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On split common solution problems: new nonlinear feasible algorithms, strong convergence results and their applications

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Abstract
In this paper, we study and give examples for classes of generalized contractive mappings. We establish some new strong convergence theorems of feasible iterative algorithms for the split common solution problem (SCSP) and give some applications of these new results.
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1 Introduction and preliminaries

Let K be a closed convex subset of a real Hilbert space H with the inner product $\langle\cdot,\cdot\rangle$ and the norm $\|\cdot\|$. The following inequalities are known and useful.

- $\|x+y\|^2 \le \|y\|^2 + 2\langle x, x+y\rangle$;
- $\|x-y\|^2 = \|x\|^2 + \|y\|^2 - 2\langle x,y\rangle$ for all $x,y \in H$;
- $\|\alpha x + (1-\alpha)y\|^2 = \alpha\|x\|^2 + (1-\alpha)\|y\|^2 - \alpha(1-\alpha)\|x-y\|^2$ for all $x,y\in H$ and $\alpha\in[0,1]$.

For each point $x \in H$, there exists a unique nearest point in K, denoted by $P_K x$, such that

$$\|x - P_K x\| \le \|x-y\| \quad \text{for all } y \in K.$$

The mapping P_K is called the *metric projection* from H onto K. It is well known that P_K has the following properties:

(i) $\langle x-y, P_K x - P_K y\rangle \ge \|P_K x - P_K y\|^2$ for every $x,y\in H$.
(ii) For $x\in H$ and $z \in K$, $z = P_K x \Leftrightarrow \langle x-z, z-y\rangle \ge 0$ for all $y \in K$.
(iii) For $x \in H$ and $y\in K$,

$$\|y - P_K x\|^2 + \|x - P_K x\|^2 \le \|x-y\|^2. \tag{1.1}$$

Let H_1 and H_2 be two Hilbert spaces. Let $A: H_1 \to H_2$ and $A^*: H_2 \to H_1$ be two bounded linear operators. A^* is called the adjoint operator (or adjoint) of A if

$$\langle Az, w\rangle = \langle z, A^* w\rangle \quad \text{for all } z \in H_1 \text{ and } w \in H_2.$$

It is known that the adjoint operator of a bounded linear operator on a Hilbert space always exists and is bounded linear and unique. Moreover, it is not hard to show that if A^* is an adjoint operator of A, then $\|A\| = \|A^*\|$. The symbols $\mathbb{N}$ and $\mathbb{R}$ are used to denote the sets of positive integers and real numbers, respectively.

Let H_1 and H_2 be two real Hilbert spaces. Let C be a closed convex subset of H_1 and K be a closed convex subset of H_2 . Let $T : C \to C$ with $\mathcal{F}(T) \neq \emptyset$ and $S : K \to K$ with $\mathcal{F}(S) \neq \emptyset$ be two mappings. Let $A : H_1 \to H_2$ be a bounded linear operator. The mathematical model of *the split common solution problem* (SCSP in short) is defined as follows:

(SCSP) Find $p \in C$ such that $Tp = p$ and $u := Ap \in K$ satisfying $Su = u$.

In fact, SCSP contains several important problems as special cases and many authors have studied and introduced some new iterative algorithms for SCSP and presented some strong and weak convergence theorems for SCSP; see, for instance, [1–24] and the references therein. Motivated and inspired by their works, in this paper, we study and establish new strong convergence results by using new iterative algorithms of SCSP for pseudocontractive mappings and k-demicontractive mappings in Hilbert spaces.

The paper is divided into four sections. In Section 2, we study and give examples for classes of generalized contractive mappings. Some new strong convergence theorems of feasible iterative algorithms for SCSP are established in Section 3. Finally, some applications and further remarks for our new results are given in Section 4. Consequently, in this paper, some of our results are original and completely different from these known related results in the literature.

2 Classes of generalized contractive mappings and their examples

Let T be a mapping with domain $\mathcal{D}(T)$ and range $\mathcal{R}(T)$ in a Hilbert space H. Recall that T is said to be

(i) *pseudocontractive* if

$$\langle Tx - Ty, x - y\rangle \le \|x - y\|^2, \quad \forall x, y \in \mathcal{D}(T),$$

or, equivalently,

$$\|Tx - Ty\|^2 \le \|x - y\|^2 + \big\|(I - T)x - (I - T)y\big\|^2, \quad \forall x, y \in \mathcal{D}(T);$$

(ii) *demicontractive* if, for all $x \in \mathcal{D}(T)$ and $p \in \mathcal{F}(T)$,

$$\langle Tx - p, x - p\rangle \le \|x - p\|^2$$

or, equivalently,

$$\|Tx - p\|^2 \le \|x - p\|^2 + \big\|(I - T)x\big\|^2;$$

(iii) *k-demicontractive* if there exists a constant $k \in [0, 1)$ such that

$$\|Tx - p\|^2 \le \|x - p\|^2 + k\big\|(I - T)x\big\|^2 \quad \text{for all } x \in \mathcal{D}(T) \text{ and } p \in \mathcal{F}(T);$$

(iv) *quasi-nonexpansive* if it is 0-demicontractive, that is,

$$\|Tx - p\| \leq \|x - p\| \quad \text{for all } x \in \mathcal{D}(T) \text{ and } p \in \mathcal{F}(T);$$

(v) *Lipschitzian* if there exists $L > 0$ such that

$$\|Tx - Ty\| \leq L\|x - y\|, \quad \forall x, y \in \mathcal{D}(T);$$

(vi) *nonexpansive* if it is Lipschitzian with $L = 1$;
(vii) *contractive* if it is Lipschitzian with $L < 1$.

A Banach space $(X, \|\cdot\|)$ is said to satisfy *Opial's condition* if, for each sequence $\{x_n\}$ in X which converges weakly to a point $x \in X$, we have

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|, \quad \forall y \in X, y \neq x.$$

It is well known that any Hilbert space satisfies Opial's condition.

Definition 2.1 (see [2]) Let K be a nonempty closed convex subset of a real Hilbert space H and T be a mapping from K into K. The mapping T is said to be *demiclosed* if, for any sequence $\{x_n\}$ which weakly converges to y, and if the sequence $\{Tx_n\}$ strongly converges to z, then $Ty = z$.

Remark 2.1 In Definition 2.1, the particular case of demiclosedness at zero is frequently used in some iterative convergence algorithms, which is the particular case when $z = \theta$, the zero vector of H; for more details, one can refer to [2].

The following concept of zero-demiclosedness was introduced as follows.

Definition 2.2 (see [25, Definition 2.3]) Let K be a nonempty closed convex subset of a real Hilbert space and T be a mapping from K into K. The mapping T is called *zero-demiclosed* if $\{x_n\}$ in K satisfying $\|x_n - Tx_n\| \to 0$ and $x_n \rightharpoonup z \in K$ implies $Tz = z$.

The following result was essentially proved in [25], but we give the proof for the sake of completeness and the reader's convenience.

Theorem 2.1 (see [25, Proposition 2.4]) *Let K be a nonempty closed convex subset of a real Hilbert space with zero vector θ. Then the following statements hold.*
(a) *Let T be a mapping from K into K. Then T is zero-demiclosed if and only if $I - T$ is demiclosed at θ.*
(b) *Let T be a nonexpansive mapping from H into itself. If there is a bounded sequence $\{x_n\} \subset H$ such that $\|x_n - Tx_n\| \to 0$ as $n \to 0$, then T is zero-demiclosed.*

Proof Obviously, the conclusion (a) holds. To see (b), since $\{x_n\}$ is bounded, there is a subsequence $\{x_{n_k}\} \subset \{x_n\}$ and $z \in H$ such that $x_{n_k} \rightharpoonup z$. One can claim $Tz = z$. Indeed, if $Tz \neq z$, it follows from Opial's condition that

$$\begin{aligned} \liminf_{k\to\infty} \|x_{n_k} - z\| &< \liminf_{k\to\infty} \|x_{n_k} - Tz\| \\ &\leq \liminf_{k\to\infty} \bigl\{ \|x_{n_k} - Tx_{n_k}\| + \|Tx_{n_k} - Tz\| \bigr\} \end{aligned}$$

$$\begin{aligned} &= \liminf_{k\to\infty} \|Tx_{n_k} - Tz\| \\ &\leq \liminf_{k\to\infty} \|x_{n_k} - z\|, \end{aligned}$$

which is a contradiction. So $Tz = z$ and hence T is zero-demiclosed. □

Now, we give some examples to show the existence of these generalized contractive mappings (i)-(vi) which also expound the relation between them.

Example A Let $H = \mathbb{R}$ with the absolute-value norm $|\cdot|$ and $C = [-2, 0]$. Let $T : C \to C$ be defined by

$$Tx = \begin{cases} x^2 - 2 & \text{if } x \in [-1, 0], \\ -1 & \text{if } x \in [-2, -1]. \end{cases}$$

Then $\mathcal{F}(T) = \{-1\}$. Since

$$\left|Tx - (-1)\right|^2 \leq \left|x - (-1)\right|^2 + \frac{1}{2}|Tx - x|^2 \quad \text{for all } x \in C,$$

we know that T is a $\frac{1}{2}$-demicontractive mapping. However, due to

$$\left|T\left(-\frac{1}{2}\right) - (-1)\right| > \left|-\frac{1}{2} - (-1)\right|,$$

T is not quasi-nonexpansive.

Example B Let $H = \mathbb{R}$ with the absolute-value norm $|\cdot|$ and $C = [\frac{1}{2}, 2]$. Let $T : C \to C$ be defined by

$$Tx = \frac{1}{x}, \quad \forall x \in C.$$

Then $\mathcal{F}(T) = \{1\}$. Since

$$|Tx - 1|^2 \leq |x - 1|^2 + \frac{3}{4}|Tx - x|^2 \quad \text{for all } x \in C,$$

T is a $\frac{3}{4}$-demicontractive mapping. Moreover, T is also a pseudocontractive mapping.

Example C Let $H = \mathbb{R}$ with the absolute-value norm $|\cdot|$. Let $T : H \to H$ be defined by

$$Tx = \begin{cases} -\sqrt{-(1 + x)} & \text{if } x \leq -2, \\ x + 1 & \text{if } x \geq -2. \end{cases}$$

It is easy to see that

$$|Tx - Ty| \leq |x - y| \quad \text{for all } x, y \in H.$$

So T is continuous nonexpansive with $\mathcal{F}(T) = \emptyset$.

The following example shows that there exists a continuous quasi-nonexpansive mapping which is not nonexpansive.

Example D (see [8]) Let $H = \mathbb{R}$ with the absolute-value norm $|\cdot|$ and $C = [0, +\infty)$. Define $T : C \to C$ by

$$Tx = \frac{x^2 + 2}{1 + x}, \quad \forall x \in C.$$

Obviously, $\mathcal{F}(T) = \{2\}$. It is easy to see that

$$|Tx - 2| = \frac{x}{1 + x}|x - 2| \leq |x - 2| \quad \text{for all } x \in C$$

and

$$\left|T(0) - T\left(\frac{1}{3}\right)\right| = \frac{5}{12} > \left|0 - \frac{1}{3}\right|.$$

Hence T is a continuous quasi-nonexpansive mapping but not nonexpansive.

The following example shows that there exists a demicontractive mapping which is neither pseudocontractive nor k-demicontractive for all $k \in [0, 1)$.

Example E Let $H = \mathbb{R}$ with the absolute-value norm $|\cdot|$. Let $T : H \to H$ be defined by

$$Tx = \begin{cases} x^2 - x + 1 & \text{if } x \in (-\infty, 1], \\ \frac{x^2+1}{1+x} & \text{if } x \in [1, +\infty). \end{cases}$$

Then $\mathcal{F}(T) = \{1\}$. Since

$$|Tx - 1|^2 \leq |x - 1|^2 + |Tx - x|^2 \quad \text{for all } x \in H,$$

T is a demicontractive mapping. However, T is not a pseudocontractive mapping due to the fact that when $x = -3$ and $y = -2.5$, we have

$$|Tx - Ty|^2 > |x - y|^2 + \left|(x - Tx) - (y - Ty)\right|^2.$$

It is easy to see that T is not a k-demicontractive mapping for all $k \in [0, 1)$.

The following example shows that there exists a discontinuous pseudocontractive mapping which is not a demicontractive mapping.

Example F Let $H = \mathbb{R}$ with the absolute-value norm $|\cdot|$. Let $T : H \to H$ be defined by

$$Tx = \begin{cases} x^2 + 1 & \text{if } x \in (-\infty, 0], \\ -1 - x^2 & \text{if } x \in (0, +\infty). \end{cases}$$

Then $\mathcal{F}(T) = \emptyset$. Due to

$$|Tx - Ty|^2 \leq |x - y|^2 + \left|(I - T)x - (I - T)y\right|^2 \quad \text{for all } x \in H,$$

we know that T is a discontinuous pseudocontractive mapping but not a demicontractive mapping.

The following example shows that there exists a pseudocontractive mapping which is not k-demicontractive for all $k \in [0,1)$.

Example G Let $H = \mathbb{R}$ with the absolute-value norm $|\cdot|$. Let $T : H \to H$ be defined by

$$Tx = \begin{cases} 2 - x^2 & \text{if } x \in [0,1], \\ 2 - x & \text{if } x \in [1,2], \\ 0 & \text{if } x \in [1,+\infty). \end{cases}$$

Then $\mathcal{F}(T) = \{1\}$. Since

$$|Tx - Ty|^2 \le |x - y|^2 + \left|(I - T)x - (I - T)y\right|^2 \quad \text{for all } x \in H,$$

T is a pseudocontractive mapping. It is easy to see that T is not a k-demicontractive mapping for all $k \in [0,1)$.

The following example shows that there exists a discontinuous k-demicontractive mapping for some $k \in [0,1)$ as well as being demiclosed at θ which is neither pseudocontractive nor quasi-nonexpansive.

Example H Let $H = \mathbb{R}$ with the absolute-value norm $|\cdot|$ and $C = [-2,0]$. Let $T : C \to C$ be defined by

$$Tx = \begin{cases} x^2 - 2 & \text{if } x \in [-1,0], \\ -\frac{1}{8} & \text{if } x = -\frac{3}{2}, \\ -1 & \text{if } x \in [-2,-\frac{3}{2}) \cup (-\frac{3}{2},-1]. \end{cases}$$

Then the following statements hold.

(a) T is discontinuous $\frac{3}{4}$-demicontractive.
(b) T is demiclosed at θ.
(c) T is not pseudocontractive.
(d) T is not quasi-nonexpansive.

Proof Clearly, $\mathcal{F}(T) = \{-1\}$. Since

$$\left|Tx - (-1)\right|^2 \le \left|x - (-1)\right|^2 + \frac{3}{4}\left|(I - T)x\right|^2 \quad \text{for all } x \in C,$$

T is a discontinuous $\frac{3}{4}$-demicontractive mapping and (a) is proved. Now, we verify (b). In fact, let $\{x_n\} \subset [-2,0]$ with $x_n \to z$ and $x_n - Tx_n \to 0$ as $n \to \infty$. If all $x_n \in [-1,0]$, we can prove $Tz = z$ and $z = -1 \in F(T)$ easily. If there exists a subsequence $\{x_{n_k}\} \subset [-2,-1]$, then, from $x_n - Tx_n \to 0$ as $n \to \infty$, we can find a subsequence $\{x_{n_{k_i}}\}$ of $\{x_{n_k}\}$ such that $x_{n_{k_i}} \neq -\frac{3}{2}$ for all i. Hence we have

$$\left|z - (-1)\right| \le |z - x_{n_{k_i}}| + |x_{n_{k_i}} - Tx_{n_{k_i}}| + \left|Tx_{n_{k_i}} - (-1)\right| \to 0 \quad \text{as } i \to \infty,$$

which implies $z = -1 \in \mathcal{F}(T)$. To see (c) and (d), note that

$$\left|T\left(-\frac{3}{2}\right) - T\left(-\frac{25}{16}\right)\right|^2 > \left|-\frac{3}{2} - \left(-\frac{25}{16}\right)\right|^2 + \left|(I-T)\left(-\frac{3}{2}\right) - (I-T)\left(-\frac{25}{16}\right)\right|^2$$

and

$$\left|T\left(-\frac{3}{2}\right) - (-1)\right| > \left|\left(-\frac{3}{2}\right) - (-1)\right|,$$

so T is neither pseudocontractive nor quasi-nonexpansive. The proof is completed. □

3 New feasible iterative algorithms for SCSP and strong convergence theorems

In this section, we establish some new strong convergence theorems by using feasible iterative algorithms for SCSP.

Theorem 3.1 *Let H_1 and H_2 be two real Hilbert spaces and θ_i be the zero vector of H_i for $i = 1, 2$. Let C be a nonempty closed convex subset of H_1 and $A : H_1 \to H_2$ be a bounded linear operator with its adjoint B. Let $T : C \to C$ be a Lipschitzian pseudocontractive mapping with Lipschitz constant $L > 0$ and $\mathcal{F}(T) \neq \emptyset$, and let $S : H_2 \to H_2$ be a k-demicontractive mapping with $\mathcal{F}(S) \neq \emptyset$ which is demiclosed at θ_2. Let $C_1 = C$ and $\{x_n\}$ be a sequence generated by the following algorithm*:

$$\begin{cases} x_1 \in C_1 \quad \text{chosen arbitrarily}, \\ y_n = (1-\alpha)x_n + \alpha T x_n, \\ z_n = \beta x_n + (1-\beta) T y_n, \\ w_n = P_C(z_n + \xi B(S-I)Az_n), \\ C_{n+1} = \{v \in C_n : \|w_n - v\| \leq \|z_n - v\| \leq \|x_n - v\|\}, \\ x_{n+1} = P_{C_{n+1}}(x_1), \quad \forall n \in \mathbb{N}, \end{cases} \tag{3.1}$$

where $0 < 1 - \beta < \alpha < \frac{1}{2\sqrt{1+L^2}}$, $\xi \in (0, \frac{1-k}{\|B\|^2})$ and P_{C_n} is the projection operator from H_1 into C_n for $n \in \mathbb{N}$. Suppose that

$$\Omega = \{p \in \mathcal{F}(T) : Ap \in \mathcal{F}(S)\} \neq \emptyset.$$

Then there exists $q \in \Omega$ such that

(a) *$x_n \to q$ as $n \to \infty$,*
(b) *$Ax_n \to Aq$ as $n \to \infty$.*

Proof We will show the conclusion by proceeding with the following steps.

Step 1. For any $p \in \Omega$, we prove

$$\|w_n - p\|^2 \leq \|z_n - p\|^2 - \xi(1 - k - \xi\|B\|^2)\|(S-I)Az_n\|^2. \tag{3.2}$$

Indeed, since

$$\begin{aligned} &\|w_n - p\|^2 \\ &\quad \leq \|z_n + \xi B(S-I)Az_n - p\|^2 \end{aligned}$$

$$
\begin{aligned}
&= \|z_n - p\|^2 + \left\|\xi B(S-I)Az_n\right\|^2 + 2\xi\langle z_n - p, B(S-I)Az_n\rangle \\
&= \|z_n - p\|^2 + \left\|\xi B(S-I)Az_n\right\|^2 + 2\xi\langle Az_n - Ap, (S-I)Az_n\rangle \\
&= \|z_n - p\|^2 + \left\|\xi B(S-I)Az_n\right\|^2 + 2\xi\langle Az_n - Ap + (S-I)Az_n - (S-I)Az_n, (S-I)Az_n\rangle \\
&= \|z_n - p\|^2 + \left\|\xi B(S-I)Az_n\right\|^2 + 2\xi\langle SAz_n - Ap, (S-I)Az_n\rangle - 2\xi\left\|(S-I)Az_n\right\|^2 \\
&\le \|z_n - p\|^2 + \xi^2\|B\|^2\left\|(S-I)Az_n\right\|^2 + 2\xi\langle SAz_n - Ap, (S-I)Az_n\rangle - 2\xi\left\|(S-I)Az_n\right\|^2
\end{aligned}
$$

and

$$
\begin{aligned}
&2\xi\langle SAz_n - Ap, (S-I)Az_n\rangle \\
&\quad = \xi\left\{\|SAz_n - Ap\|^2 + \left\|(S-I)Az_n\right\|^2 - \|Az_n - Ap\|^2\right\} \\
&\quad \le \xi\left\{\|Az_n - Ap\|^2 + k\left\|(S-I)Az_n\right\|^2 + \left\|(S-I)Az_n\right\|^2 - \|Az_n - Ap\|^2\right\} \\
&\quad \le \xi\left\{-\|Az_n - Az_n\|^2 + k\left\|(S-I)Az_n\right\|^2 + \left\|(S-I)Az_n\right\|^2\right\} \\
&\quad = \xi\left\{k\left\|(S-I)Az_n\right\|^2 + \left\|(S-I)Az_n\right\|^2\right\},
\end{aligned}
$$

we get

$$\|w_n - p\|^2 \le \|z_n - p\|^2 - \xi\left(1 - k - \xi\|B\|^2\right)\left\|(S-I)Az_n\right\|^2,$$

and our desired result is proved.

Step 2. We prove

$$\|z_n - p\| \le \|x_n - p\| \quad \text{for all } n \in \mathbb{N}. \tag{3.3}$$

For any $n \in \mathbb{N}$, by (3.1), we have

$$
\begin{aligned}
&\|z_n - p\|^2 \\
&\quad = \beta\|x_n - p\|^2 + (1-\beta)\|Ty_n - p\|^2 - (1-\beta)\beta\|Ty_n - x_n\|^2 \\
&\quad \le \beta\|x_n - p\|^2 + (1-\beta)\|y_n - p\|^2 + (1-\beta)\|Ty_n - y_n\|^2 - (1-\beta)\beta\|Ty_n - x_n\|^2 \\
&\quad \le \beta\|x_n - p\|^2 + (1-\beta)\big((1-\alpha)\|x_n - p\|^2 + \alpha\|Tx_n - x_n\|^2 - (1-\alpha)\alpha\|Tx_n - x_n\|^2\big) \\
&\qquad + (1-\beta)\|Ty_n - y_n\|^2 - (1-\beta)\beta\|Ty_n - x_n\|^2 \\
&\quad \le \|x_n - p\|^2 + (1-\beta)\big(\alpha\|Tx_n - x_n\|^2 - (1-\alpha)\alpha\|Tx_n - x_n\|^2\big) - (1-\beta)\beta\|Ty_n - x_n\|^2 \\
&\qquad + (1-\beta)\left\|(1-\alpha)(x_n - Ty_n) + \alpha(Tx_n - Ty_n)\right\|^2 \\
&\quad \le \|x_n - p\|^2 + (1-\beta)\big(\alpha\|Tx_n - x_n\|^2 - (1-\alpha)\alpha\|Tx_n - x_n\|^2\big) - (1-\beta)\beta\|Ty_n - x_n\|^2 \\
&\qquad + (1-\beta)\big((1-\alpha)\|x_n - Ty_n\|^2 + \alpha\|Tx_n - Ty_n\|^2 - (1-\alpha)\alpha\|Tx_n - x_n\|^2\big) \\
&\quad \le \|x_n - p\|^2 + (1-\beta)\big(\alpha\|Tx_n - x_n\|^2 - (1-\alpha)\alpha\|Tx_n - x_n\|^2\big) - (1-\beta)\beta\|Ty_n - x_n\|^2 \\
&\qquad + (1-\beta)\big((1-\alpha)\|x_n - Ty_n\|^2 + \alpha L^2\|x_n - y_n\|^2 - (1-\alpha)\alpha\|Tx_n - x_n\|^2\big) \\
&\quad \le \|x_n - p\|^2 + (1-\beta)\big(\alpha\|Tx_n - x_n\|^2 - (1-\alpha)\alpha\|Tx_n - x_n\|^2\big) - (1-\beta)\beta\|Ty_n - x_n\|^2
\end{aligned}
$$

$$\begin{aligned}&+(1-\beta)\big((1-\alpha)\|x_n-Ty_n\|^2+\alpha^3L^2\|x_n-Tx_n\|^2-(1-\alpha)\alpha\|Tx_n-x_n\|^2\big)\\ &=\|x_n-p\|^2-(1-\beta)(\alpha+\beta-1)\|Ty_n-x_n\|^2\\ &\quad-(1-\beta)\alpha\big(1-2\alpha-\alpha^2L^2\big)\|Tx_n-x_n\|^2. \qquad (3.4)\end{aligned}$$

Since $\alpha+\beta>1$ and $\alpha<\frac{1}{2\sqrt{1+L^2}}$, from (3.4), we have $\|z_n-p\|^2\le\|x_n-p\|^2$, or, equivalently,

$$\|z_n-p\|\le\|x_n-p\|. \qquad (3.5)$$

Step 3. We show that C_n is a nonempty closed convex set for any $n\in\mathbb{N}$.

For any $p\in\Omega$, by taking into account (3.2) and (3.5), we obtain

$$\|w_n-p\|\le\|z_n-p\|\le\|x_n-p\| \quad \text{for all } n\in\mathbb{N}.$$

So we know $\Omega\subset C_n$ and hence $C_n\neq\emptyset$ for all $n\in\mathbb{N}$. It is easy to verify that C_n is closed and convex for all $n\in\mathbb{N}$.

Step 4. We prove that $\{x_n\}$ is a Cauchy sequence in C and $x_n\to q$ as $n\to\infty$ for some $q\in C$.

Since $\Omega\subset C_{n+1}\subset C_n$ and $x_{n+1}=P_{C_{n+1}}(x_1)\subset C_n$, we get

$$\|x_{n+1}-x_1\|\le\|p-x_1\| \quad \text{for all } p\in\Omega$$

and

$$\|x_n-x_1\|\le\|x_{n+1}-x_1\| \quad \text{for all } n\in\mathbb{N},$$

which show that $\{x_n\}$ is bounded and $\{\|x_n-x_1\|\}$ is nondecreasing in $[0,\infty)$. So

$$\lim_{n\to\infty}\|x_n-x_1\|\ge 0$$

exists. For any $m,n\in\mathbb{N}$ with $m>n$, from $x_m=P_{C_m}(x_1)\subset C_n$ and (1.1), we have

$$\|x_m-x_n\|^2+\|x_1-x_n\|^2=\big\|x_m-P_{C_n}(x_1)\big\|^2+\big\|x_1-P_{C_n}(x_1)\big\|^2\le\|x_m-x_1\|^2. \qquad (3.6)$$

Inequality (3.6) implies

$$\lim_{m,n\to\infty}\|x_n-x_m\|=0.$$

So $\{x_n\}$ is a Cauchy sequence. Clearly,

$$\lim_{n\to\infty}\|x_{n+1}-x_n\|=0. \qquad (3.7)$$

By the completeness of C, there exists $q\in C$ such that $x_n\to q$ as $n\to\infty$.

Step 5. Finally, we show that the following hold:

(i) $q\in\Omega$,

(ii) $Ax_n\to Aq$ as $n\to\infty$.

For any $n \in \mathbb{N}$, since $x_{n+1} = P_{C_{n+1}}(x_1) \in C_{n+1} \subset C_n$, from (3.1), we have

$$\|z_n - x_n\| \leq \|z_n - x_{n+1}\| + \|x_{n+1} - x_n\| \leq 2\|x_{n+1} - x_n\| \tag{3.8}$$

and

$$\|w_n - x_n\| \leq \|w_n - x_{n+1}\| + \|x_{n+1} - x_n\| \leq 2\|x_{n+1} - x_n\|. \tag{3.9}$$

From inequalities (3.7), (3.8) and (3.9), we deduce

$$\begin{aligned} &\lim_{n\to\infty} \|z_n - x_n\| = 0, \\ &\lim_{n\to\infty} \|w_n - x_n\| = 0 \end{aligned} \tag{3.10}$$

and hence

$$\lim_{n\to\infty} \|w_n - z_n\| = 0. \tag{3.11}$$

By taking into account (3.4) and (3.10), we get

$$\begin{aligned} &\alpha\big(1 - 2\alpha - \alpha^2 L^2\big)\|Tx_n - x_n\|^2 + (\alpha + \beta - 1)\|Ty_n - x_n\|^2 \\ &\quad\leq \frac{1}{1-\beta}\big(\|x_n - p\|^2 - \|z_n - p\|^2\big) \\ &\quad\leq \frac{2}{1-\beta}\|x_n - z_n\|\|x_n - p\| \to 0 \quad \text{as } n \to \infty. \end{aligned}$$

So, we obtain

$$\lim_{n\to\infty} \|Tx_n - x_n\| = \lim_{n\to\infty} \|Ty_n - x_n\| = 0. \tag{3.12}$$

Since $x_n \to q$ as $n \to \infty$, from (3.12) and the continuity of the norm $\|\cdot\|$ and the Lipschitzian pseudocontractive mapping T, we can deduce that $Tq = q$, namely $q \in \mathcal{F}(T)$. On the other hand, from (3.2) and (3.11), we have

$$\begin{aligned} &\xi\big(1 - k - \xi\|B\|^2\big)\big\|(S-I)Az_n\big\|^2 \\ &\quad\leq \|z_n - p\|^2 - \|w_n - p\|^2 \\ &\quad\leq \|z_n - w_n\|\big(\|z_n - p\| - \|w_n - p\|\big) \to 0 \quad \text{as } n \to \infty, \end{aligned}$$

which yields that

$$\lim_{n\to\infty} \big\|(S-I)Az_n\big\| = 0. \tag{3.13}$$

Since the k-demicontractive mapping S is demiclosed at θ_2, taking into account $x_n \to q$, $Ax_n \to Aq$, $\|z_n - x_n\| \to 0$ and (3.13), we have

$$Az_n \to Aq$$

and

$$Aq \in \mathcal{F}(S).$$

Hence we confirm $q \in \Omega$. The proof is completed. □

By virtue of Theorem 3.1, we can establish the following:

(i) Strong convergence algorithms for the split common solution problem for Lipschitzian pseudocontractive mappings and nonexpansive mappings (see Corollary 3.1 below).

(ii) Strong convergence algorithms for the split common solution problem for Lipschitzian pseudocontractive mappings and quasi-nonexpansive mappings (see Corollary 3.2 below).

Corollary 3.1 *Let H_1 and H_2 be two real Hilbert spaces and θ_i be the zero vector of H_i for $i = 1, 2$. Let C be a nonempty closed convex subset of H_1 and $A : H_1 \to H_2$ be a bounded linear operator with its adjoint B. Let $T : C \to C$ be a Lipschitzian pseudocontractive mapping with Lipschitz constant $L > 0$ and $\mathcal{F}(T) \neq \emptyset$, and let $S : H_2 \to H_2$ be a nonexpansive mapping with $\mathcal{F}(S) \neq \emptyset$. Let $C_1 = C$ and $\{x_n\}$ be a sequence generated by the following algorithm*:

$$\begin{cases} x_1 \in C_1 \quad \text{chosen arbitrarily}, \\ y_n = (1-\alpha)x_n + \alpha T x_n, \\ z_n = \beta x_n + (1-\beta) T y_n, \\ w_n = P_C(z_n + \xi B(S-I)Az_n), \\ C_{n+1} = \{v \in C_n : \|w_n - v\| \leq \|z_n - v\| \leq \|x_n - v\|\}, \\ x_{n+1} = P_{C_{n+1}}(x_1), \quad \forall n \in \mathbb{N}, \end{cases}$$

where $0 < 1 - \beta < \alpha < \frac{1}{2\sqrt{1+L^2}}$, $\xi \in (0, \frac{1}{\|B\|^2})$ and P_{C_n} is the projection operator from H_1 into C_n for $n \in \mathbb{N}$. Suppose that

$$\Omega = \{p \in \mathcal{F}(T) : Ap \in \mathcal{F}(S)\} \neq \emptyset.$$

Then there exists $q \in \Omega$ such that

(a) *$x_n \to q$ as $n \to \infty$,*

(b) *$Ax_n \to Aq$ as $n \to \infty$.*

Proof Since the mapping S is nonexpansive, it is 0-demicontractive. Hence the desired conclusion follows from Theorem 3.1 immediately by taking $k = 0$. □

Corollary 3.2 *Let H_1 and H_2 be two real Hilbert spaces and θ_i be the zero vector of H_i for $i = 1, 2$. Let C be a nonempty closed convex subset of H_1 and $A : H_1 \to H_2$ be a bounded linear operator with its adjoint B. Let $T : C \to C$ be a Lipschitzian pseudocontractive mapping with Lipschitz constant $L > 0$ and $\mathcal{F}(T) \neq \emptyset$, and let $S : H_2 \to H_2$ be a quasi-nonexpansive mapping with $\mathcal{F}(S) \neq \emptyset$ which is demiclosed at θ_2. Let $C_1 = C$ and $\{x_n\}$ be a sequence gen-*

erated by the following algorithm:

$$\begin{cases} x_1 \in C_1 \quad \textit{chosen arbitrarily}, \\ y_n = (1-\alpha)x_n + \alpha T x_n, \\ z_n = \beta x_n + (1-\beta) T y_n, \\ w_n = P_C(z_n + \xi B(S-I)Az_n), \\ C_{n+1} = \{v \in C_n : \|w_n - v\| \le \|z_n - v\| \le \|x_n - v\|\}, \\ x_{n+1} = P_{C_{n+1}}(x_1), \quad \forall n \in \mathbb{N}, \end{cases}$$

where $0 < 1-\beta < \alpha < \frac{1}{2\sqrt{1+L^2}}$, $\xi \in (0, \frac{1}{\|B\|^2})$ *and* P_{C_n} *is the projection operator from* H_1 *into* C_n *for* $n \in \mathbb{N}$. *Suppose that*

$$\Omega = \{p \in \mathcal{F}(T) : Ap \in \mathcal{F}(S)\} \neq \emptyset.$$

Then there exists $q \in \Omega$ *such that*

(a) $x_n \to q$ *as* $n \to \infty$,

(b) $Ax_n \to Aq$ *as* $n \to \infty$.

Example 3.1 Let $H_1 = \mathbb{R}$ with the absolute-value norm $|\cdot|$. Let $H_2 = [\frac{1}{\sqrt{2}}, \sqrt{2}]^2$ with the norm $\|\alpha\| = (a_1^2 + a_2^2)^{\frac{1}{2}}$ for $\alpha = (a_1, a_2) \in H_2$ and the inner product $\langle \alpha, \beta \rangle = \sum_{i=1}^2 a_i b_i$ for $\alpha = (a_1, a_2)$ and $\beta = (b_1, b_2) \in H_2$. Let $A : H_1 \to H_2$ be defined by $Ax = (x, x)$ for $x \in \mathbb{R}$. Then A is a bounded linear operator with its adjoint operator $Bz = z_1 + z_2$ for $z = (z_1, z_2) \in H_2$. Clearly, $\|A\| = \|B\| = \sqrt{2}$. Let $C = [\frac{1}{\sqrt{2}}, \sqrt{2}]$. Let $T : C \to C$ and $S : H_2 \to H_2$ be defined by

$$Tx = \frac{1}{x} \quad \text{for } x \in C$$

and

$$Sz = \left(\frac{1}{z_1}, \frac{1}{z_2}\right) \quad \text{for } z = (z_1, z_2) \in H_2,$$

respectively. It is easy to see that

- $\mathcal{F}(T) = \{1\}$;
- $\mathcal{F}(S) = \{(1,1)\}$;
- $\Omega = \{p \in \mathcal{F}(T) : Ap \in \mathcal{F}(S)\} = \{1\} \neq \emptyset$;
- T is a Lipschitzian pseudocontractive mapping with Lipschitz constant $L = \sqrt{2}$;
- T and S both are $\frac{3}{4}$-demicontractive mappings.

By using algorithm (3.1) with $0 < 1-\beta < \alpha < \frac{1}{2\sqrt{3}}$ and $\xi \in (0, \frac{1}{8})$, we can verify $x_n \to 1$ and $Ax_n \to A(1) = (1,1) \in \mathcal{F}(S)$ as $n \to \infty$.

4 Some applications and further remarks for Theorem 3.1

Let C be a nonempty subset of a Hilbert space H. Recall that a mapping $U : C \to C$ is said to be *accretive* if

$$\langle Ux - Uy, x - y \rangle \ge 0 \quad \text{for all } x, y \in C.$$

Obviously, $U : C \to C$ is accretive if and only if $I - U : C \to C$ is pseudocontractive. Moreover,

$$\mathcal{F}(I - U) = U^{-1}(\theta) := \{x \in C : Ux = \theta\},$$

where θ is the zero vector of H.

At the end of this paper, by applying Theorem 3.1, we obtain the following:

(i) Strong convergence algorithms for the split common solution problem for Lipschitzian accretive mappings and demicontractive nonexpansive mappings (see Theorem 4.1 below).
(ii) Strong convergence algorithms for the split common solution problem for Lipschitzian accretive mappings and nonexpansive mappings (see Corollary 4.1 below).
(iii) Strong convergence algorithms for the split common solution problem for Lipschitzian accretive mappings and quasi-nonexpansive mappings (see Corollary 4.2 below).

Theorem 4.1 *Let H_1 and H_2 be two real Hilbert spaces and θ_i be the zero vector of H_i for $i = 1, 2$. Let $A : H_1 \to H_2$ be a bounded linear operator with its adjoint B and $U : H_1 \to H_1$ be a Lipschitzian accretive mapping with Lipschitz constant $L > 0$ and $U^{-1}(\theta_1) \neq \emptyset$. Let $S : H_2 \to H_2$ be a k-demicontractive mapping with $\mathcal{F}(S) \neq \emptyset$ which is demiclosed at θ_2. Let $\{x_n\}$ be a sequence generated by the following algorithm*:

$$\begin{cases} x_1 \in H_1 \quad \text{chosen arbitrarily}, \\ y_n = x_n - \alpha U x_n, \\ z_n = \beta x_n + (1 - \beta)(I - U) y_n, \\ w_n = z_n + \xi B(S - I) A z_n, \\ C_{n+1} = \{v \in C_n : \|w_n - v\| \leq \|z_n - v\| \leq \|x_n - v\|\}, \\ x_{n+1} = P_{C_{n+1}}(x_1), \quad \forall n \in \mathbb{N}, \end{cases} \tag{4.1}$$

where $0 < 1 - \beta < \alpha < \frac{1}{2\sqrt{1+L^2}}$, $\xi \in (0, \frac{1-k}{\|B\|^2})$ and P_{C_n} is the projection operator from H_1 into C_n for $n \in \mathbb{N}$. Suppose that

$$\Omega = \{p \in U^{-1}(\theta_1) : Ap \in \mathcal{F}(S)\} \neq \emptyset.$$

Then there exists $q \in \Omega$ such that

(a) $x_n \to q$ *as* $n \to \infty$,
(b) $Ax_n \to Aq$ *as* $n \to \infty$.

Proof Let $C_1 = H_1$. Then the iterative process (4.1) can be rewritten as follows:

$$\begin{cases} x_1 \in C_1 \quad \text{chosen arbitrarily}, \\ y_n = (1 - \alpha) x_n + \alpha (I - U) x_n, \\ z_n = \beta x_n + (1 - \beta)(I - U) y_n, \\ w_n = z_n + \xi B(S - I) A z_n, \\ C_{n+1} = \{v \in C_n : \|w_n - v\| \leq \|z_n - v\| \leq \|x_n - v\|\}, \\ x_{n+1} = P_{C_{n+1}}(x_1), \quad \forall n \in \mathbb{N}. \end{cases}$$

Set $T := I - U$, then $\mathcal{F}(T) = U^{-1}(\theta_1)$ and T is a Lipschitzian pseudocontractive mapping with Lipschitz constant $1 + L$. Therefore the desired conclusion follows from Theorem 3.1 immediately. □

The following interesting results are immediate from Theorem 4.1.

Corollary 4.1 *Let H_1 and H_2 be two real Hilbert spaces and θ_i be the zero vector of H_i for $i = 1, 2$. Let $A : H_1 \to H_2$ be a bounded linear operator with its adjoint B and $U : H_1 \to H_1$ be a Lipschitzian accretive mapping with Lipschitz constant $L > 0$ and $U^{-1}(\theta_1) \neq \emptyset$. Let $S : H_2 \to H_2$ be a quasi-nonexpansive mapping with $\mathcal{F}(S) \neq \emptyset$ which is demiclosed at θ_2. Let $\{x_n\}$ be a sequence generated by the following algorithm*:

$$\begin{cases} x_1 \in H_1 \quad \textit{chosen arbitrarily}, \\ y_n = x_n - \alpha U x_n, \\ z_n = \beta x_n + (1 - \beta)(I - U) y_n, \\ w_n = z_n + \xi B(S - I) A z_n, \\ C_{n+1} = \{v \in C_n : \|w_n - v\| \leq \|z_n - v\| \leq \|x_n - v\|\}, \\ x_{n+1} = P_{C_{n+1}}(x_1), \quad \forall n \in \mathbb{N}, \end{cases}$$

where $0 < 1 - \beta < \alpha < \frac{1}{2\sqrt{1+L^2}}$, $\xi \in (0, \frac{1}{\|B\|^2})$ and P_{C_n} is the projection operator from H_1 into C_n for $n \in \mathbb{N}$. Suppose that

$$\Omega = \{p \in U^{-1}(\theta_1) : Ap \in \mathcal{F}(S)\} \neq \emptyset.$$

Then there exists $q \in \Omega$ such that

(a) $x_n \to q$ *as* $n \to \infty$,

(b) $Ax_n \to Aq$ *as* $n \to \infty$.

Corollary 4.2 *Let H_1 and H_2 be two real Hilbert spaces and θ_i be the zero vector of H_i for $i = 1, 2$. Let $A : H_1 \to H_2$ be a bounded linear operator with its adjoint B. Let $U : H_1 \to H_1$ be a Lipschitzian accretive mapping with Lipschitz constant $L > 0$ and $U^{-1}(\theta_1) \neq \emptyset$. Let $S : H_2 \to H_2$ be a nonexpansive mapping with $\mathcal{F}(S) \neq \emptyset$. Let $\{x_n\}$ be a sequence generated by the following algorithm*:

$$\begin{cases} x_1 \in H_1 \quad \textit{chosen arbitrarily}, \\ y_n = x_n - \alpha U x_n, \\ z_n = \beta x_n + (1 - \beta)(I - U) y_n, \\ w_n = z_n + \xi B(S - I) A z_n, \\ C_{n+1} = \{v \in C_n : \|w_n - v\| \leq \|z_n - v\| \leq \|x_n - v\|\}, \\ x_{n+1} = P_{C_{n+1}}(x_1), \quad \forall n \in \mathbb{N}, \end{cases}$$

where $0 < 1 - \beta < \alpha < \frac{1}{2\sqrt{1+L^2}}$, $\xi \in (0, \frac{1}{\|B\|^2})$ and P_{C_n} is the projection operator from H_1 into C_n for $n \in \mathbb{N}$. Suppose that

$$\Omega = \{p \in U^{-1}(\theta_1) : Ap \in \mathcal{F}(S)\} \neq \emptyset.$$

Then there exists $q \in \Omega$ such that

(a) $x_n \to q$ *as* $n \to \infty$,
(b) $Ax_n \to Aq$ *as* $n \to \infty$.

Remark 4.1 In Theorems 3.1 and 4.1, the control coefficients α and β can be respectively replaced with the sequences $\{\alpha_n\}$ and $\{\beta_n\}$ satisfying $0 < \varepsilon < 1 - \beta_n < \alpha_n < \frac{1}{2\sqrt{1+L^2}}$ for some positive real number ε.

Remark 4.2 Obviously, all results in this paper are true if $H_1 = H_2$. They generalize and improve many results in the literature; see, for instance, [23, 24, 26–29].

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
Both authors contributed equally and significantly in writing this paper. Both authors read and approved the final manuscript.

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