

**CORRECTION**

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# Correction: Some new theorems of expanding mappings without continuity in cone metric spaces

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## Abstract

In this note we correct some errors that appeared in the article (Han and Xu in *Fixed Point Theory Appl.* 2013:3, 2013) by modifying some conditions in the main theorems and corresponding corollaries.

**MSC:** 47H10; 54H25

**Keywords:** cone metric space; expanding mapping; common fixed point

## Correction

Upon critical examination of the main results and their proofs in [1], we note several critical errors in the conditions of the main theorems in [1]. These errors lead to subsequent errors in the corresponding corollaries in [1].

In this note, we would like to supplement several conditions, which are used in their proofs but not referred to in the conditions of the main results, to achieve our claim.

The following theorem is a modification to [1, Theorem 2.1]. The proof is the same as that in [1]. We will attain the desired goal by adding two conditions to that in [1, Theorem 2.1]. We state that Theorem 2.1 in [1] is replaced by the following theorem.

**Theorem 2.1** *Let  $(X, d)$  be a complete cone metric space. Suppose that the mapping  $f : X \rightarrow X$  is onto and such that*

$$d(fx, fy) \geq a_1 d(x, y) + a_2 d(x, fx) + a_3 d(y, fy) + a_4 d(x, fy) + a_5 d(y, fx), \quad (2.1)$$

*for all  $x, y \in X$ , where  $a_i$  ( $i = 1, 2, 3, 4, 5$ ) satisfy  $a_2 \geq 0$ ,  $a_4 \geq 0$ ,  $a_1 + a_2 + a_3 > 1$  and  $a_3 \leq 1 + a_4$ . Then  $f$  has a fixed point.*

**Remark 2.1** Compared to Theorem 2.1 in [1], Theorem 2.1 mentioned above possesses the conditions  $a_2 \geq 0$  and  $a_4 \geq 0$  while, unluckily, Theorem 2.1 in [1] does not. The reason for supplementing these conditions is the fact that in the proof of [1, Theorem 2.1] we have used the conditions  $a_2 \geq 0$  and  $a_4 \geq 0$  to ensure that the two deductions

(i)  $d(x_{n+1}, x_{n-1}) \geq d(x_{n+1}, x_n) - d(x_{n-1}, x_n)$  implies

$$a_4 d(x_{n+1}, x_{n-1}) \geq a_4 d(x_{n+1}, x_n) - a_4 d(x_{n-1}, x_n)$$

and

(ii)  $d(p, q) \geq d(p, x_{n+1}) - d(q, x_{n-1})$  implies  $a_2 d(p, q) \geq a_2 d(p, x_{n+1}) - a_2 d(q, x_{n-1})$  must be valid.

Similarly, the following theorem is a modification to [1, Theorem 2.5]. The proof is the same as that in [1]. We state that Theorem 2.5 in [1] is replaced by the following theorem.

**Theorem 2.5** *Let  $(X, d)$  be a complete cone metric space. Suppose the mappings  $f, g : X \rightarrow X$  are onto and satisfy*

$$d(fx, gy) \geq a_1 d(x, y) + a_2 d(x, fx) + a_3 d(y, gy) + a_4 d(x, gy) + a_5 d(y, fx), \quad (2.2)$$

*for all  $x, y \in X$ , where  $a_i$  ( $i = 1, 2, 3, 4, 5$ ) satisfy  $a_3 \geq 0$ ,  $a_5 \geq 0$ ,  $a_1 + a_2 + a_3 > 1$  and  $a_2 \leq 1 + a_5$ ,  $a_3 \leq 1 + a_4$ . Then  $f$  and  $g$  have a common fixed point.*

Accordingly, the following two corollaries are modifications to Corollary 2.2 and Corollary 2.6 in [1], and we state that the latter corollaries are replaced by the former ones, respectively.

**Corollary 2.2** *Let  $(X, d)$  be a complete cone metric space. Suppose the mapping  $f : X \rightarrow X$  is onto and such that*

$$d(fx, fy) \geq kd(x, y) + ld(x, fx) + pd(y, fy),$$

*for all  $x, y \in X$ , where  $p \leq 1$ ,  $l \geq 0$  and  $k + l + p > 1$ . Then  $f$  has a fixed point.*

**Corollary 2.6** *Let  $(X, d)$  be a complete cone metric space. Suppose the mappings  $f, g : X \rightarrow X$  are onto and such that*

$$d(fx, gy) \geq \alpha d(x, y) + \beta [d(x, fx) + d(y, gy)] + \gamma [d(x, gy) + d(y, fx)],$$

*for all  $x, y \in X$ , where  $\beta \geq 0$ ,  $\gamma \geq 0$ ,  $\beta \leq 1 + \gamma$  and  $\alpha + 2\beta > 1$ . Then  $f$  and  $g$  have a common fixed point.*

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