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Strong convergence theorems for common zeros of a family of accretive operators

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Abstract
In this paper, common zeros of a family of accretive operators are investigated based on the Kirk-like proximal point algorithm. A strong convergence theorem is established in a reflexive Banach space.

Keywords: accretive operator; resolvent; strict convexness; zero

1 Introduction

In the real world, many important problems have reformulations which require finding common zero (fixed) points of nonlinear operators, for instance, image recovery, inverse problems, transportation problems and optimization problems. It is well known that the convex feasibility problem is a special case of the common zero (fixed) points of nonlinear operators. In 1971, Kirk [1] introduced a parallel iterative process for finding a family of nonexpansive mappings. Common fixed point theorems were established in a Banach space; for more details, see [1]. For studying zero points of monotone operators, the most well-known algorithm is the proximal point algorithm; see [2, 3] and the references therein. It is known that Rockfellar's proximal point algorithm is, in general, weak convergence; see [4] and the references therein.

Recently, many authors have been devoted to investigating the strong convergence of a proximal point algorithm. Strong convergence theorems for zero points of accretive operators were established; see, for example, [5–29] and the references therein.

In this paper, we are concerned with the problem of finding a common zero of a family of accretive operators based on the Kirk-like proximal point algorithm. A strong convergence theorem is established in a reflexive Banach space.

2 Preliminaries

Let E be a Banach space with the dual E^*. Let $\langle\cdot,\cdot\rangle$ denote the pairing between E and E^*. The normalized duality mapping $J : E \to 2^{E^*}$ is defined by

$$J(x) = \left\{f \in E^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2\right\}, \quad \forall x \in E.$$

Let $U_E = \{x \in E : \|x\| = 1\}$. E is said to be smooth or to have a Gâteaux differentiable norm if the limit $\lim_{t\to 0} \frac{\|x+ty\|-\|x\|}{t}$ exists for each $x, y \in U_E$. E is said to have a uniformly Gâteaux differentiable norm if for each $y \in U_E$, the limit is attained uniformly for all $x \in U_E$. E is said to be uniformly smooth or to have a uniformly Fréchet differentiable norm if the limit is attained uniformly for $x, y \in U_E$.

It is well known that (uniform) Fréchet differentiability of the norm of E implies (uniform) Gâteaux differentiability of the norm of E. It is known that if the norm of E is uniformly Gâteaux differentiable, then the duality mapping j is single-valued and uniformly norm to weak* continuous on each bounded subset of E. In the sequel, we use j to denote the single-valued normalized duality mapping.

A Banach space E is said to be strictly convex if and only if

$$\|x\| = \|y\| = \big\|(1-\lambda)x + \lambda y\big\|$$

for $x, y \in E$, and $0 < \lambda < 1$ implies that $x = y$.

Recall that a closed convex subset C of a Banach space E is said to have a normal structure if for each bounded closed convex subset K of C which contains at least two points, there exists an element x of K which is not a diametral point of K, *i.e.*, $\sup\{\|x-y\| : y \in K\} < d(K)$, where $d(K)$ is the diameter of K.

Let C be a nonempty closed convex subset of E. Let $T : C \to C$ be a mapping. In this paper, we use $F(T)$ to denote the set of fixed points of T. Recall that T is said to be nonexpansive iff $\|Tx - Ty\| \leq \|x - y\|, \forall x, y \in C$. For the existence of fixed points of a nonexpansive mapping, we refer readers to [30].

Let I denote the identity operator on E. An operator $A \subset E \times E$ with the domain $D(A) = \{z \in E : Az \neq \emptyset\}$ and the range $R(A) = \bigcup\{Az : z \in D(A)\}$ is said to be accretive if for each $x_i \in D(A)$ and $y_i \in Ax_i$, $i = 1, 2$, there exists $j(x_1 - x_2) \in J(x_1 - x_2)$ such that $\langle y_1 - y_2, j(x_1 - x_2)\rangle \geq 0$. An accretive operator A is said to be m-accretive if $R(I + rA) = E$ for all $r > 0$. In this paper, we use $A^{-1}(0)$ to denote the set of zero points of A. For an accretive operator A, we can define a nonexpansive single-valued mapping $J_r : R(I + rA) \to D(A)$ by $J_r = (I + rA)^{-1}$ for each $r > 0$, which is called the resolvent of A.

Next we give the following lemmas which play an important role in this article.

Lemma 2.1 [31] *Let E be a real Banach space and let J be the normalized duality mapping. Then there exists $j(x + y) \in J(x + y)$ such that*

$$\|x + y\|^2 \leq \|x\|^2 + 2\big\langle y, j(x + y)\big\rangle, \quad \forall x, y \in E.$$

Lemma 2.2 [32] *Let C be a closed convex subset of a strictly convex Banach space E. Let $S : C \to C$ and $T : C \to c$ be two nonexpansive mappings. Suppose that $F(S) \cap F(T)$ is nonempty. Then the mapping $wS + (1 - w)T$, where $s \in (0, 1)$ is a real number, is well defined nonexpansive with $F(wS + (1 - w)T) = F(S) \cap F(T)$.*

Lemma 2.3 [33] *Let E be a real reflexive Banach space with the uniformly Gâteaux differentiable norm and the normal structure, and let C be a nonempty closed convex subset of E. Let $T : C \to C$ be a nonexpansive mapping with a fixed point. Let $\{x_t\}$ be a sequence generated by the following $x_t = tu + (1 - t)Tx_t$, where $t \in (0, 1)$ and $u \in C$ is a fixed element. Then $\{x_t\}$ converges strongly as $t \to 0$ to a fixed point x^* of T, which is the unique solution in $F(T)$ to the following variational inequality $\langle u - x^*, j(x^* - p)\rangle \geq 0, \forall p \in F(T)$.*

Lemma 2.4 [34] *Let $\{a_n\}$, $\{b_n\}$, $\{c_n\}$ and $\{d_n\}$ be three nonnegative real sequences satisfying $a_{n+1} \le (1-b_n)a_n + b_n c_n + d_n$, $\forall n \ge n_0$, where n_0 is some positive integer, $\{b_n\}$ is a number sequence in $(0,1)$ such that $\sum_{n=n_0}^{\infty} b_n = \infty$, $\{c_n\}$ is a number sequence such that $\limsup_{n\to\infty} c_n \le 0$, and $\{d_n\}$ is a positive number sequence such that $\sum_{n=n_0}^{\infty} d_n < \infty$. Then $\lim_{n\to\infty} a_n = 0$.*

3 Main results

Theorem 3.1 *Let E be a real reflexive, strictly convex Banach space with the uniformly Gâteaux differentiable norm. Let $N \ge 1$ be some positive integer. Let A_m be an m-accretive operator in E for each $m \in \{1,2,\dots,N\}$. Assume that $C := \bigcap_{m=1}^{N} \overline{D(A_m)}$ is convex and has the normal structure. Let $\{\alpha_n\}$ be a real number sequence in $(0,1)$ such that $\lim_{n\to\infty} \alpha_n = 0$, $\sum_{n=1}^{\infty} \alpha_n = \infty$ and $\sum_{n=1}^{\infty} |\alpha_{n+1} - \alpha_n| < \infty$, let $\{\beta_{n,m}\}$ be a real number sequence in $(0,1)$ such that $\sum_{m=1}^{N} \beta_{n,m} = 1$, $\lim_{n\to\infty} \beta_{n,m} = \beta_m$ and $\sum_{n=1}^{\infty} |\beta_{n+1,m} - \beta_{n,m}| < \infty$, let $\{r_m\}$ be a positive real number sequence, and let $\{e_{n,m}\}$ be a sequence in E such that $\sum_{n=1}^{\infty} \|e_{n,m}\| < \infty$ for each $m \in \{1,2,\dots,N\}$. Assume that $\bigcap_{m=1}^{N} A_m^{-1}(0)$ is not empty. Let $\{x_n\}$ be a sequence generated in the following manner*:

$$x_1 \in C, \quad x_{n+1} = \alpha_n u + (1-\alpha_n) \sum_{m=1}^{N} \beta_{n,m} J_{r_m}(x_n + e_{n,m}), \quad \forall n \ge 1,$$

where u is a fixed element in C and $J_{r_m} = (I + r_m A_m)^{-1}$. Then the sequence $\{x_n\}$ converges strongly to $\bar{x}$, which is the unique solution to the following variational inequality $\langle u - \bar{x}, j(p - \bar{x})\rangle \le 0$, $\forall p \in \bigcap_{m=1}^{N} A_m^{-1}(0)$.

Proof The proof is split into five steps.

Step 1. Show that $\{x_n\}$ is bounded.

Put $y_n = \sum_{m=1}^{N} \beta_{n,m} J_{r_m}(x_n + e_{n,m})$. Fixing $p \in \bigcap_{m=1}^{N} A_m^{-1}(0)$, we find that

$$\|y_n - p\| \le \sum_{m=1}^{N} \beta_{n,m} \big\| J_{r_m}(x_n + e_{n,m}) - p \big\| \le \|x_n - p\| + \sum_{m=1}^{N} \|e_{n,m}\|.$$

It follows that

$$\begin{aligned}
\|x_{n+1} - p\| &\le \alpha_n \|u - p\| + (1-\alpha_n)\|y_n - p\| \\
&\le \alpha_n \|u - p\| + (1-\alpha_n)\|x_n - p\| + \sum_{m=1}^{N} \|e_{n,m}\| \\
&\le \max\big\{\|u - p\|, \|x_n - p\|\big\} + \sum_{m=1}^{N} \|e_{n,m}\|.
\end{aligned}$$

By induction, we find that

$$\|x_{n+1} - p\| \le \max\big\{\|u - p\|, \|x_1 - p\|\big\} + \sum_{i=1}^{\infty} \sum_{m=1}^{N} \|e_{i,m}\| < \infty.$$

This proves Step 1.

Step 2. Show that $\lim_{n\to\infty}\|x_{n+1}-x_n\|=0$.

Note that

$$\begin{aligned} y_n-y_{n-1} &= \sum_{m=1}^{N}\beta_{n,m}\big(J_{r_m}(x_n+e_{n,m})-J_{r_m}(x_{n-1}+e_{n-1,m})\big) \\ &\quad+\sum_{m=1}^{N}(\beta_{n,m}-\beta_{n-1,m})J_{r_m}(x_{n-1}+e_{n-1,m}). \end{aligned}$$

It follows that

$$\begin{aligned} \|y_n-y_{n-1}\| &\le \sum_{m=1}^{N}\beta_{n,m}\big\|J_{r_m}(x_n+e_{n,m})-J_{r_m}(x_{n-1}+e_{n-1,m})\big\| \\ &\quad+\sum_{m=1}^{N}|\beta_{n,m}-\beta_{n-1,m}|\big\|J_{r_m}(x_{n-1}+e_{n-1,m})\big\| \\ &\le \|x_n-x_{n-1}\|+\sum_{m=1}^{N}\|e_{n,m}\|+\sum_{m=1}^{N}\|e_{n-1,m}\| \\ &\quad+\sum_{m=1}^{N}|\beta_{n,m}-\beta_{n-1,m}|\big\|J_{r_m}(x_{n-1}+e_{n-1,m})\big\| \\ &\le \|x_n-x_{n-1}\|+M_1\sum_{m=1}^{N}|\beta_{n,m}-\beta_{n-1,m}|+\sum_{m=1}^{N}\|e_{n,m}\|+\sum_{m=1}^{N}\|e_{n-1,m}\|, \end{aligned}$$

where M_1 is an appropriate constant such that

$$M_1=\max\Big\{\sup_{n\ge1}\big\|J_{r_1}(x_n+e_{n,1})\big\|,\sup_{n\ge1}\big\|J_{r_2}(x_n+e_{n,2})\big\|,\dots,\sup_{n\ge1}\big\|J_{r_N}(x_n+e_{n,N})\big\|\Big\}.$$

It follows that

$$\begin{aligned} \|x_{n+1}-x_n\| &\le (1-\alpha_n)\|y_n-y_{n-1}\|+|\alpha_n-\alpha_{n-1}|\|u-y_{n-1}\| \\ &\le (1-\alpha_n)\|x_n-x_{n-1}\|+M_2\Bigg(\sum_{m=1}^{N}|\beta_{n,m}-\beta_{n-1,m}|+|\alpha_n-\alpha_{n-1}|\Bigg) \\ &\quad+\sum_{m=1}^{N}\|e_{n,m}\|+\sum_{m=1}^{N}\|e_{n-1,m}\|, \end{aligned}$$

where $M_2=\max\{M_1,\sup_{n\ge1}\|u-y_n\|\}$. In view of Lemma 2.4, we conclude Step 2.

Step 3. Show that $\lim_{n\to\infty}\|Tx_n-x_n\|=0$, where $T=\sum_{m=1}^{N}\beta_mJ_{r_m}$. In light of Lemma 2.2, we see that T is nonexpansive with $F(T)=\bigcap_{m=1}^{N}F(J_{r_m})=\bigcap_{m=1}^{N}A_m^{-1}(0)$. Since

$$\begin{aligned} \|y_n-Tx_n\| &\le \Bigg\|\sum_{m=1}^{N}\beta_{n,m}J_{r_m}(x_n+e_{n,m})-\sum_{m=1}^{N}\beta_mJ_{r_m}(x_n+e_{n,m})\Bigg\| \\ &\quad+\Bigg\|\sum_{m=1}^{N}\beta_mJ_{r_m}(x_n+e_{n,m})-\sum_{m=1}^{N}\beta_mJ_{r_m}x_n\Bigg\| \\ &\le \sum_{m=1}^{N}|\beta_{n,m}-\beta_m|\big\|J_{r_m}(x_n+e_{n,m})\big\|+\sum_{m=1}^{N}\|e_{n,m}\|, \end{aligned}$$

we find from the restrictions imposed on the control sequences that $\lim_{n\to\infty}\|Tx_n - y_n\| = 0$. Since

$$\begin{aligned}\|Tx_n - x_n\| &\le \|x_n - x_{n+1}\| + \|x_{n+1} - Tx_n\| \\ &\le \|x_n - x_{n+1}\| + \alpha_n\|u - Jx_n\| + (1-\alpha_n)\|y_n - Tx_n\|,\end{aligned}$$

we conclude Step 3.

Step 4. Show that $\limsup_{n\to\infty}\langle u-\bar{x}, j(x_n-\bar{x})\rangle \le 0$, where $\bar{x} = \lim_{t\to 0} x_t$, and x_t solves the fixed point equation

$$x_t = tu + (1-t)Tx_t, \quad \forall t \in (0,1).$$

It follows that

$$\begin{aligned}\|x_t - x_n\|^2 &= t\langle u - x_n, j(x_t - x_n)\rangle + (1-t)\langle Tx_t - x_n, j(x_t - x_n)\rangle \\ &= t\langle u - x_t, j(x_t - x_n)\rangle + t\langle x_t - x_n, j(x_t - x_n)\rangle \\ &\quad + (1-t)\langle Tx_t - Tx_n, j(x_t - x_n)\rangle + (1-t)\langle Tx_n - x_n, j(x_t - x_n)\rangle \\ &\le t\langle u - x_t, j(x_t - x_n)\rangle + \|x_t - x_n\|^2 + \|Tx_n - x_n\|\|x_t - x_n\|, \quad \forall t \in (0,1).\end{aligned}$$

This implies that

$$\langle x_t - u, j(x_t - x_n)\rangle \le \frac{1}{t}\|Tx_n - x_n\|\|x_t - x_n\|, \quad \forall t \in (0,1).$$

Since $\lim_{n\to\infty}\|Tx_n - x_n\| = 0$, we find that $\limsup_{n\to\infty}\langle x_t - u, j(x_t - x_n)\rangle \le 0$. In view of the fact that j is strong to weak* uniformly continuous on bounded subsets of E, we find that

$$\begin{aligned}&\bigl|\langle u-\bar{x}, j(x_n-\bar{x})\rangle - \langle x_t - u, j(x_t - x_n)\rangle\bigr| \\ &\quad\le \bigl|\langle u-\bar{x}, j(x_n-\bar{x})\rangle - \langle u-\bar{x}, j(x_n - x_t)\rangle\bigr| \\ &\qquad + \bigl|\langle u-\bar{x}, j(x_n - x_t)\rangle - \langle x_t - u, j(x_t - x_n)\rangle\bigr| \\ &\quad\le \bigl|\langle u-\bar{x}, j(x_n-\bar{x}) - j(x_n - x_t)\rangle\bigr| + \bigl|\langle u-\bar{x}+x_t-u, j(x_n - x_t)\rangle\bigr| \\ &\quad\le \|u-\bar{x}\|\bigl\|j(x_n-\bar{x}) - j(x_n - x_t)\bigr\| + \|x_t-\bar{x}\|\|x_n - x_t\|.\end{aligned}$$

Since $x_t \to \bar{x}$, as $t \to 0$, we have

$$\lim_{t\to 0}\bigl|\langle x_t - u, j(x_t - x_n)\rangle - \langle u-\bar{x}, j(x_n-\bar{x})\rangle\bigr| = 0.$$

For $\epsilon > 0$, there exists $\delta > 0$ such that $\forall t \in (0,\delta)$, we have

$$\langle u-\bar{x}, j(x_n-\bar{x})\rangle \le \langle x_t - u, j(x_t - x_n)\rangle + \epsilon.$$

This implies that

$$\limsup_{n\to\infty}\langle u-\bar{x}, j(x_n-\bar{x})\rangle \le \limsup_{n\to\infty}\langle x_t - u, j(x_t - x_n)\rangle + \epsilon.$$

Since ϵ is arbitrarily chosen, we find that $\limsup_{n\to\infty}\langle u-\bar{x}, j(x_n-\bar{x})\rangle \le 0$. This implies that $\limsup_{n\to\infty}\langle u-\bar{x}, j(x_{n+1}-\bar{x})\rangle \le 0$. This proves Step 4.

Step 5. Show that $x_n \to \bar{x}$ as $n \to \infty$.

Using Lemma 2.1, we find that

$$\begin{aligned}
\|x_{n+1}-\bar{x}\|^2 &= \left\|\alpha_n(u-\bar{x}) + (1-\alpha_n)\left(\sum_{m=1}^{N}\beta_{n,m}\bigl(J_{r_m}(x_n+e_{n,m})-\bar{x}\bigr)\right)\right\|^2 \\
&\le (1-\alpha_n)^2\left\|\sum_{m=1}^{N}\beta_{n,m}\bigl(J_{r_m}(x_n+e_{n,m})-\bar{x}\bigr)\right\|^2 + 2\alpha_n\bigl\langle u-\bar{x}, j(x_{n+1}-\bar{x})\bigr\rangle \\
&\le (1-\alpha_n)^2\sum_{m=1}^{N}\beta_{n,m}\|x_n+e_{n,m}-\bar{x}\|^2 + 2\alpha_n\bigl\langle u-\bar{x}, j(x_{n+1}-\bar{x})\bigr\rangle \\
&\le (1-\alpha_n)\|x_n-\bar{x}\|^2 + \lambda_n + 2\alpha_n\bigl\langle u-\bar{x}, j(x_{n+1}-\bar{x})\bigr\rangle,
\end{aligned}$$

where $\lambda_n = \sum_{m=1}^{N}(\|e_{n,m}\|^2 + 2\|e_{n,m}\|\|x_n-\bar{x}\|)$. We, therefore, find that $\sum_{n=1}^{\infty}\lambda_n < \infty$. From Lemma 2.4, we find the desired conclusion. This proves the proof. □

Remark 3.2 Theorem 3.1 is still valid in the framework of the space which is uniformly convex and the norm is uniformly Gâteaux differentiable.

4 Applications

In this section, we consider an application of Theorem 3.1. Let $A : C \to E^*$ be a single-valued monotone operator which is hemicontinuous; that is, continuous along each line segment in C with respect to the weak* topology of E^*. Consider the following variational inequality:

$$\text{find } x \in C \text{ such that } \langle y-x, Ax\rangle \ge 0, \quad \forall y \in C.$$

The solution set of the variational inequality is denoted by $VI(C,A)$. Recall that the normal cone $N_C(x)$ for C at a point $x \in C$ is defined by

$$N_C(x) = \bigl\{x^* \in E^* : \bigl\langle y-x, x^*\bigr\rangle \le 0, \forall y \in C\bigr\}.$$

Now, we are in a position to give the result on the variational inequality.

Theorem 4.1 *Let E be a real reflexive, strictly convex Banach space with the uniformly Gâteaux differentiable norm. Let $N \ge 1$ be some positive integer and let C be a nonempty closed and convex subset of E. Let $A_m : C \to E^*$ be a single-valued, monotone and hemicontinuous operator. Assume that $\bigcap_{m=1}^{N} VI(C, A_m)$ is not empty and C has the normal structure. Let $\{\alpha_n\}$ be a real number sequence in $(0,1)$ such that $\lim_{n\to\infty}\alpha_n = 0$, $\sum_{n=1}^{\infty}\alpha_n = \infty$ and $\sum_{n=1}^{\infty}|\alpha_{n+1}-\alpha_n| < \infty$, let $\{\beta_{n,m}\}$ be a real number sequence in $(0,1)$ such that $\sum_{m=1}^{N}\beta_{n,m} = 1$, $\lim_{n\to\infty}\beta_{n,m} = \beta_m$ and $\sum_{n=1}^{\infty}|\beta_{n+1,m}-\beta_{n,m}| < \infty$, let $\{r_m\}$ be a positive real number sequence for each $m \in \{1,2,\ldots,N\}$. Assume that $\{x_n\}$ is a sequence generated in the following manner*:

$$x_1 \in C, \quad x_{n+1} = \alpha_n u + (1-\alpha_n)\sum_{m=1}^{N}\beta_{n,m} VI\left(C, A_m + \frac{1}{r_m}(I - x_n)\right), \quad \forall n \ge 1,$$

where u is a fixed element in C. Then the sequence $\{x_n\}$ converges strongly to $\bar{x}$, which is the unique solution to the following variational inequality $\langle u - \bar{x}, j(p - \bar{x})\rangle \leq 0$, $\forall p \in \bigcap_{m=1}^{N} VI(C, A_m)$.

Proof First, we define a mapping $T_m \subset E \times E^*$ by

$$T_m x = \begin{cases} A_m x + N_C x, & x \in C, \\ \emptyset, & x \notin C. \end{cases}$$

From Rockafellar [33], we find that T_m is maximal monotone and $T_m^{-1}(0) = VI(C, A_m)$. For each $r_m > 0$ and $x_n \in E$, we see that there exists a unique $x_{r_m} \in D(T_m)$ such that $x_n \in x_{r_m} + r_m T_m(x_{r_m})$, where $x_{r_m} = (I + r_m T_m)^{-1} x_n$. Notice that

$$y_{n,m} = VI\left(C, A_m + \frac{1}{r_m}(I - x_n)\right),$$

which is equivalent to

$$\left\langle y - y_{n,m}, A_m y_{n,m} + \frac{1}{r_m}(y_{n,m} - x_n)\right\rangle \geq 0, \quad \forall y \in C,$$

that is, $-A_m y_{n,m} + \frac{1}{r_m}(x_n - y_{n,m}) \in N_C(y_{n,m})$. This implies that $y_{n,m} = (I + r_m T_m)^{-1} x_n$. In light of Theorem 3.1, we draw the desired conclusion immediately. □

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors contributed equally to this manuscript. Both authors read and approved the final manuscript.

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