big((u), g(v)\big)\big) d\big(F(x,y), F(u,v)\big)$$

$$\leq \frac{1}{4}\big(d(x,u) + d(y,v)\big) \leq \frac{1}{2}\big(d(x,u) + d(y,v)\big)$$

$$= \big(d\big(g(x), g(u)\big) + d\big(g(y), g(v)\big)\big).$$

Thus (3.1) holds for $\psi(t) = t$ for all $t > 0$, and we also see that F satisfies the g-mixed monotone property as well as F and g commute. Therefore, all the hypotheses of Theorem 3.2 are fulfilled. Then there exists a coupled coincidence point of F and g. In this case, $(0,0)$ is a coupled coincidence point of F and g.

Authors' contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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