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Remarks on 'Coupled coincidence point results for a generalized compatible pair with applications'

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Abstract

Very recently, Hussain *et al.* (Fixed Point Theory Appl. 2014:62, 2014) announced the existence and uniqueness of some coupled coincidence point. In this short note we remark that the announced results can be derived from the coincidence point results in the literature.

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1 Introduction

Recently, a number of studies related to fixed points, coupled fixed points and coupled coincidence points of maps defined via auxiliary functions have appeared in the literature. In particular, the so-called weak φ-contractions, contractions defined by means of altering distance functions, $\alpha - \psi$-type contractions have been a subject of considerable interest. Studies of this type aim to generalize and improve contractive condition on the maps (see, *e.g.*, [1–15]).

A great deal of these studies investigate contractions on partially ordered metric spaces because of their applicability to initial value problems defined by differential or integral equations. This is the case of the following result.

Theorem 1.1 (Hussain *et al.* [16], Theorem 15) *Let $(X, \preceq)$ be a partially ordered set such that there exists a complete metric d on X. Assume that $F, G : X \times X \to X$ are two generalized compatible mappings such that F is G-increasing with respect to $\preceq$, G is continuous and has the mixed monotone property, and there exist two elements $x_0, y_0 \in X$ such that*

$$G(x_0, y_0) \preceq F(x_0, y_0) \quad \text{and} \quad G(y_0, x_0) \succeq F(y_0, x_0). \tag{1}$$

Suppose that there exist $\phi \in \Phi$ and $\psi \in \Psi$ such that

$$\phi\big(d\big(F(x,y), F(u,v)\big)\big) \le \frac{1}{2}\phi\big(d\big(G(x,y), G(u,v)\big) + d\big(G(y,x), G(v,u)\big)\big) - \psi\left(\frac{d(G(x,y), G(u,v)) + d(G(y,x), G(v,u))}{2}\right) \tag{2}$$

for all $x, y, u, v \in X$ *with* $G(x,y) \preceq G(u,v)$ *and* $G(y,x) \succeq G(v,u)$. *Suppose that for any* $x, y \in X$, *there exist* $u, v \in X$ *such that*

$$F(x,y) = G(u,v) \quad \text{and} \quad F(y,x) = G(v,u). \tag{3}$$

Also suppose that either

(a) F *is continuous, or*
(b) X *has the following property*:
  (i) *if a* $\preceq$*-non-decreasing sequence* $\{x_n\} \to x$, *then* $x_n \preceq x$ *for all* $n \in \mathbb{N}$,
  (ii) *if a* $\preceq$*-non-increasing sequence* $\{y_n\} \to y$, *then* $y \preceq y_n$ *for all* $n \in \mathbb{N}$.

Then F *and* G *have a coupled coincidence point in* X.

In this paper we show that the previous result can be easily improved because of the following facts.

(1) The mixed monotone property is not necessary since F is G-increasing with respect to $\preceq$.
(2) It is possible to consider a pair of mappings satisfying a weaker condition than the generalized compatible property (using monotone sequences).
(3) In fact, Theorem 1.1 is not a true advance because it can be reduced to its corresponding unidimensional coincidence point theorem.

To prove our main claims, we will show a unidimensional proof of the mentioned theorem.

2 Preliminaries

Firstly, we recall some basic definitions and elementary results needed throughout the paper. Some of them can be found in [17]. In the sequel, we denote by X a nonempty set. Given a natural number $n \in \mathbb{N}$, let X^n be the *nth Cartesian product* $X \times X \times \cdots \times X$ (n times). We employ mappings $T, g : X \to X$ and $F : X^n \to X$. For simplicity, if $x \in X$, we denote $T(x)$ by Tx.

Definition 2.1 (Khan *et al.* [18]) An *altering distance function* is a continuous, non-decreasing function $\phi : [0,\infty) \to [0,\infty)$ such that $\phi(t) = 0$ if and only if $t = 0$. Let $\mathcal{F}_{\text{alt}}$ denote the family of all altering distance functions.

A function $\phi : [0,\infty) \to [0,\infty)$ is said to be *subadditive* if $\phi(t+s) \leq \phi(t) + \phi(s)$ for all $t, s \geq 0$. Following [16], we introduce the following families of control functions. Let Φ denote the family of all subadditive altering distance functions, that is, functions $\phi : [0,\infty) \to [0,\infty)$ which satisfy the following:

(ϕ_1) ϕ is continuous and non-decreasing;
(ϕ_2) $\phi(t) = 0$ if and only if $t = 0$;
(ϕ_3) $\phi(t+s) \leq \phi(t) + \phi(s)$ for all $t, s \in [0,\infty)$.

We denote by Ψ the family of all functions $\psi : [0,\infty) \to [0,\infty)$ which satisfy the following:

(1) $\lim_{t \to r} \psi(t) > 0$ for all $r > 0$;
(2) $\lim_{t \to 0^+} \psi(t) = 0$.

Remark 2.1 Let $\psi \in \Psi$, $c > 0$ and define $\psi_c : [0,\infty) \to [0,\infty)$ by $\psi_c(t) = c\psi(t/c)$ for all $t \geq 0$. Then $\psi_c \in \Psi$.

Definition 2.2 (see [19, 20]) A *coincidence point* of two mappings $T, g : X \to X$ is a point $x \in X$ such that $Tx = gx$.

Definition 2.3 (Hussain *et al.* [16], Definition 10) A *coupled coincidence point* of two mappings $F, G : X^2 \to X$ is a point $(x,y) \in X$ such that

$$F(x,y) = G(x,y) \quad \text{and} \quad F(y,x) = G(y,x).$$

Definition 2.4 An *ordered metric space* $(X,d,\preceq)$ is a metric space (X,d) provided with a partial order $\preceq$.

Definition 2.5 ([16, 21]) An ordered metric space $(X,d,\preceq)$ is said to be *non-decreasing-regular* (respectively, *non-increasing-regular*) if for every sequence $\{x_m\} \subseteq X$ such that $\{x_m\} \to x$ and $x_m \preceq x_{m+1}$ (respectively, $x_m \succeq x_{m+1}$) for all m, we have that $x_m \preceq x$ (respectively, $x_m \succeq x$) for all m. $(X,d,\preceq)$ is said to be *regular* if it is both non-decreasing-regular and non-increasing-regular.

Remark 2.2 Notice that condition (b) in Theorem 1.1 means that $(X,d,\preceq)$ is regular.

Definition 2.6 Let $(X,\preceq)$ be a partially ordered set, and let $T, g : X \to X$ be two mappings. We say that T is *$(g,\preceq)$-non-decreasing* if $Tx \preceq Ty$ for all $x,y \in X$ such that $gx \preceq gy$. If g is the identity mapping on X, we say that T is *$\preceq$-non-decreasing*.

Remark 2.3 If T is *$(g,\preceq)$-non-decreasing* and $gx = gy$, then $Tx = Ty$. It follows that

$$gx = gy \quad \Rightarrow \quad \begin{Bmatrix} gx \preceq gy \\ gy \preceq gx \end{Bmatrix} \quad \Rightarrow \quad \begin{Bmatrix} Tx \preceq Ty \\ Ty \preceq Tx \end{Bmatrix} \quad \Rightarrow \quad Tx = Ty.$$

Definition 2.7 (Hussain *et al.* [16], Definition 7) Suppose that $F, G : X \times X \to X$ are two mappings, and let $\preceq$ be a partial order on X. The mapping F is said to be *G-increasing with respect to* $\preceq$ if for all $x,y,u,v \in X$ with $G(x,y) \preceq G(u,v)$ we have $F(x,y) \preceq F(u,v)$.

Lemma 2.1 (see [22]) *Let (X,d) be a metric space and define $\Delta_n : X^n \times X^n \to [0,\infty)$, for all $A = (a_1,a_2,\ldots,a_n), B = (b_1,b_2,\ldots,b_n) \in X^n$, by*

$$\Delta_n(A,B) = \sum_{i=1}^{n} d(a_i,b_i).$$

Then Δ_n is metric on X^n and (X,d) is complete if and only if (X,Δ_n) is complete.

Consider on the product space X^2 the following partial order: for $(x,y),(u,v) \in X^2$,

$$(x,y) \sqsubseteq (u,v) \quad \Leftrightarrow \quad [x \preceq u \text{ and } y \succeq v]. \tag{4}$$

Definition 2.8 ([17, 23–25]) Let $(X, d, \preceq)$ be an ordered metric space. Two mappings $T, g : X \to X$ are said to be *O-compatible* if

$$\lim_{m\to\infty} d(gTx_m, Tgx_m) = 0$$

provided that $\{x_m\}$ is a sequence in X such that $\{gx_m\}$ is $\preceq$-monotone, that is, it is either non-increasing or non-decreasing with respect to $\preceq$, and

$$\lim_{m\to\infty} Tx_m = \lim_{m\to\infty} gx_m \in X.$$

Definition 2.9 (Hussain *et al.* [16], Definition 12) Let $F, G : X \times X \to X$. We say that the pair $\{F, G\}$ is *generalized compatible* if for all sequences $\{x_n\}, \{y_n\} \subseteq X$ such that

$$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} G(x_n, y_n) = t_1 \in X \quad \text{and} \quad \lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} G(y_n, x_n) = t_2 \in X,$$

we have that

$$\lim_{n\to\infty} d\big(F\big(G(x_n, y_n), G(y_n, x_n)\big), G\big(F(x_n, y_n), F(y_n, x_n)\big)\big) = 0 \quad \text{and}$$
$$\lim_{n\to\infty} d\big(F\big(G(y_n, x_n), G(x_n, y_n)\big), G\big(F(y_n, x_n), F(x_n, y_n)\big)\big) = 0.$$

3 Main results

To start with, we highlight the weakness of Theorem 1.1 using the following example.

Example 3.1 Let $X = [0, \infty)$ endowed with the standard metric $d(x, y) = |x - y|$ for all $x, y \in X$. Consider the maps $F, G : X \times X \to X$ defined by

$$F(x, y) = \frac{3}{5}x - \frac{1}{5}y \quad \text{and} \quad G(x, y) = \frac{x - y}{2} \quad \text{for all } x, y \in X.$$

Then, for all $x, y, u, v \in X$ with $y = v$, we have

$$d\big(F(x, y), F(u, v)\big) = \frac{3}{5}|x - u| \quad \text{and} \quad d\big(G(x, y), G(u, v)\big) + d\big(G(y, x), G(v, u)\big) = |x - u|.$$

Thus,

$$d\big(F(x, y), F(u, v)\big) > \frac{1}{2}\big(d\big(G(x, y), G(u, v)\big) + d\big(G(y, x), G(v, u)\big)\big).$$

Regarding the properties of the functions in Φ, we derive that

$$\varphi\big(d\big(F(x, y), F(u, v)\big)\big) > \frac{1}{2}\varphi\big(d\big(G(x, y), G(u, v)\big) + d\big(G(y, x), G(v, u)\big)\big).$$

Since the function in the class Ψ takes values on $[0, \infty)$, it is impossible to verify inequality (2). Hence, Theorem 1.1 cannot be applied to get a coupled coincidence point. However, it is easy to see that $(0, 0)$ is a coupled coincidence point of F and G.

Next, we show a unidimensional version of Theorem 1.1. Notice that, indeed, the following result is better than Theorem 1.1 because we reorder the hypotheses obtaining that, in some cases, neither the continuity of, at least, one mapping (T or g) nor the O-compatibility of the pair (T,g) is necessary. In fact, both hypotheses are omitted in case (c).

Theorem 3.1 *Let $(X,d,\preceq)$ be an ordered metric space, and let $T,g:X\to X$ be two mappings such that the following properties are fulfilled*:

(i) $T(X)\subseteq g(X)$;
(ii) *T is $(g,\preceq)$-non-decreasing*;
(iii) *there exists $x_0\in X$ such that $gx_0\preceq Tx_0$*;
(iv) *there exist $\phi\in\Phi$ and $\psi\in\Psi$ verifying*

$$\phi\big(d(Tx,Ty)\big)\le\phi\big(d(gx,gy)\big)-\psi\big(d(gx,gy)\big)\quad \textit{for all } x,y\in X \textit{ such that } gx\preceq gy.$$

Also assume that, at least, one of the following conditions holds.

(a) *(X,d) is complete, T and g are continuous and the pair (T,g) is O-compatible*;
(b) *(X,d) is complete and T and g are continuous and commuting*;
(c) *$(g(X),d)$ is complete and $(X,d,\preceq)$ is non-decreasing-regular*;
(d) *(X,d) is complete, $g(X)$ is closed and $(X,d,\preceq)$ is non-decreasing-regular*;
(e) *(X,d) is complete, g is continuous and monotone $\preceq$-non-decreasing, the pair (T,g) is O-compatible and $(X,d,\preceq)$ is non-decreasing-regular.*

Then T and g have, at least, a coincidence point.

We omit the proof of the previous result since its proof is similar to the main theorem in [17] and it can be concluded by following, point by point, all of its arguments.

Next, we show how to deduce an appropriate version of Theorem 1.1 from Theorem 3.1. Given the ordered metric space $(X,d,\preceq)$, let us consider the ordered metric space $(X^2,\Delta_2,\sqsubseteq)$, where Δ_2 was defined in Lemma 2.1 and $\sqsubseteq$ was introduced in (4). We define the mappings $T_F,T_G:X^2\to X^2$, for all $(x,y)\in X^2$, by

$$T_F(x,y)=\big(F(x,y),F(y,x)\big)\quad\text{and}\quad T_G(x,y)=\big(G(x,y),G(y,x)\big).$$

Under these conditions, the following properties hold.

Lemma 3.1 *Let $(X,d,\preceq)$ be an ordered metric space, and let $F,G:X^2\to X$ be two mappings. Then the following properties hold.*

(1) *(X,d) is complete if and only if (X^2,Δ_2) is complete.*
(2) *If $(X,d,\preceq)$ is regular, then $(X^2,\Delta_2,\sqsubseteq)$ is also regular.*
(3) *If F is d-continuous, then T_F is Δ_2-continuous.*
(4) *If F is G-increasing with respect to $\preceq$, then T_F is $(T_G,\sqsubseteq)$-non-decreasing.*
(5) *Condition* (1) *is equivalent to the existence of a point $(x_0,y_0)\in X^2$ such that $T_G(x_0,y_0)\sqsubseteq T_F(x_0,y_0)$.*
(6) *Condition* (3) *is equivalent to $T_F(X^2)\subseteq T_G(X^2)$.*
(7) *If there exist $\phi\in\Phi$ and $\psi\in\Psi$ such that* (2) *holds, then*

$$\phi\big(\Delta_2\big(T_F(x,y),T_F(u,v)\big)\big)\le\phi\big(\Delta_2\big(T_G(x,y),T_G(u,v)\big)\big)-\psi_2\big(\Delta_2\big(T_G(x,y),T_G(u,v)\big)\big)$$

for all $(x,y),(u,v)\in X^2$ *such that* $T_G(x,y)\sqsubseteq T_G(u,v)$, *where* $\psi_2\in\Psi$ *was defined in Remark* 2.1.

(8) *If the pair* $\{F,G\}$ *is generalized compatible, then the mappings* T_F *and* T_G *are O-compatible in* $(X^2,\Delta_2,\sqsubseteq)$.

(9) *A point* $(x,y)\in X^2$ *is a coupled coincidence point of F and G if and only if it is a coincidence point of* T_F *and* T_G.

Proof Item (1) follows from Lemma 2.1 and items (2), (3), (5), (6) and (9) are obvious.

(4) Assume that F is G-increasing with respect to $\preceq$, and let $(x,y),(u,v)\in X^2$ be such that $T_G(x,y)\sqsubseteq T_G(u,v)$. Then $G(x,y)\preceq G(u,v)$ and $G(y,x)\succeq G(v,u)$. Since F is G-increasing with respect to $\preceq$, we deduce that $F(x,y)\preceq F(u,v)$ and $F(y,x)\succeq F(v,u)$. Therefore, $T_F(x,y)\sqsubseteq T_F(u,v)$ and this means that T_F is $(T_G,\sqsubseteq)$-non-decreasing.

(7) Suppose that there exist $\phi\in\Phi$ and $\psi\in\Psi$ such that (2) holds, and let $(x,y),(u,v)\in X^2$ be such that $T_G(x,y)\sqsubseteq T_G(u,v)$. Therefore $G(x,y)\preceq G(u,v)$ and $G(y,x)\succeq G(v,u)$. Using (2), we have that

$$\begin{aligned}\phi\bigl(d\bigl(F(x,y),F(u,v)\bigr)\bigr)\le{}&\frac{1}{2}\phi\bigl(d\bigl(G(x,y),G(u,v)\bigr)+d\bigl(G(y,x),G(v,u)\bigr)\bigr)\\&-\psi\left(\frac{d(G(x,y),G(u,v))+d(G(y,x),G(v,u))}{2}\right).\end{aligned}\tag{5}$$

Furthermore, taking into account that $G(v,u)\preceq G(y,x)$ and $G(u,v)\succeq G(x,y)$, the contractivity condition (2) also guarantees that

$$\begin{aligned}\phi\bigl(d\bigl(F(v,u),F(y,x)\bigr)\bigr)\le{}&\frac{1}{2}\phi\bigl(d\bigl(G(v,u),G(y,x)\bigr)+d\bigl(G(u,v),G(x,y)\bigr)\bigr)\\&-\psi\left(\frac{d(G(v,u),G(y,x))+d(G(u,v),G(x,y))}{2}\right).\end{aligned}\tag{6}$$

Since ϕ is subadditive, it follows from (5) and (6) that

$$\begin{aligned}&\phi\bigl(\Delta_2\bigl(T_F(x,y),T_F(u,v)\bigr)\bigr)\\&\quad=\phi\bigl(\Delta_2\bigl[\bigl(F(x,y),F(y,x)\bigr),\bigl(F(u,v),F(v,u)\bigr)\bigr]\bigr)\\&\quad=\phi\bigl(d\bigl(F(x,y),F(u,v)\bigr)+d\bigl(F(y,x),F(v,u)\bigr)\bigr)\\&\quad\le\phi\bigl(d\bigl(F(x,y),F(u,v)\bigr)\bigr)+\phi\bigl(d\bigl(F(y,x),F(v,u)\bigr)\bigr)\\&\quad\le\phi\bigl(d\bigl(G(x,y),G(u,v)\bigr)+d\bigl(G(y,x),G(v,u)\bigr)\bigr)\\&\qquad-2\psi\left(\frac{d(G(v,u),G(y,x))+d(G(u,v),G(x,y))}{2}\right)\\&\quad=\phi\bigl(\Delta_2\bigl(T_G(x,y),T_G(u,v)\bigr)\bigr)-\psi_2\bigl(\Delta_2\bigl(T_G(x,y),T_G(u,v)\bigr)\bigr).\end{aligned}$$

(8) Let $\{(x_m,y_m)\}\subseteq X^2$ be any sequence such that $\{T_F(x_m,y_m)\}\xrightarrow{\Delta_2}(x,y)$ and $\{T_G(x_m,y_m)\}\xrightarrow{\Delta_2}(x,y)$ (notice that we do not need to suppose that $\{T_G(x_m,y_m)\}$ is $\sqsubseteq$-monotone). Therefore,

$$\bigl\{\bigl(F(x_m,y_m),F(y_m,x_m)\bigr)\bigr\}\xrightarrow{\Delta_2}(x,y)\quad\Rightarrow\quad\bigl[\bigl\{F(x_m,y_m)\bigr\}\xrightarrow{d}x\text{ and }\bigl\{F(y_m,x_m)\bigr\}\xrightarrow{d}y\bigr];$$

$$\begin{aligned}&\bigl\{\bigl(G(x_m,y_m),G(y_m,x_m)\bigr)\bigr\}\xrightarrow{\Delta_2}(x,y)\\&\quad\Rightarrow\quad\bigl[\bigl\{G(x_m,y_m)\bigr\}\xrightarrow{d}x\text{ and }\bigl\{G(y_m,x_m)\bigr\}\xrightarrow{d}y\bigr].\end{aligned}$$

Therefore

$$\begin{aligned}&\lim_{m\to\infty}F(x_m,y_m)=\lim_{m\to\infty}G(x_m,y_m)=x\in X\quad\text{and}\\&\lim_{m\to\infty}F(y_m,x_m)=\lim_{m\to\infty}G(y_m,x_m)=y\in X.\end{aligned}$$

Since the pair $\{F,G\}$ is generalized compatible, we deduce that

$$\begin{aligned}&\lim_{m\to\infty}d\bigl(F\bigl(G(x_m,y_m),G(y_m,x_m)\bigr),G\bigl(F(x_m,y_m),F(y_m,x_m)\bigr)\bigr)=0\quad\text{and}\\&\lim_{m\to\infty}d\bigl(F\bigl(G(y_m,x_m),G(x_m,y_m)\bigr),G\bigl(F(y_m,x_m),F(x_m,y_m)\bigr)\bigr)=0.\end{aligned}$$

In particular,

$$\begin{aligned}&\lim_{m\to\infty}\Delta_2\bigl(T_GT_F(x_m,y_m),T_FT_G(x_m,y_m)\bigr)\\&\quad=\lim_{m\to\infty}\Delta_2\bigl(T_G\bigl(F(x_m,y_m),F(y_m,x_m)\bigr),T_F\bigl(G(x_m,y_m),G(y_m,x_m)\bigr)\bigr)\\&\quad=\lim_{m\to\infty}\Delta_2\bigl(\bigl(G\bigl(F(x_m,y_m),F(y_m,x_m)\bigr),G\bigl(F(y_m,x_m),F(x_m,y_m)\bigr)\bigr),\\&\qquad\bigl(F\bigl(G(x_m,y_m),G(y_m,x_m)\bigr),F\bigl(G(y_m,x_m),G(x_m,y_m)\bigr)\bigr)\bigr)\\&\quad=\lim_{m\to\infty}\bigl[d\bigl(\bigl(G\bigl(F(x_m,y_m),F(y_m,x_m)\bigr),F\bigl(G(x_m,y_m),G(y_m,x_m)\bigr)\bigr)\bigr)\\&\qquad+d\bigl(\bigl(G\bigl(F(y_m,x_m),F(x_m,y_m)\bigr),F\bigl(G(y_m,x_m),G(x_m,y_m)\bigr)\bigr)\bigr)\bigr]\\&\quad=0.\end{aligned}$$

Hence, the mappings T_F and T_G are O-compatible in $(X^2,\Delta_2,\sqsubseteq)$. □

As a consequence, we conclude that Hussain *et al.*'s result can be deduced from the corresponding unidimensional result. Furthermore, as we have pointed out, it is not necessary for G to have the mixed monotone property because F is G-increasing with respect to $\preceq$.

Corollary 3.1 *Theorem* 1.1, *even avoiding the assumption that G has the mixed monotone property, is a consequence of Theorem* 3.1.

Proof It is only necessary to apply Theorem 3.1 to the mappings $T=T_F$ and $g=T_G$ in the ordered metric space $(X^2,\Delta_2,\sqsubseteq)$, taking into account all items of Lemma 3.1. □

The following result is an improved version of Theorem 1.1 in which the contractivity condition is replaced by a more convenient one, which is symmetric on the variables (x,y) and (u,v).

Corollary 3.2 *Let* $(X,\preceq)$ *be a partially ordered set such that there exists a complete metric* d *on* X. *Assume that* $F,G:X\times X\to X$ *are two generalized compatible mappings such that*

F is G-increasing with respect to $\preceq$, *G is continuous and there exist two elements* $x_0, y_0 \in X$ *such that*

$$G(x_0, y_0) \preceq F(x_0, y_0) \quad \textit{and} \quad G(y_0, x_0) \succeq F(y_0, x_0).$$

Suppose that there exist $\phi \in \Phi$ *and* $\psi \in \Psi$ *such that*

$$\begin{aligned}
&\phi\left(\frac{d(F(x,y),F(u,v)) + d(F(y,x),F(v,u))}{2}\right) \\
&\quad \le \phi\left(\frac{d(G(x,y),G(u,v)) + d(G(y,x) + G(v,u))}{2}\right) \\
&\qquad - \psi\left(\frac{d(G(x,y),G(u,v)) + d(G(y,x) + G(v,u))}{2}\right)
\end{aligned} \tag{7}$$

for all $x, y, u, v \in X$ *with* $G(x,y) \preceq G(u,v)$ *and* $G(y,x) \succeq G(v,u)$. *Suppose that for any* $x, y \in X$, *there exist* $u, v \in X$ *such that*

$$F(x,y) = G(u,v) \quad \textit{and} \quad G(y,x) \succeq F(y,x).$$

Also assume that either

(a) *F is continuous, or*

(b) $(X, d, \preceq)$ *is regular.*

Then F and G have, at least, a coupled coincidence point, that is, there exist $x, y \in X$ *such that* $G(x,y) = F(x,y)$ *and* $G(y,x) = F(y,x)$.

Proof It is only necessary to apply Theorem 3.1 to the mappings $T = T_F$ and $g = T_G$ in the ordered metric space $(X^2, \Delta_2', \sqsubseteq)$, where $\Delta_2' = \Delta_2/2$, taking into account all items of Lemma 3.1. □

In the following example we show that Corollary 3.2 is applicable to the mappings of Example 3.1, when Theorem 1.1 is not useful.

Example 3.2 Let $X = [0, \infty)$ endowed with the Euclidean metric $d(x,y) = |x - y|$ for all $x, y \in X$. Consider the maps $F, G : X \times X \to X$ defined by

$$F(x,y) = \frac{3}{5}x - \frac{1}{5}y \quad \text{and} \quad G(x,y) = \frac{x-y}{2} \quad \text{for all } x, y \in X.$$

Then, for all $x, y, u, v \in X$ with $y = v$, we have

$$d\big(F(x,y),F(u,v)\big) + d\big(F(y,x),F(v,u)\big) = \frac{4}{5}\big(|x-u| + |y-v|\big) \quad \text{and}$$

$$d\big(G(x,y),G(u,v)\big) + d\big(G(y,x),G(v,u)\big) = |x-u| + |y-v|.$$

Thus,

$$d\big(F(x,y),F(u,v)\big) < d\big(G(x,y),G(u,v)\big) + d\big(G(y,x),G(v,u)\big).$$

Regarding the properties of the functions in Φ, we derive that

$$\phi\left(\frac{d(F(x,y),F(u,v))+d(F(y,x),F(v,u))}{2}\right) \leq \phi\left(\frac{d(G(x,y),G(u,v))+d(G(y,x)+G(v,u))}{2}\right).$$

To provide inequality (7), it is sufficient to choose $\psi(t)=\frac{t}{10}$. Hence, Theorem 3.2 can be applied in order to guarantee that F and G have a coupled coincidence point. Indeed, it is easy to check that $(0,0)$ is a coupled coincidence point of F and G.

To finish the paper, we want to point out a pair of details.

(1) The function in ϕ in Theorem 3.1 is not a true generalization because if $\phi\in\Phi$, then the mapping $d_\phi : X\times X\to[0,\infty)$, defined by $d_\phi(x,y)=\phi(d(x,y))$ for all $x,y\in X$, is also a metric on X. For more details, see [26]. Notice also that the assumption of sub-additivity (ϕ_2) is superfluous in most of the published results (see, *e.g.*, [27]).

(2) Using the same techniques that can be found in [17, 22, 28–31], it is possible to deduce, from Theorem 3.1, tripled, quadrupled and, in general, multidimensional coincidence point theorems.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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