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Fixed points by certain iterative schemes with applications

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Abstract

The main aim of this paper is to present the concept of general Mann and general Ishikawa type double-sequences iterations with errors to approximate fixed points. We prove that the general Mann type double-sequence iteration process with errors converges strongly to a coincidence point of two continuous pseudo-contractive mappings, each of which maps a bounded closed convex nonempty subset of a real Hilbert space into itself. Moreover, we discuss equivalence from the S, T -stabilities point of view under certain restrictions between the general Mann type double-sequence iteration process with errors and the general Ishikawa iterations with errors. An application is also given to support our idea using compatible-type mappings.

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1 Introduction

In the last few decades investigations of fixed points by some iterative schemes have attracted many mathematicians. With the recent rapid developments in fixed point theory, there has been a renewed interest in iterative schemes. The properties of iterations between the type of sequences and kind of operators have not been completely studied and are now under discussion. The theory of operators has occupied a central place in modern research using iterative schemes because of its promise of enormous utility in fixed point theory and its applications. There are a number of papers that have studied fixed points by some iterative schemes (see [1]). It is rather interesting to note that the type of operators play a crucial role in investigations of fixed points.

The Mann iterative scheme was invented in 1953 (see [1–3]), and it is used to obtain convergence to a fixed point for many classes of mappings (see [4–16] and others). The idea of considering fixed point iteration procedures with errors comes from practical numerical computations. This topic of research plays an important role in the stability problem of fixed point iterations. In 1995, Liu [17] initiated a study of fixed point iterations with errors. Several authors have proved some fixed point theorems for Mann-type iterations with errors using several classes of mappings (see [18–28] and others).

Suppose that H is a real Hilbert space and A is a nonlinear mapping of H into itself. The map A is said to be accretive if $\forall x, y \in D(A)$, we have that

$$\langle Ax - Ay, x - y \rangle \geq 0, \tag{1}$$

and it is said to be strongly accretive if $A - kI$ is accretive, where $k \in (0, 1)$ is a constant and I denotes the identity operator on H .

The map A is said to be ϕ -strongly accretive if $\forall x, y \in E$, exists a strictly increasing function $\phi : [0, \infty) \rightarrow [0, \infty)$ with $\phi(0) = 0$ such that

$$\langle Ax - Ay, x - y \rangle \geq \phi(\|x - y\|) \|x - y\|,$$

and it is called uniformly accretive if there exists a strictly increasing function $\psi : [0, \infty) \rightarrow [0, \infty)$ with $\psi(0) = 0$ such that $\langle Ax - Ay, x - y \rangle \geq \psi(\|x - y\|)$.

Let $N(A) = \{x^* \in H : Ax^* = 0\}$ denote the null space (set of zero) of A . If $N(A) \neq \emptyset$ and (1) holds for all $x \in D(A)$ and $y \in N(A)$, then A is said to be quasi-accretive. The notions of strongly, ϕ -strongly, uniformly quasi-accretive are similarly defined. A is said to be m -accretive if $\forall r > 0$ the operator $(I + rA)$ is surjective. Closely related to the class of accretive maps is the class of pseudo-contractive maps.

A map $T : H \rightarrow H$ is said to be pseudo-contractive if $\forall x, y \in D(T)$ we have that

$$\langle (I - T)x - (I - T)y, x - y \rangle \geq 0, \tag{2}$$

observe that T is pseudo-contractive if and only if $A = (I - T)$ is accretive.

A mapping $T : H \rightarrow H$ is called Lipschitzian (or L -Lipschitzian) if there exists $L > 0$ such that

$$\|Tx - Ty\| \leq L\|x - y\|, \quad \forall x, y \in H.$$

In the sequel we use $L > 1$.

Definition 1.1 (see e.g. [24]) Let \mathbb{N} denote the set of all natural numbers, and let E be a normed linear space. By a double sequence in E we mean a function $f : \mathbb{N} \times \mathbb{N} \rightarrow E$ defined by $f(n, m) = x_{n,m} \in E$.

The double sequence $\{x_{n,m}\}$ is said to converge strongly to x^* if for a given $\epsilon > 0$, there exist integers $N, M > 0$ such that $\forall n \geq N, m \geq M$, we have that

$$\|x_{n,m} - x^*\| < \epsilon.$$

If $\forall n, r \geq N, m, t \geq M$, we have that

$$\|x_{n,r} - x_{m,t}\| < \epsilon,$$

then the double sequence is said to be Cauchy. Furthermore, if for each fixed $n, x_{n,m} \rightarrow x_n^*$ as $m \rightarrow \infty$ and then $x_n^* \rightarrow x^*$ as $n \rightarrow \infty$, so $x_{n,m} \rightarrow x^*$ as $n, m \rightarrow \infty$.

In 2002, Moore [24] introduced the following theorem.

Theorem A Let C be a bounded closed convex nonempty subset of a (real) Hilbert space H , and let $T : C \rightarrow C$ be a continuous pseudo-contractive map. Let $\{\alpha_n\}_{n \geq 0}, \{a_k\}_{k \geq 0} \subset (0, 1)$ be real sequences satisfying the following conditions:

- (i) $\lim_{k \rightarrow \infty} a_k = 1$ (monotonically);

- (ii) $\lim_{k,r \rightarrow \infty} \frac{a_k - a_r}{1 - a_k} = 0, \forall 0 < r \leq k;$
- (iii) $\lim_{n \rightarrow \infty} \alpha_n = 0;$
- (iv) $\sum_{n \geq 0} \alpha_n = \infty.$

For an arbitrary but fixed $w \in C$, and for each $k \geq 0$, define $T_k : C \rightarrow C$ by

$$T_k x := (1 - a_k)w + a_k T x, \quad \forall x \in C.$$

Then the double sequence $\{x_{k,n}\}_{k \geq 0, n \geq 0}$ generated from an arbitrary $x_{0,0} \in C$ by

$$x_{k,n+1} = (1 - \alpha_n)x_{k,n} + \alpha_n T_k x_{k,n}, \quad k, n \geq 0, \tag{3}$$

converges strongly to a fixed point x_{∞}^* of T in C .

The two most popular iteration procedures for obtaining fixed points of T , when the Banach principle fails, are doubly Mann iterations with errors [29] defined by

$$u_{k,n+1} = (1 - \alpha_n)u_{k,n} + \alpha_n T u_{k,n} + \alpha_n u_n,$$

and doubly Ishikawa iterations with errors defined by

$$\begin{aligned} x_{k,n+1} &= (1 - \alpha_n)x_{k,n} + \alpha_n T z_{k,n} + \alpha_n v_n, \\ z_{k,n} &= (1 - \beta_n)x_{k,n} + \beta_n T x_{k,n} + \beta_n w_n. \end{aligned}$$

The sequences $\{\alpha_n\} \subset (0, 1), \{\beta_n\} \subset [0, 1)$ satisfy

$$\lim_{n \rightarrow \infty} \alpha_n = \lim_{n \rightarrow \infty} \beta_n = 0, \quad \sum_{n=1}^{\infty} \alpha_n = \infty.$$

A reasonable conjecture is that the doubly Ishikawa iteration with error and the corresponding doubly Mann iteration with error are equivalent for all maps for which either method provides convergence to a fixed point.

In the present paper, we define the following iteration which will be called the general Mann iteration process with errors:

$$S u_{k,n+1} = (1 - \alpha_n)S u_{k,n} + \alpha_n T u_{k,n} + \alpha_n u_n. \tag{4}$$

Using this general Mann iteration process, we give a strong convergence theorem in the double-sequence setting.

It should be remarked that in (4), if we put $S = I$, where I denotes the identity mapping, then we obtain the Mann iteration process with errors (see [30]).

The general doubly Ishikawa iteration with error is defined by

$$S x_{k,n+1} = (1 - \alpha_n)S x_{k,n} + \alpha_n T z_{k,n} + \alpha_n v_n, \tag{5}$$

$$S z_{k,n} = (1 - \beta_n)S x_{k,n} + \beta_n T x_{k,n} + \beta_n w_n. \tag{6}$$

The sequences $\{\alpha_n\} \subset (0, 1), \{\beta_n\} \subset [0, 1)$ satisfy

$$\lim_{n \rightarrow \infty} \alpha_n = \lim_{n \rightarrow \infty} \beta_n = 0, \quad \sum_{n=1}^{\infty} \alpha_n = \infty. \tag{7}$$

It should be remarked that in (5) and (6), if we put $S = I$, where I denotes the identity mapping, then we obtain the Ishikawa iteration process with errors (see [31, 32]).

2 A strong convergence theorem

In this section, it is proved that a general Mann-type double-sequence iteration process with error converges strongly to a coincidence point of the continuous pseudo-contractive mappings S and T both of them map C into C (where C is a bounded closed convex nonempty subset of a (real) Hilbert space). Now, we give the following theorem.

Theorem 2.1 *Let C be a bounded closed convex nonempty subset of a (real) Hilbert space H , and let $S, T : C \rightarrow C$ be continuous pseudo-contractive maps. Let $\{\alpha_n\}_{n \geq 0}, \{a_k\}_{k \geq 0} \subset (0, 1)$ be real sequences satisfying the following conditions:*

- (i) $\lim_{k \rightarrow \infty} a_k = 1$ (monotonically);
- (ii) $\lim_{k,r \rightarrow \infty} \frac{a_k - a_r}{1 - a_k} = 0, \forall 0 < r \leq k$;
- (iii) $\lim_{n \rightarrow \infty} \alpha_n = 0$;
- (iv) $\sum_{n \geq 0} \alpha_n = \infty$.

For an arbitrary but fixed $w \in C$, and for each $k \geq 0$, define $T_k : C \rightarrow C$ by

$$T_k x = (1 - a_k)w + a_k T x + (1 - a_k)u_k, \quad \forall x \in C.$$

Then the double sequence $\{x_{k,n}\}_{k \geq 0, n \geq 0}$ generated from an arbitrary $x_{0,0} \in C$ by

$$Sx_{k,n+1} = (1 - \alpha_n)Sx_{k,n} + \alpha_n T_k x_{k,n} + (1 - \alpha_n)u_{k,n}, \quad k, n \geq 0, \tag{8}$$

converges strongly to a coincidence point x_∞^* of S and $T \in C$.

Proof Clearly, $CF(T) \neq \emptyset$ and $CF(S) \neq \emptyset$ (see e.g. [33]), where the set of coincidence points of T is denoted by $CF(T)$ and the set of coincidence points of S is denoted by $CF(S)$.

Now, we have

$$\langle T_k x - T_k y, Sx - Sy \rangle = a_k \langle Tx - Ty, Sx - Sy \rangle \leq a_k \|Sx - Sy\|^2$$

so that for all $k \geq 0$, T_k is continuous and strongly pseudo-contractive. Also, C is invariant under T_k for all k by convexity. Hence, T_k has a unique fixed point $x_k^* \in C, \forall k \geq 0$. It thus suffices to prove the following:

- (1) for each fixed $k \geq 0, Sx_{k,n} \rightarrow Sx_k^* \in C$ as $n \rightarrow \infty$;
- (2) $Sx_k^* \rightarrow Sx_\infty^* \in C$ as $k \rightarrow \infty$;
- (3) $x_\infty^* \in CF(S) \cap CF(T)$.

The first is known, but for completeness we give the details.

Now, let $d = \text{diam } C$ and $b_k = 1 - a_k \in (0, 1), \forall k$. Then

$$\begin{aligned} & \|Sx_{k,n+1} - Sx_k^*\|^2 \\ &= \|(1 - \alpha_n)Sx_{k,n} + \alpha_n T_k x_{k,n} + (1 - \alpha_n)u_{k,n} - Sx_k^*\|^2 \\ &= \|Sx_{k,n} - Sx_k^* - \alpha_n(Sx_{k,n} - T_k x_{k,n}) + (1 - \alpha_n)u_{k,n}\|^2 \\ &= \|Sx_{k,n} - Sx_k^*\|^2 - 2\alpha_n \langle Sx_{k,n} - T_k x_{k,n}, Sx_{k,n} - Sx_k^* \rangle - 2(1 - \alpha_n) \langle u_{k,n}, Sx_{k,n} - Sx_k^* \rangle \end{aligned}$$

$$\begin{aligned}
 & + \alpha_n^2 \|Sx_{k,n} - T_k x_{k,n}\|^2 + 2\alpha_n(1 - \alpha_n) \langle x_{k,n} - T_k x_{k,n}, u_n \rangle + (1 - \alpha_n)^2 \|u_{k,n}\|^2 \\
 \leq & \|Sx_{k,n} - Sx_k^*\|^2 - 2\alpha_n a_k \|Sx_{k,n} - Sx_k^*\|^2 - 2(1 - \alpha_n) \|u_{k,n}\| \|Sx_{k,n} - Sx_k^*\| \alpha_n^2 d^2 \\
 & + 2\alpha_n(1 - \alpha_n) \|Sx_{k,n} - T_k x_{k,n}\| \|u_{k,n}\| + (1 - \alpha_n)^2 \|u_{k,n}\|^2.
 \end{aligned} \tag{9}$$

If we set

$$\theta_{k,n} = \|Sx_{k,n} - Sx_k^*\|, \quad \delta_{k,n} = 2a_k \alpha_n,$$

then from (5) we obtain

$$\begin{aligned}
 \theta_{k,n+1}^2 & \leq (1 - \delta_{k,n}) \theta_{k,n}^2 + d^2 \alpha_n^2 + (2\alpha_n(1 - \alpha_n)d - 2(1 - \alpha_n)\theta_{k,n}) \|u_{k,n}\| \\
 & + (1 - \alpha_n)^2 \|u_{k,n}\|^2 \\
 & = (1 - \delta_{k,n}) \theta_{k,n}^2 + d^2 \alpha_n^2 + \{(1 - \alpha_n)(2\alpha_n d + (1 - \alpha_n) \|u_{k,n}\| - 2\theta_{k,n})\} \|u_{k,n}\|.
 \end{aligned}$$

Observing that

$$d^2 \alpha_n^2 = O(\delta_{k,n}), \quad \lim_{n \rightarrow \infty} \delta_{k,n} = 0 \quad \text{and} \quad \sum_{n \geq 0} d^2 \alpha_n^2 = \infty,$$

we obtain $\theta_{k,n} \rightarrow 0$ as $n \rightarrow \infty$. So the first part is proved. Now, we have

$$\begin{aligned}
 \|Sx_k^* - Tx_k^*\| & = \|Sx_k^* - a_k^{-1} Sx_k^* - a_k^{-1}(1 - a_k)w - a_k^{-1}(1 - a_k)u_k\| \\
 & = \left\| \left(1 - \frac{1}{a_k}\right) Sx_k^* - \left(\frac{1 - a_k}{a_k}\right) (w + u_k) \right\| \\
 & = \left\| -\left(\frac{1 - a_k}{a_k}\right) Sx_k^* - \left(\frac{1 - a_k}{a_k}\right) (w + u_k) \right\| \\
 & = \left(\frac{1 - a_k}{a_k}\right) \|-(Sx_k^* + w + u_k)\| \\
 & \leq \left(\frac{1 - a_k}{a_k}\right) (\|Sx_k^*\| + \|w\| + \|u_k\|) \\
 & \leq \left(\frac{1 - a_k}{a_k}\right) (2d + \|u_k\|),
 \end{aligned}$$

which implies that

$$\lim_{k \rightarrow \infty} \|Sx_k^* - Tx_k^*\| \leq 0.$$

Then

$$\lim_{k \rightarrow \infty} \|Sx_k^* - Tx_k^*\| = 0,$$

hence $\{x_k^*\}$ is a coincidence point sequence for S and T . Also, assuming that x_∞^* is a coincidence point of S and T , then

$$\|Sx_\infty^* - Tx_\infty^*\| \leq \lim_{k \rightarrow \infty} (2d + \|u_k\|) \left(\frac{1 - a_k}{a_k}\right) = 0.$$

Now, for all $0 < r \leq k$, we have

$$\begin{aligned} & \|Sx_k^* - Sx_r^*\|^2 \\ &= \langle Sx_k^* - Sx_r^*, Sx_k^* - Sx_r^* \rangle = \langle Tx_k^* - Tx_r^*, Sx_k^* - Sx_r^* \rangle \\ &= \langle 7w + a_k Tx_k - a_r Tx_r + (1 - a_k)u_k - (1 - a_r)u_r, Sx_k^* - Sx_r^* \rangle \\ &= (a_r - a_k) \langle w, Sx_k^* - Sx_r^* \rangle + \langle a_k Tx_k^* - a_r Tx_r^*, Sx_k^* - Sx_r^* \rangle \\ &\quad + (1 - a_k) \langle u_k, Sx_k^* - Sx_r^* \rangle - (1 - a_r) \langle u_r, Sx_k^* - Sx_r^* \rangle \\ &= (a_r - a_k) \langle w, Sx_k^* - Sx_r^* \rangle + (a_k - a_r) \langle Tx_r^*, Sx_k^* - Sx_r^* \rangle + a_k \langle Tx_k^* - Tx_r^*, Sx_k^* - Sx_r^* \rangle \\ &\quad + (1 - a_k) \langle u_k, Sx_k^* - Sx_r^* \rangle - (1 - a_r) \langle u_r, Sx_k^* - Sx_r^* \rangle \\ &\leq (a_k - a_r) \|w\| \|Sx_k^* - Sx_r^*\| + (a_k - a_r) \|Tx_r^*\| \|Sx_k^* - Sx_r^*\| + a_k \|Sx_k^* - Sx_r^*\|^2 \\ &\quad + (1 - a_k) \|u_k\| \|Sx_k^* - Sx_r^*\| - (1 - a_r) \|u_r\| \|Sx_k^* - Sx_r^*\| \\ &\leq (a_k - a_r) \|Sx_k^* - Sx_r^*\| (\|w\| + \|Tx_r^*\|) + a_k \|Sx_k^* - Sx_r^*\|^2 \\ &\quad + ((1 - a_k) \|u_k\| - (1 - a_r) \|u_r\|) \|Sx_k^* - Sx_r^*\|. \end{aligned}$$

Then we obtain

$$\begin{aligned} (1 - a_k) \|Sx_k^* - Sx_r^*\|^2 &\leq (a_k - a_r) (\|w\| + \|Tx_r^*\|) \|Sx_k^* - Sx_r^*\| \\ &\quad + ((1 - a_k) \|u_k\| - (1 - a_r) \|u_r\|) \|Sx_k^* - Sx_r^*\|. \end{aligned}$$

Then

$$\begin{aligned} \|Sx_k^* - Sx_r^*\|^2 &\leq \frac{a_k - a_r}{1 - a_k} (\|w\| + \|Tx_r^*\|) \|Sx_k^* - Sx_r^*\| \\ &\quad + \left(\frac{1 - a_k}{1 - a_k} \|u_k\| - \frac{1 - a_r}{1 - a_k} \|u_r\| \right) \|Sx_k^* - Sx_r^*\| \\ &\leq \frac{a_k - a_r}{1 - a_k} (2d) \|Sx_k^* - Sx_r^*\| + \left(\|u_k\| - \frac{1 - a_r}{1 - a_k} \|u_r\| \right) \|Sx_k^* - Sx_r^*\|, \end{aligned}$$

which implies that

$$\|Sx_k^* - Sx_r^*\| \leq \frac{a_k - a_r}{1 - a_k} (2d) + \|u_k\| - \frac{1 - a_r}{1 - a_k} \|u_r\|.$$

Hence,

$$\lim_{k,r \rightarrow \infty} \|Sx_k^* - Sx_r^*\| \leq 2d \lim_{k,r \rightarrow \infty} \left(\frac{a_k - a_r}{1 - a_k} \right) + \lim_{k \rightarrow \infty} \|u_k\| - \lim_{k,r \rightarrow \infty} \left(\frac{1 - a_r}{1 - a_k} \cdot \|u_r\| \right) = 0.$$

Thus $\{Sx_k^*\}$ is a Cauchy sequence, and hence there exists $\{Sx_\infty^*\} \in C$ such that $Sx_k^* \rightarrow Sx_\infty^*$ as $k \rightarrow \infty$. Therefore, the second part is proved. By continuity, $Tx_k^* \rightarrow Tx_\infty^*$ as $k \rightarrow \infty$. But $Sx_k^* - Tx_k^* \rightarrow 0$ as $k \rightarrow \infty$. Hence, $x_\infty^* \in CF(S) \cap CF(T)$. This completes the proof. \square

Corollary 2.1 *Let C be a bounded closed convex nonempty subset of a Hilbert space H with $0 \in C$. Let $S, T, \{a_k\}, \{\alpha_n\}, \{x_{k,n}\}$ be as in Theorem 2.1 and $\forall k \geq 0$ define $T_k = a_k T +$*

$(1 - a_k)Su_k$. Then T_k maps C into itself and $\{x_{k,n}\}$ converges strongly to a coincidence point of S and T .

Proof The proof follows from Theorem 2.1 by setting $w = 0 \in C$. □

Corollary 2.2 *In Theorem 2.1, let S, T be two nonexpansive self-mappings. Then the same conclusion is obtained.*

Proof The proof of this corollary can be followed directly by observing that every nonexpansive mapping is a continuous pseudo-contraction. □

Remark 2.1 If we put $u_k = 0$ in Theorem 2.1, we obtain the result of Moore in [24].

3 The equivalence between S, T -stabilities

In this section, we give the concept of S, T -stabilities, then we show that S, T -stabilities of general doubly Mann and general doubly Ishikawa iterations are equivalent.

Let $\{Sx_{k,n}\}$ be the doubly general Ishikawa iteration with errors and $\{Su_{k,n}\}$ be the general doubly Mann iteration with errors. Let $\{q_{k,n}\}, \{p_{k,n}\} \subset E$ be such that $q_{0,0} = p_{0,0}$, and let $(\alpha_n)_n \subset (0, 1), (\beta_n)_n \subset [0, 1]; n \in \mathbb{N}$ satisfy (7) and

$$Sy_{k,n} = (1 - \beta_n)Sq_{k,n} + \beta_n Tq_{k,n}. \tag{10}$$

We consider the following nonnegative sequences for all $n \in \mathbb{N}$:

$$\epsilon_{k,n} := \|Sq_{k,n+1} - (1 - \alpha_n)Sq_{k,n} - \alpha_n Ty_{k,n} + \alpha_n v_n\| \tag{11}$$

and

$$\delta_{k,n} := \|Sp_{k,n+1} - (1 - \alpha_n)Sp_{k,n} - \alpha_n Tp_{k,n} + \alpha_n v_n\|. \tag{12}$$

Let E be a normed space and T be a self-map of E . Let $x_{0,0}$ be a point of E , and assume that $x_{k,n+1} = f(T, x_{k,n})$ is an iteration procedure, involving T , which yields a sequence $\{x_{k,n}\}$ of points from E . Suppose that $x_{k,n}$ converges to a fixed point x^* of T . Let $\xi_{k,n}$ be an arbitrary sequence in E , and set

$$\epsilon_n = \|\xi_{k,n+1} - f(T, \xi_{k,n})\|, \quad \forall n \in \mathbb{N}.$$

Definition 3.1 If $\lim_{n \rightarrow \infty} \epsilon = 0 \Rightarrow \lim_{n \rightarrow \infty} \xi_{k,n} = p$, then the iteration procedure $x_{k,n+1} = f(T, x_{k,n})$ is said to be T -stable with respect to T .

Remark 3.1 In practice, such a sequence $\{\xi_{k,n}\}$ could arise in the following way. Let $x_{0,0}$ be a point in E . Set $x_{k,n+1} = f(T, x_{k,n})$. Let $\xi_{0,0} = x_{0,0}$. Now $x_{0,1} = f(T, x_{0,0})$. Because of rounding in the function T , a new value $\xi_{0,1}$ approximately equal to $x_{0,1}$ might be computed to yield $\xi_{1,2}$, an approximation of $f(T, \xi_{0,1})$. This computation is continued to obtain $\{\xi_{k,n}\}$ an approximate sequence of $\{x_{k,n}\}$.

Definition 3.2 Let E be a normed space and $S, T : E \rightarrow E$.

- (i) If $\lim_{k,n \rightarrow \infty} \epsilon_{k,n} = 0$ implies that $\lim_{k,n \rightarrow \infty} Sq_{k,n} = Sx^*$, then the general Ishikawa iteration as defined in (5) and (6) is said to be S, T -stable.
- (ii) If $\lim_{k,n \rightarrow \infty} \delta_{k,n} = 0$ implies that $\lim_{k,n \rightarrow \infty} Sp_{k,n} = Sx^*$, then the general Mann iteration process as defined in (4) is said to be S, T -stable.

Remark 3.2 Let E be a normed space and $S, T : E \rightarrow E$. The following are equivalent:

- (a) for all $\{\alpha_n\} \subset (0, 1)$, $\{\beta_n\} \subset [0, 1)$ satisfying (7), the Ishikawa iteration is S, T -stable,
- (b) for all $\{\alpha_n\} \subset (0, 1)$, $\{\beta_n\} \subset [0, 1)$ satisfying (7), $\forall \{q_{k,n}\} \subset E$,

$$\begin{aligned} \lim_{k,n \rightarrow \infty} \epsilon_{k,n} &= \lim_{k,n \rightarrow \infty} \|Sq_{k,n+1} - (1 - \alpha_n)Sq_{k,n} - \alpha_n Ty_{k,n} + \alpha_n v_n\| = 0 \\ \Rightarrow \lim_{k,n \rightarrow \infty} Sq_{k,n} &= Sx^*. \end{aligned} \tag{13}$$

Remark 3.3 Let E be a normed space and $S, T : E \rightarrow E$. Then the following are equivalent:

- (a₁) for all $\{\alpha_n\} \subset (0, 1)$ satisfying (7), the general Mann iteration is S, T -stable,
- (a₂) for all $\{\alpha_n\} \subset (0, 1)$ satisfying (7), $\forall \{p_{k,n}\} \subset E$,

$$\begin{aligned} \lim_{k,n \rightarrow \infty} \delta_{k,n} &= \lim_{k,n \rightarrow \infty} \|Sp_{k,n+1} - (1 - \alpha_n)Sp_{k,n} - \alpha_n Tp_{k,n} + \alpha_n v_n\| = 0 \\ \Rightarrow \lim_{k,n \rightarrow \infty} Sp_{k,n} &= Sx^*. \end{aligned} \tag{14}$$

The next result states that these two methods of iterations with errors are equivalent from the S, T -stability point of view under certain restrictions.

Theorem 3.1 Let E be a normed space and $S, T : E \rightarrow E$. Then the following are equivalent:

- (I) For all $\{\alpha_n\} \subset (0, 1)$, $\{\beta_n\} \subset [0, 1)$ satisfying (7), the general Ishikawa iteration process as defined by (5) and (6) is S, T -stable.
- (II) For all $\{\alpha_n\} \subset (0, 1)$, satisfying (7), the general Mann iteration process as defined in (4) is S, T -stable.

Proof Let

$$M := \max \left\{ \sup_{k,n \in \mathbb{N}} \{ \|T(y_{k,n})\| \}, \sup_{k,n \in \mathbb{N}} \{ \|T(q_{k,n})\| \}, \sup_{k,n \in \mathbb{N}} \{ \|T(p_{k,n})\| \}, \sup_{n \in \mathbb{N}} \{ \|u_n\| \} \right\}.$$

Since the general Mann and general Ishikawa iterations converge and $M < \infty$, Remarks 3.1 and 3.2 assure that (I) \Leftrightarrow (II) is equivalent to (b) \Leftrightarrow (a₂). We shall prove that (b) \Rightarrow (a₂).

In (b) and (13) set $Sq_{k,n} := Sp_{k,n}$, we obtain

$$\begin{aligned} &\|Sp_{k,n+1} - (1 - \alpha_n)Sp_{k,n} - \alpha_n Tp_{k,n} + \alpha_n u_n\| \\ &\leq \|Sp_{k,n+1} - (1 - \alpha_n)Sp_{k,n} - \alpha_n Ty_{k,n}\| + \|\alpha_n Ty_{k,n} - \alpha_n Tp_{k,n} + \alpha_n u_n\| \\ &\leq \|Sp_{k,n+1} - (1 - \alpha_n)Sp_{k,n} - \alpha_n Ty_{k,n}\| + \alpha_n (\|Ty_{k,n}\| + \|Tp_{k,n}\| + \|u_n\|) \\ &\leq \|Sp_{k,n+1} - (1 - \alpha_n)Sp_{k,n} - \alpha_n Ty_{k,n}\| + 3\alpha_n M \rightarrow 0 \quad \text{as } n \rightarrow \infty. \end{aligned} \tag{15}$$

Condition (b) assures that

$$\lim_{k,n \rightarrow \infty} \|Sp_{k,n+1} - (1 - \alpha_n)Sp_{k,n} - \alpha_n Ty_{k,n} + \alpha_n u_n\| = 0 \quad \Rightarrow \quad \lim_{k,n \rightarrow \infty} Sp_{k,n} = Sx^*.$$

Thus, for $\{Sp_{k,n}\}$ satisfying

$$\lim_{k,n \rightarrow \infty} \|Sp_{k,n+1} - (1 - \alpha_n)Sp_{k,n} - \alpha_n Ty_{k,n} + \alpha_n u_n\| = 0,$$

we have shown that

$$\lim_{k,n \rightarrow \infty} Sp_{k,n} = Sx^*.$$

Conversely, we prove (a₂) ⇒ (b). In (a₂) and (14) set $Sp_{k,n} = Sq_{k,n}$ to obtain

$$\begin{aligned} & \|Sq_{k,n+1} - (1 - \alpha_n)Sq_{k,n} - \alpha_n Ty_{k,n} + \alpha_n u_n\| \\ & \leq \|Sq_{k,n+1} - (1 - \alpha_n)Sq_{k,n} - \alpha_n Ts_{k,n}\| + \|\alpha_n Ty_{k,n} - \alpha_n Ts_n + \alpha_n u_n\| \\ & \leq \|Sq_{k,n+1} - (1 - \alpha_n)Sq_{k,n} - \alpha_n TSq_{k,n}\| + 3\alpha_n M \rightarrow 0 \quad \text{as } n \rightarrow \infty. \end{aligned} \tag{16}$$

Condition (a₂) assures that

$$\lim_{k,n \rightarrow \infty} \|Sq_{k,n+1} - (1 - \alpha_n)Sq_{k,n} - \alpha_n TSq_{k,n} + \alpha_n u_n\| = 0 \quad \Rightarrow \quad \lim_{k,n \rightarrow \infty} Sq_{k,n} = Sx^*.$$

Thus, for $\{Sq_{k,n}\}$ satisfying

$$\lim_{k,n \rightarrow \infty} \|Sq_{k,n+1} - (1 - \alpha_n)Sq_{k,n} - \alpha_n Ty_{k,n} + \alpha_n u_n\| = 0,$$

we have shown that

$$\lim_{k,n \rightarrow \infty} Sq_{k,n} = Sx^*.$$

This completes the proof of the theorem. □

Corollary 3.1 *Let E be a normed space and $S, T : E \rightarrow E$. Then the following are equivalent:*

- (i) *For all $\{\alpha_n\} \subset (0, 1)$, $\{\beta_n\} \subset [0, 1)$ satisfying (7), the Ishikawa iteration process defined by*

$$\begin{aligned} x_{k,n+1} &= (1 - \alpha_n)x_{k,n} + \alpha_n Tz_{k,n} + \alpha_n v_n, \\ z_{k,n} &= (1 - \beta_n)x_{k,n} + \beta_n Tx_{k,n} + \beta_n w_n \end{aligned}$$

is T -stable.

- (ii) *For all $\{\alpha_n\} \subset (0, 1)$, satisfying (7), the Mann iteration process defined by*

$$u_{k,n+1} = (1 - \alpha_n)u_{k,n} + \alpha_n Tu_{k,n} + \alpha_n u_n \tag{17}$$

is T -stable.

Proof The proof of this result can be obtained directly by setting $S = I$ in Theorem 3.1, where I denotes the identity mapping. □

4 Application

In this section, we investigate the solvability of certain nonlinear functional equations in a Banach space X by the help of compatible mappings of type (B) in the double-sequence setting.

The concept of compatible mappings of type (B) was introduced by Pathak and Khan (see [34]).

Definition 4.1 (see [34] and [29]) Let S and T be mappings from a normed space E into itself. The mappings S and T are said to be compatible mappings of type (B) if

$$\lim_{n \rightarrow \infty} \|STx_n - TTx_n\| \leq \frac{1}{2} \left[\lim_{n \rightarrow \infty} \|STx_n - St\| + \lim_{n \rightarrow \infty} \|St - SSx_n\| \right]$$

and

$$\lim_{n \rightarrow \infty} \|TSx_n - SSx_n\| \leq \frac{1}{2} \left[\lim_{n \rightarrow \infty} \|TSx_n - Tt\| + \lim_{n \rightarrow \infty} \|Tt - TTx_n\| \right]$$

whenever $\{x_n\}$ is a sequence in E such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = t$ for some $t \in E$.

Now, we extend the above definition to double-sequence setting as follows.

Definition 4.2 Let S and T be mappings from a normed space E into itself. The mappings S and T are said to be compatible mappings of type (B) if

$$\lim_{n,m \rightarrow \infty} \|STx_{n,m} - TTx_{n,m}\| \leq \frac{1}{2} \left[\lim_{n,m \rightarrow \infty} \|STx_{n,m} - St\| + \lim_{n,m \rightarrow \infty} \|St - SSx_{n,m}\| \right]$$

and

$$\lim_{n,m \rightarrow \infty} \|TSx_{n,m} - SSx_{n,m}\| \leq \frac{1}{2} \left[\lim_{n,m \rightarrow \infty} \|TSx_{n,m} - Tt\| + \lim_{n,m \rightarrow \infty} \|Tt - TTx_{n,m}\| \right]$$

whenever $\{x_{n,m}\}$ is a sequence in E such that $\lim_{n,m \rightarrow \infty} Sx_{n,m} = \lim_{n,m \rightarrow \infty} Tx_{n,m} = t$ for some $t \in E$.

Now, we state and prove the following result.

Theorem 4.1 Let $\{f_{n,m}\}$, $\{g_{n,m}\}$, $\{t_{n,m}\}$ and $\{r_{n,m}\}$ be sequences of elements in a Banach space X . Let $\{v_{n,m}\}$ be the unique solution of the system of equations

$$\begin{cases} Fv - ABv = f_{n,m}, \\ Fv - BBv = g_{n,m}, \\ Fv - STv = t_{n,m}, \\ Fv - TTv = r_{n,m}, \end{cases}$$

where $F, A, B, S, T : X \rightarrow X$ satisfy the following conditions:

- (d₁) The pairs $\{A, S\}$ and $\{B, T\}$ are compatible of type (B),
- (d₂) $A^2 = B^2 = S^2 = T^2 = I$, where I denotes the identity mapping, and

(d₃)

$$\|Ax - By\|^2 \leq q \max \left\{ \|Sx - Ty\|^2, \|Sx - Ax\|^2, \|Sx - Ax\| \times \|Ty - By\|, \right. \\ \left. \|Ty - Ax\| \times \|Sx - By\|, \frac{1}{2} [\|Ty - Ax\|^2 + \|Sx - By\|^2] \right\}$$

for all $x, y \in X$, where $q \in (0, 1)$. If $Fv = v$ and

$$\lim_{n,m \rightarrow \infty} \|f_{n,m}\| = \lim_{n,m \rightarrow \infty} \|g_{n,m}\| = \lim_{n,m \rightarrow \infty} \|t_{n,m}\| = \lim_{n,m \rightarrow \infty} \|r_{n,m}\| = 0,$$

then the sequence $\{v_{n,m}\}$ converges to the solution of the equation

$$v = Fv = Av = Bv = Sv = Tv.$$

Proof We will show that $\{v_{n,m}\}$ is a Cauchy sequence. Since

$$\begin{aligned} & \|v_{n,m} - v_{n_1,m_1}\|^2 \\ &= [\|v_{n,m} - STv_{n,m}\| + \|STv_{n,m} - TTv_{n,m}\| + \|TTv_{n,m} - ABv_{n,m}\| \\ &\quad + \|ABv_{n,m} - BBv_{n_1,m_1}\| + \|BBv_{n_1,m_1} - v_{n_1,m_1}\|]^2 \\ &\leq [\|v_{n,m} - STv_{n,m}\| + \|STv_{n,m} - TTv_{n,m}\| + \|TTv_{n,m} - ABv_{n,m}\| \\ &\quad + \|BBv_{n_1,m_1} - v_{n_1,m_1}\|]^2 + 2[\|v_{n,m} - STv_{n,m}\| + \|STv_{n,m} - TTv_{n,m}\| \\ &\quad + \|TTv_{n,m} - ABv_{n,m}\| + \|BBv_{n_1,m_1} - v_{n_1,m_1}\|][\|ABv_{n,m} - v_{n,m}\| \\ &\quad + \|v_{n,m} - v_{n_1,m_1}\| + \|v_{n_1,m_1} - BBv_{n_1,m_1}\|] + \|ABv_{n,m} - BBv_{n_1,m_1}\|^2 \\ &\leq [\|v_{n,m} - STv_{n,m}\| + \|STv_{n,m} - TTv_{n,m}\| + \|TTv_{n,m} - ABv_{n,m}\| \\ &\quad + \|BBv_{n_1,m_1} - v_{n_1,m_1}\|]^2 + 2[\|v_{n,m} - STv_{n,m}\| + \|STv_{n,m} - TTv_{n,m}\| \\ &\quad + \|TTv_{n,m} - ABv_{n,m}\| + \|BBv_{n_1,m_1} - v_{n_1,m_1}\|][\|ABv_{n,m} - v_{n,m}\| \\ &\quad + \|v_{n,m} - v_{n_1,m_1}\| + \|v_{n_1,m_1} - BBv_{n_1,m_1}\|] + q \max \left\{ \|SBv_{n,m} - TBv_{n_1,m_1}\|^2, \right. \\ &\quad \|SBv_{n,m} - ABv_{n,m}\|^2, \|SBv_{n,m} - ABv_{n,m}\| \times \|TBv_{n_1,m_1} - BBv_{n_1,m_1}\|, \\ &\quad \|TBv_{n_1,m_1} - ABv_{n,m}\| \times \|SBv_{n,m} - BBv_{n_1,m_1}\|, \frac{1}{2} [\|TBv_{n_1,m_1} - ABv_{n,m}\|^2 \\ &\quad \left. + \|SBv_{n,m} - BBv_{n_1,m_1}\|^2] \right\}. \end{aligned}$$

Letting $n, n_1 \rightarrow \infty$ with $m > n$ and $m_1 > n_1$, we deduce

$$\lim_{n,n_1 \rightarrow \infty} \|v_{n,m} - v_{n_1,m_1}\|^2 \leq q \lim_{n,n_1 \rightarrow \infty} \|v_{n,m} - v_{n_1,m_1}\|^2,$$

which implies that

$$\lim_{n,n_1 \rightarrow \infty} \|v_{n,m} - v_{n_1,m_1}\|^2 = 0.$$

Thus $\{v_{n,m}\}$ is a Cauchy sequence and converges to a point v in X . Further,

$$\begin{aligned} \|v - ABv\| &\leq \|v - v_{n,m}\| + \|v_n - BBv_{n,m}\| + \|BBv_{n,m} - ABv\| \\ &\leq \|v - v_{n,m}\| + \|v_{n,m} - BBv_{n,m}\| + q \max \left\{ \|SBv_{n,m} - TBv\|^2, \right. \\ &\quad \|SBv - ABv\|^2, \|SBv - ABv\| \times \|TBv_{n,m} - BBv_{n,m}\|, \|TBv_{n,m} - ABv\| \\ &\quad \left. \times \|SBv - BBv_{n,m}\|, \frac{1}{2} \|TBv_n - ABv\|^2 + \|SBv - BBv_n\|^2 \right\}^{\frac{1}{2}} \\ &\leq \|v - v_{n,m}\| + \|v_{n,m} - BBv_{n,m}\| + q \max \left\{ [\|SBv_{n,m} - v_{n,m}\| + \|v_{n,m} - v\|]^2 \right. \\ &\quad \times [\|SBv - v\| + \|v - ABv\|]^2, \|SBv - ABv\| \times \|TBv_n - BBv_{n,m}\|, \\ &\quad [\|TBv_{n,m} - v_{n,m}\| + \|v_{n,m} - ABv\|] \times [\|SBv - v\| + \|v - BBv_{n,m}\|], \\ &\quad \left. \frac{1}{2} [\|TBv_{n,m} - ABv\|^2 + \|SBv - BBv_{n,m}\|^2] \right\}^{\frac{1}{2}}. \end{aligned}$$

Letting $n \rightarrow \infty$, we get $v = ABv$, which from (d_2) implies that $Av = Tv$. Similarly, $Tv = Sv$. From (d_1) , we now have

$$ABv = BA v = v = SBv = BSv = TBv = BTv.$$

Using (i) and (d_2) , we have

$$\begin{aligned} \|v - Bv\|^2 &= \|A^2v - Bv\|^2 \\ &\leq q \max \left\{ \|SAv - Tv\|^2, \|SAv - A^2v\|^2, \|SAv - A^2v\| \times \|Tv - Bv\|, \right. \\ &\quad \left. \|Tv - A^2v\| \times \|SAv - Bv\|, \frac{1}{2} [\|Tv - A^2v\|^2 + \|SAv - Bv\|^2] \right\} \\ &\leq q \max \left\{ \|v - Tv\|^2, 0, 0, \|Tv - v\| \times \|v - Bv\|, \frac{1}{2} \|Tv - v\|^2 + \|v - Bv\| \right\} \\ &\leq q \max \{ \|v - Tv\|^2, 0, 0, \|v - Tv\|^2, \|Tv - v\|^2 \}, \end{aligned}$$

which implies that $v = Tv$. It follows that

$$Tv = TSv = STv = v = ABv = BA v = BTv = TBv,$$

completing the proof of the theorem. □

As a consequence of Theorem 4.1, we have the following corollary.

Corollary 4.1 *Let $\{f_{n,m}\}$, $\{g_{n,m}\}$, $\{t_{n,m}\}$ and $\{r_{n,m}\}$ be sequences of elements in a Banach space X . Let $\{v_{n,m}\}$ be the unique solution of the system of equations*

$$\begin{cases} v - ABv = f_{n,m}, \\ v - BBv = g_{n,m}, \\ v - STv = t_{n,m}, \\ v - TTv = r_{n,m}, \end{cases}$$

where $A, B, S, T : X \rightarrow X$ satisfy the following conditions:

- (d₁) The pairs $\{A, S\}$ and $\{B, T\}$ are compatible of type (B),
- (d₂) $A^2 = B^2 = S^2 = T^2 = I$, where I denotes the identity mapping, and
- (d₃)

$$\|Ax - By\|^2 \leq q \max \left\{ \|Sx - Ty\|^2, \|Sx - Ax\|^2, \|Sx - Ax\| \times \|Ty - By\|, \|Ty - Ax\| \times \|Sx - By\|, \frac{1}{2} [\|Ty - Ax\|^2 + \|Sx - By\|^2] \right\}$$

for all $x, y \in X$, where $q \in (0, 1)$. If

$$\lim_{n,m \rightarrow \infty} \|f_{n,m}\| = \lim_{n,m \rightarrow \infty} \|g_{n,m}\| = \lim_{n,m \rightarrow \infty} \|t_{n,m}\| = \lim_{n,m \rightarrow \infty} \|r_{n,m}\| = 0,$$

then the sequence $\{v_{n,m}\}$ converges to the solution of the equation

$$v = Av = Bv = Sv = Tv.$$

Proof The proof can be obtained by putting $F = I$ in Theorem 4.1, where I denotes the identity mapping. □

Open problem It is still an open problem to extend some defined iterative schemes in the sense of double-sequence setting. For some recent studies on various iterative schemes, we refer to [1, 35–39] and others.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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