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Some common tripled fixed point results in two quasi-partial metric spaces

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Abstract

In this paper, we establish some new common tripled fixed point theorems for mappings defined on a set equipped with two quasi-partial metrics. We also provide illustrative examples in support of our new results. The results presented in this paper generalize the well-known comparable results in the literature due to Karapinar *et al.* [Math. Comput. Model. 57:2442-2448, 2013], and Shatanawi and Pitea [Fixed Point Theory Appl. 2013:153, 2013].

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1 Introduction and preliminaries

In 1994, Matthews [1] introduced the notion of partial metric spaces and extended the Banach contraction principle from metric spaces to partial metric spaces. Based on the notion of partial metric spaces, several authors (for example, [2–32]) obtained some fixed point results for mappings satisfying different contractive conditions. Very recently, Haghi *et al.* [33] showed in their interesting paper that some of fixed point theorems in partial metric spaces can be obtained from metric spaces.

In 2013, Karapinar *et al.* [34] introduced the concept of quasi-partial metric spaces and studied some fixed point problems on quasi-partial metric spaces.

The notion of partial metric space is given as follows.

Definition 1.1 (Matthews [1]) A *partial metric* on a nonempty set X is a function $p: X \times X \longrightarrow \mathbb{R}^+$ such that for all $x, y, z \in X$:

(p1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,

(p2) $p(x,x) \le p(x,y)$,

(p3) $p(x,y) = p(y,x)$,

(p4) $p(x,y) \le p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X.

Following Karapinar *et al.* [34], the notion of quasi-partial metric spaces is given as follows.

Definition 1.2 (Karapinar *et al.* [34]) A *quasi-partial metric* on nonempty set X is a function $q: X \times X \to \mathbb{R}^+$ which satisfies:

(QPM_1) If $q(x,x) = q(x,y) = q(y,y)$, then $x = y$,
(QPM_2) $q(x,x) \leq q(x,y)$,
(QPM_3) $q(x,x) \leq q(y,x)$, and
(QPM_4) $q(x,y) + q(z,z) \leq q(x,z) + q(z,y)$ for all $x, y, z \in X$.

A *quasi-partial metric space* is a pair (X,q) such that X is a nonempty set and q is a quasi-partial metric on X.

Let q be a quasi-partial metric on set X. Then

$$d_q(x,y) = q(x,y) + q(y,x) - q(x,x) - q(y,y)$$

is a metric on X.

Definition 1.3 (Karapinar *et al.* [34]) Let (X,q) be a quasi-partial metric space. Then we have the following.

(i) A sequence $\{x_n\}$ *converges* to a point $x \in X$ if and only if

$$q(x,x) = \lim_{n\to\infty} q(x,x_n) = \lim_{n\to\infty} q(x_n,x).$$

(ii) A sequence $\{x_n\}$ is called a *Cauchy sequence* if $\lim_{n,m\to\infty} q(x_n,x_m)$ and $\lim_{n,m\to\infty} q(x_m,x_n)$ exist (and are finite).

(iii) The quasi-partial metric space (X,q) is said to be *complete* if every Cauchy sequence $\{x_n\}$ in X converges, with respect to τ_q, to a point $x \in X$ such that

$$q(x,x) = \lim_{n,m\to\infty} q(x_n,x_m) = \lim_{n,m\to\infty} q(x_n,x_m).$$

Bhaskar and Lakshmikantham [35] introduced the concept of coupled fixed point and studied some nice coupled fixed point theorems. Later, Lakshmikantham and Ćirić [36] introduced the notion of a coupled coincidence point of mappings. For some works on a coupled fixed point, we refer the reader to [37–68].

For simplicity, we denote from now on $\underbrace{X \times X \times \cdots \times X}_{k \text{ terms}}$ by X^k where $k \in \mathbb{N}$ and X is a nonempty set. We start by recalling some definitions.

Definition 1.4 (Bhaskar and Lakshmikantham [35]) An element $(x,y) \in X^2$ is called a *coupled fixed point* of the mapping $F : X^2 \to X$ if $F(x,y) = x$ and $F(y,x) = y$.

Definition 1.5 (Lakshmikantham and Ćirić [36]) An element $(x,y) \in X^2$ is called

(i) a *coupled coincidence point* of the mappings $F : X^2 \to X$ and $g : X \to X$ if $F(x,y) = gx$ and $F(y,x) = gy$, and (gx,gy) is called a *coupled point of coincidence*;

(ii) a *common coupled fixed point* of mappings $F : X^2 \to X$ and $g : X \to X$ if $F(x,y) = gx = x$ and $F(y,x) = gy = y$.

Definition 1.6 (Abbas *et al.* [37]) The mappings $F : X^2 \to X$ and $g : X \to X$ are called *w-compatible* if $gF(x,y) = F(gx,gy)$ whenever $F(x,y) = gx$ and $F(y,x) = gy$.

In 2010, Samet and Vetro [38] introduced a fixed point of order $N \geq 3$. In particular, for $N = 3$. we have the following definition.

Definition 1.7 (Samet and Vetro [38]) An element $(x, y, z) \in X^3$ is called a *tripled fixed point* of a given mapping $F : X^3 \to X$ if $F(x, y, z) = x$, $F(y, z, x) = y$, and $F(z, x, y) = z$.

Note that Berinde and Borcut [39] defined differently the notion of tripled fixed point in the case of ordered sets in order to keep true the mixed monotone property. For more details, see [39].

Definition 1.8 (Aydi *et al.* [40]) An element $(x, y, z) \in X^3$ is called

(i) a *tripled coincidence point* of mappings $F : X^3 \to X$ and $g : X \to X$ if $F(x, y, z) = gx$, $F(y, z, x) = gy$, and $F(z, x, y) = gz$. In this case (gx, gy, gz) is called a *tripled point of coincidence*;

(ii) a *common tripled fixed point* of mappings $F : X^3 \to X$ and $g : X \to X$ if $F(x, y, z) = gx = x$, $F(y, z, x) = gy = y$, and $F(z, x, y) = gz = z$.

Definition 1.9 (Aydi *et al.* [40]) The mappings $F : X^3 \to X$ and $g : X \to X$ are called *w-compatible* if $gF(x, y, z) = F(gx, gy, gz)$ whenever $F(x, y, z) = gx$, $F(y, z, x) = gy$, and $F(z, x, y) = gz$.

Recently, Aydi and Abbas [41] obtained some tripled coincidence and fixed point results in partial metric space.

Very recently, Shatanawi and Pitea [42] obtained some common coupled fixed point results for a pair of mappings in quasi-partial metric space.

Theorem 1.1 (Shatanawi and Pitea [42]) *Let (X, q) be a quasi-partial metric space, $g : X \to X$ and $F : X^2 \to X$ be two mappings. Suppose that there exist k_1, k_2, and k_3 in $[0, 1)$ with $k_1 + k_2 + k_3 < 1$ such that the condition*

$$\begin{aligned} &q\bigl(F(x,y),F(u,v)\bigr) + q\bigl(F(y,x),F(v,u)\bigr) \\ &\quad \le k_1\bigl[q(gx,gu) + q(gy,gv)\bigr] + k_2\bigl[q\bigl(gx,F(x,y)\bigr) + q\bigl(gy,F(y,x)\bigr)\bigr] \\ &\qquad + k_3\bigl[q\bigl(gu,F(u,v)\bigr) + q\bigl(gv,F(v,u)\bigr)\bigr] \end{aligned} \tag{1.1}$$

holds for all $x, y, u, v \in X$. Also, suppose we have the following hypotheses:

(i) *$F(X \times X) \subset g(X)$.*

(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial metric q.*

Then the mappings F and g have a coincidence point (x, y) satisfying $gx = F(x, y)$ and $gy = F(y, x)$.

Moreover, if F and g are w-compatible, then F and g have a unique common coupled fixed point of the form (x, x).

The aim of this article is to prove some new common tripled fixed point theorems for mappings defined on a set equipped with two quasi-partial metrics.

The following lemma is crucial in our work.

Lemma 1.1 (Shatanawi and Pitea [42]) *Let (X, q) be a quasi-partial metric space. Then the following statements hold true*:

(i) *If $q(x, y) = 0$, then $x = y$.*

(ii) *If $x \neq y$, then $q(x, y) > 0$ and $q(y, x) > 0$.*

In this manuscript, we generalize, improve, enrich, and extend the above coupled common fixed point results. We also state some examples to illustrate our results. This paper can be considered as a continuation of the remarkable works of Karapinar *et al.* [34] and Shatanawi and Pitea [42].

2 Main results

Theorem 2.1 *Let q_1 and q_2 be two quasi-partial metrics on X such that $q_2(x,y) \le q_1(x,y)$, for all $x,y \in X$, and $F : X^3 \to X$, $g : X \to X$ be two mappings. Suppose that there exist k_1, k_2, k_3, k_4, and k_5 in $[0,1)$ with*

$$k_1 + k_2 + k_3 + 2k_4 + k_5 < 1 \tag{2.1}$$

such that the condition

$$\begin{aligned} &q_1\bigl(F(x,y,z),F(u,v,w)\bigr) + q_1\bigl(F(y,z,x),F(v,w,u)\bigr) + q_1\bigl(F(z,x,y),F(w,u,v)\bigr) \\ &\quad\le k_1\bigl[q_2(gx,gu) + q_2(gy,gv) + q_2(gz,gw)\bigr] \\ &\qquad + k_2\bigl[q_2\bigl(gx,F(x,y,z)\bigr) + q_2\bigl(gy,F(y,z,x)\bigr) + q_2\bigl(gz,F(z,x,y)\bigr)\bigr] \\ &\qquad + k_3\bigl[q_2\bigl(gu,F(u,v,w)\bigr) + q_2\bigl(gv,F(v,w,u)\bigr) + q_2\bigl(gw,F(w,u,v)\bigr)\bigr] \\ &\qquad + k_4\bigl[q_2\bigl(gx,F(u,v,w)\bigr) + q_2\bigl(gy,F(v,w,u)\bigr) + q_2\bigl(gz,F(w,u,v)\bigr)\bigr] \\ &\qquad + k_5\bigl[q_2\bigl(gu,F(x,y,z)\bigr) + q_2\bigl(gv,F(y,z,x)\bigr) + q_2\bigl(gw,F(z,x,y)\bigr)\bigr] \end{aligned} \tag{2.2}$$

holds for all $x,y,z,u,v,w \in X$. Also, suppose we have the following hypotheses:

(i) *$F(X^3) \subset g(X)$.*
(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.*

Then the mappings F and g have a tripled coincidence point (x,y,z) satisfying

$$gx = F(x,y,z) = gy = F(y,z,x) = gz = F(z,x,y).$$

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u,u,u).

Proof Let $x_0,y_0,z_0 \in X$. Since $F(X^3) \subset g(X)$, we can choose $x_1,y_1,z_1 \in X$ such that $gx_1 = F(x_0,y_0,z_0)$, $gy_1 = F(y_0,z_0,x_0)$ and $gz_1 = F(z_0,x_0,y_0)$. Similarly, we can choose $x_2,y_2,z_2 \in X$ such that $gx_2 = F(x_1,y_1,z_1)$, $gy_2 = F(y_1,z_1,x_1)$, and $gz_2 = F(z_1,x_1,y_1)$. Continuing in this way we construct three sequences $\{x_n\}$, $\{y_n\}$, and $\{z_n\}$ in X such that

$$\begin{aligned} &gx_{n+1} = F(x_n,y_n,z_n), \qquad gy_{n+1} = F(y_n,z_n,x_n) \quad \text{and} \\ &gz_{n+1} = F(z_n,x_n,y_n), \quad \forall n \ge 0. \end{aligned} \tag{2.3}$$

It follows from (2.2), (2.3), (QPM2), and (QMP4) that

$$\begin{aligned} &q_1(gx_n,gx_{n+1}) + q_1(gy_n,gy_{n+1}) + q_1(gz_n,gz_{n+1}) \\ &\quad = q_1\bigl(F(x_{n-1},y_{n-1},z_{n-1}),F(x_n,y_n,z_n)\bigr) + q_1\bigl(F(y_{n-1},z_{n-1},x_{n-1}),F(y_n,z_n,x_n)\bigr) \end{aligned}$$

$$
\begin{aligned}
&+ q_1\big(F(z_{n-1},x_{n-1},y_{n-1}),F(z_n,x_n,y_n)\big)\\
&\le k_1\big[q_2(gx_{n-1},gx_n)+q_2(gy_{n-1},gy_n)+q_2(gz_{n-1},gz_n)\big]\\
&\quad + k_2\big[q_2\big(gx_{n-1},F(x_{n-1},y_{n-1},z_{n-1})\big)+q_2\big(gy_{n-1},F(y_{n-1},z_{n-1},x_{n-1})\big)\\
&\quad + q_2\big(gz_{n-1},F(z_{n-1},x_{n-1},y_{n-1})\big)\big]\\
&\quad + k_3\big[q_2\big(gx_n,F(x_n,y_n,z_n)\big)+q_2\big(gy_n,F(y_n,z_n,x_n)\big)+q_2\big(gz_n,F(z_n,x_n,y_n)\big)\big]\\
&\quad + k_4\big[q_2\big(gx_{n-1},F(x_n,y_n,z_n)\big)+q_2\big(gy_{n-1},F(y_n,z_n,x_n)\big)+q_2\big(gz_{n-1},F(z_n,x_n,y_n)\big)\big]\\
&\quad + k_5\big[q_2\big(gx_n,F(x_{n-1},y_{n-1},z_{n-1})\big)+q_2\big(gy_n,F(y_{n-1},z_{n-1},x_{n-1})\big)\\
&\quad + q_2\big(gz_n,F(z_{n-1},x_{n-1},y_{n-1})\big)\big]\\
&= (k_1+k_2)\big[q_2(gx_{n-1},gx_n)+q_2(gy_{n-1},gy_n)+q_2(gz_{n-1},gz_n)\big]\\
&\quad + k_3\big[q_2(gx_n,gx_{n+1})+q_2(gy_n,gy_{n+1})+q_2(gz_n,gz_{n+1})\big]\\
&\quad + k_4\big[q_2(gx_{n-1},gx_{n+1})+q_2(gy_{n-1},gy_{n+1})+q_2(gz_{n-1},gz_{n+1})\big]\\
&\quad + k_5\big[q_2(gx_n,gx_n)+q_2(gy_n,gy_n)+q_2(gz_n,gz_n)\big]\\
&\le (k_1+k_2)\big[q_2(gx_{n-1},gx_n)+q_2(gy_{n-1},gy_n)+q_2(gz_{n-1},gz_n)\big]\\
&\quad + k_3\big[q_2(gx_n,gx_{n+1})+q_2(gy_n,gy_{n+1})+q_2(gz_n,gz_{n+1})\big]\\
&\quad + k_4\big[q_2(gx_{n-1},gx_n)+q_2(gx_n,gx_{n+1})-q_2(gx_n,gx_n)\\
&\quad + q_2(gy_{n-1},gy_n)+q_2(gy_n,gy_{n+1})-q_2(gy_n,gy_n)\\
&\quad + q_2(gz_{n-1},gz_n)+q_2(gz_n,gz_{n+1})-q_2(gz_n,gz_n)\big]\\
&\quad + k_5\big[q_2(gx_n,gx_{n+1})+q_2(gy_n,gy_{n+1})+q_2(gz_n,gz_{n+1})\big]\\
&\le (k_1+k_2+k_4)\big[q_2(gx_{n-1},gx_n)+q_2(gy_{n-1},gy_n)+q_2(gz_{n-1},gz_n)\big]\\
&\quad + (k_3+k_4+k_5)\big[q_2(gx_n,gx_{n+1})+q_2(gy_n,gy_{n+1})+q_2(gz_n,gz_{n+1})\big]\\
&\le (k_1+k_2+k_4)\big[q_1(gx_{n-1},gx_n)+q_1(gy_{n-1},gy_n)+q_1(gz_{n-1},gz_n)\big]\\
&\quad + (k_3+k_4+k_5)\big[q_1(gx_n,gx_{n+1})+q_1(gy_n,gy_{n+1})+q_1(gz_n,gz_{n+1})\big],
\end{aligned}
$$

which implies that

$$
\begin{aligned}
&q_1(gx_n,gx_{n+1})+q_1(gy_n,gy_{n+1})+q_1(gz_n,gz_{n+1})\\
&\quad\le \frac{k_1+k_2+k_4}{1-k_3-k_4-k_5}\big[q_1(gx_{n-1},gx_n)+q_1(gy_{n-1},gy_n)+q_1(gz_{n-1},gz_n)\big].
\end{aligned}
\tag{2.4}
$$

Put $k=\frac{k_1+k_2+k_4}{1-k_3-k_4-k_5}$. Obviously, $0\le k<1$. Repetition of the above inequality (2.4) n times, we get

$$
\begin{aligned}
&q_1(gx_n,gx_{n+1})+q_1(gy_n,gy_{n+1})+q_1(gz_n,gz_{n+1})\\
&\quad\le k^n\big[q_1(gx_0,gx_1)+q_1(gy_0,gy_1)+q_1(gz_0,gz_1)\big].
\end{aligned}
\tag{2.5}
$$

Next, we shall prove that $\{gx_n\}$, $\{gy_n\}$, and $\{gz_n\}$ are Cauchy sequences in $g(X)$.

In fact, for each $n, m \in \mathbb{N}$, $m > n$, from (QPM4) and (2.5) we have

$$\begin{aligned} &q_1(gx_n, gx_m) + q_1(gy_n, gy_m) + q_1(gz_n, gz_m) \\ &\quad \le \sum_{i=n}^{m-1} \bigl[q_1(gx_i, gx_{i+1}) + q_1(gy_i, gy_{i+1}) + q_1(gz_i, gz_{i+1})\bigr] \\ &\quad \le \sum_{i=n}^{m-1} k^i \bigl[q_1(gx_0, gx_1) + q_1(gy_0, gy_1) + q_1(gz_0, gz_1)\bigr] \\ &\quad \le \frac{k^n}{1-k} \bigl[q_1(gx_0, gx_1) + q_1(gy_0, gy_1) + q_1(gz_0, gz_1)\bigr]. \end{aligned} \tag{2.6}$$

This implies that

$$\lim_{n,m\to\infty} \bigl[q_1(gx_n, gx_m) + q_1(gy_n, gy_m) + q_1(gz_n, gz_m)\bigr] = 0,$$

and so

$$\begin{aligned} &\lim_{n,m\to\infty} q_1(gx_n, gx_m) = 0, \qquad \lim_{n,m\to\infty} q_1(gy_n, gy_m) = 0 \quad \text{and} \\ &\lim_{n,m\to\infty} q_1(gz_n, gz_m) = 0. \end{aligned} \tag{2.7}$$

By similar arguments as above, we can show that

$$\begin{aligned} &\lim_{n,m\to\infty} q_1(gx_m, gx_n) = 0, \qquad \lim_{n,m\to\infty} q_1(gy_m, gy_n) = 0 \quad \text{and} \\ &\lim_{n,m\to\infty} q_1(gz_m, gz_n) = 0. \end{aligned} \tag{2.8}$$

Hence $\{gx_n\}$, $\{gy_n\}$, and $\{gz_n\}$ are Cauchy sequences in (gX, q_1). Since (gX, q_1) is complete, there exist $gx, gy, gz \in g(X)$ such that $\{gx_n\}$, $\{gy_n\}$, and $\{gz_n\}$ converge to gx, gy, and gz with respect to τ_{q_1}, that is,

$$\begin{aligned} q_1(gx, gx) &= \lim_{n\to\infty} q_1(gx, gx_n) = \lim_{n\to\infty} q_1(gx_n, gx) \\ &= \lim_{n,m\to\infty} q_1(gx_m, gx_n) = \lim_{n,m\to\infty} q_1(gx_n, gx_m), \end{aligned} \tag{2.9}$$

$$\begin{aligned} q_1(gy, gy) &= \lim_{n\to\infty} q_1(gy, gy_n) = \lim_{n\to\infty} q_1(gy_n, gy) \\ &= \lim_{n,m\to\infty} q_1(gy_m, gy_n) = \lim_{n,m\to\infty} q_1(gy_n, gy_m), \end{aligned} \tag{2.10}$$

and

$$\begin{aligned} q_1(gz, gz) &= \lim_{n\to\infty} q_1(gz, gz_n) = \lim_{n\to\infty} q_1(gz_n, gz) \\ &= \lim_{n,m\to\infty} q_1(gz_m, gz_n) = \lim_{n,m\to\infty} q_1(gz_n, gz_m). \end{aligned} \tag{2.11}$$

Combining (2.7)-(2.11), we have

$$\begin{aligned} q_1(gx,gx) &= \lim_{n\to\infty} q_1(gx,gx_n) = \lim_{n\to\infty} q_1(gx_n,gx) \\ &= \lim_{n,m\to\infty} q_1(gx_m,gx_n) = \lim_{n,m\to\infty} q_1(gx_n,gx_m) = 0, \end{aligned} \tag{2.12}$$

$$\begin{aligned} q_1(gy,gy) &= \lim_{n\to\infty} q_1(gy,gy_n) = \lim_{n\to\infty} q_1(gy_n,gy) \\ &= \lim_{n,m\to\infty} q_1(gy_m,gy_n) = \lim_{n,m\to\infty} q_1(gy_n,gy_m) = 0, \end{aligned} \tag{2.13}$$

and

$$\begin{aligned} q_1(gz,gz) &= \lim_{n\to\infty} q_1(gz,gz_n) = \lim_{n\to\infty} q_1(gz_n,gz) \\ &= \lim_{n,m\to\infty} q_1(gz_m,gz_n) = \lim_{n,m\to\infty} q_1(gz_n,gz_m) = 0. \end{aligned} \tag{2.14}$$

On the other hand, by (QMP4) we obtain

$$\begin{aligned} q_1\big(gx_{n+1},F(x,y,z)\big) &\le q_1(gx_{n+1},gx) + q_1\big(gx,F(x,y,z)\big) - q_1(gx,gx) \\ &\le q_1(gx_{n+1},gx) + q_1\big(gx,F(x,y,z)\big) \\ &\le q_1(gx_{n+1},gx) + q_1(gx,gx_{n+1}) + q_1\big(gx_{n+1},F(x,y,z)\big) - q_1(gx_{n+1},gx_{n+1}) \\ &\le q_1(gx_{n+1},gx) + q_1(gx,gx_{n+1}) + q_1\big(gx_{n+1},F(x,y,z)\big). \end{aligned}$$

Letting $n\to\infty$ in the above inequalities and using (2.12), we have

$$\lim_{n\to\infty} q_1\big(gx_{n+1},F(x,y,z)\big) \le q_1\big(gx,F(x,y,z)\big) \le \lim_{n\to\infty} q_1\big(gx_{n+1},F(x,y,z)\big).$$

That is,

$$\lim_{n\to\infty} q_1\big(gx_{n+1},F(x,y,z)\big) = q_1\big(gx,F(x,y,z)\big). \tag{2.15}$$

Similarly, we have

$$\lim_{n\to\infty} q_1\big(gy_{n+1},F(y,z,x)\big) = q_1\big(gy,F(y,z,x)\big) \tag{2.16}$$

and

$$\lim_{n\to\infty} q_1\big(gy_{n+1},F(z,x,y)\big) = q_1\big(gz,F(z,x,y)\big). \tag{2.17}$$

Now we prove that $F(x,y,z)=gx$, $F(y,z,x)=gy$, and $F(z,x,y)=gz$. In fact, it follows from (2.2) and (2.3) that

$$\begin{aligned} &q_1\big(gx_{n+1},F(x,y,z)\big) + q_1\big(gy_{n+1},F(y,z,x)\big) + q_1\big(gz_{n+1},F(z,x,y)\big) \\ &\quad = q_1\big(F(x_n,y_n,z_n),F(x,y,z)\big) + q_1\big(F(y_n,z_n,x_n),F(y,z,x)\big) + q_1\big(F(z_n,x_n,y_n),F(z,x,y)\big) \\ &\quad \le k_1\big[q_2(gx_n,gx) + q_2(gy_n,gy) + q_2(gz_n,gz)\big] \end{aligned}$$

$$\begin{aligned}
&+ k_2\bigl[q_2\bigl(gx_n, F(x_n, y_n, z_n)\bigr) + q_2\bigl(gy_n, F(y_n, z_n, x_n)\bigr) + q_2\bigl(gz_n, F(z_n, x_n, y_n)\bigr)\bigr] \\
&+ k_3\bigl[q_2\bigl(gx, F(x, y, z)\bigr) + q_2\bigl(gy, F(y, z, x)\bigr) + q_2\bigl(gz, F(z, x, y)\bigr)\bigr] \\
&+ k_4\bigl[q_2\bigl(gx_n, F(x, y, z)\bigr) + q_2\bigl(gy_n, F(y, z, x)\bigr) + q_2\bigl(gz_n, F(z, x, y)\bigr)\bigr] \\
&+ k_5\bigl[q_2\bigl(gx, F(x_n, y_n, z_n)\bigr) + q_2\bigl(gy, F(y_n, z_n, x_n)\bigr) + q_2\bigl(gz, F(z_n, x_n, y_n)\bigr)\bigr] \\
= {}& k_1\bigl[q_2(gx_n, gx) + q_2(gy_n, gy) + q_2(gz_n, gz)\bigr] \\
&+ k_2\bigl[q_2(gx_n, gx_{n+1}) + q_2(gy_n, gy_{n+1}) + q_2(gz_n, gz_{n+1})\bigr] \\
&+ k_3\bigl[q_2\bigl(gx, F(x, y, z)\bigr) + q_2\bigl(gy, F(y, z, x)\bigr) + q_2\bigl(gz, F(z, x, y)\bigr)\bigr] \\
&+ k_4\bigl[q_2\bigl(gx_n, F(x, y, z)\bigr) + q_2\bigl(gy_n, F(y, z, x)\bigr) + q_2\bigl(gz_n, F(z, x, y)\bigr)\bigr] \\
&+ k_5\bigl[q_2(gx, gx_{n+1}) + q_2(gy, gy_{n+1}) + q_2(gz, gz_{n+1})\bigr] \\
\leq {}& k_1\bigl[q_1(gx_n, gx) + q_1(gy_n, gy) + q_1(gz_n, gz)\bigr] \\
&+ k_2\bigl[q_1(gx_n, gx_{n+1}) + q_1(gy_n, gy_{n+1}) + q_1(gz_n, gz_{n+1})\bigr] \\
&+ k_3\bigl[q_1\bigl(gx, F(x, y, z)\bigr) + q_1\bigl(gy, F(y, z, x)\bigr) + q_1\bigl(gz, F(z, x, y)\bigr)\bigr] \\
&+ k_4\bigl[q_1\bigl(gx_n, F(x, y, z)\bigr) + q_1\bigl(gy_n, F(y, z, x)\bigr) + q_1\bigl(gz_n, F(z, x, y)\bigr)\bigr] \\
&+ k_5\bigl[q_1(gx, gx_{n+1}) + q_1(gy, gy_{n+1}) + q_1(gz, gz_{n+1})\bigr].
\end{aligned}$$

Letting $n \to \infty$ in the above inequality, using (2.12)-(2.17), we obtain

$$\begin{aligned}
&q_1\bigl(gx, F(x, y, z)\bigr) + q_1\bigl(gy, F(y, z, x)\bigr) + q_1\bigl(gz, F(z, x, y)\bigr) \\
&\quad \leq (k_3 + k_4)\bigl[q_1\bigl(gx, F(x, y, z)\bigr) + q_1\bigl(gy, F(y, z, x)\bigr) + q_1\bigl(gz, F(z, x, y)\bigr)\bigr].
\end{aligned} \tag{2.18}$$

By (2.1) we have $k_3 + k_4 < 1$. Hence, it follows from (2.18) that

$$q_1\bigl(gx, F(x, y, z)\bigr) + q_1\bigl(gy, F(y, z, x)\bigr) + q_1\bigl(gz, F(z, x, y)\bigr) = 0.$$

This implies that

$$q_1\bigl(gx, F(x, y, z)\bigr) = q_1\bigl(gy, F(y, z, x)\bigr) = q_1\bigl(gz, F(z, x, y)\bigr) = 0.$$

By Lemma 1.1, we get $F(x, y, z) = gx$, $F(y, z, x) = gy$, and $F(z, x, y) = gz$. Hence, (gx, gy, gz) is a tripled point of coincidence of mappings F and g.

Next, we will show that the tripled point of coincidence is unique. Suppose that $(x^*, y^*, z^*) \in X^3$ with $F(x^*, y^*, z^*) = gx^*$, $F(y^*, z^*, x^*) = gy^*$, and $F(z^*, x^*, y^*) = gz^*$. Using (2.2), (2.12), (2.13), (2.14), and (QPM_3), we obtain

$$\begin{aligned}
&q_1\bigl(gx, gx^*\bigr) + q_1\bigl(gy, gy^*\bigr) + q_1\bigl(gz, gz^*\bigr) \\
&\quad = q_1\bigl(F(x, y, z), F\bigl(x^*, y^*, z^*\bigr)\bigr) + q_1\bigl(F(y, z, x), F\bigl(y^*, z^*, x^*\bigr)\bigr) + q_1\bigl(F(z, x, y), F\bigl(z^*, x^*, y^*\bigr)\bigr) \\
&\quad \leq k_1\bigl[q_2\bigl(gx, gx^*\bigr) + q_2\bigl(gy, gy^*\bigr) + q_2\bigl(z, z^*\bigr)\bigr] \\
&\qquad + k_2\bigl[q_2\bigl(gx, F(x, y, z)\bigr) + q_2\bigl(gy, F(y, z, x)\bigr) + q_2\bigl(gz, F(z, x, y)\bigr)\bigr] \\
&\qquad + k_3\bigl[q_2\bigl(gx^*, F\bigl(x^*, y^*, z^*\bigr)\bigr) + q_2\bigl(gy^*, F\bigl(y^*, z^*, x^*\bigr)\bigr) + q_2\bigl(gz^*, F\bigl(z^*, x^*, y^*\bigr)\bigr)\bigr]
\end{aligned}$$

$$
\begin{aligned}
&\quad + k_4\bigl[q_2\bigl(gx, F\bigl(x^*, y^*, z^*\bigr)\bigr) + q_2\bigl(gy, F\bigl(y^*, z^*, x^*\bigr)\bigr) + q_2\bigl(gz^*, F\bigl(z^*, x^*, y^*\bigr)\bigr)\bigr] \\
&\quad + k_5\bigl[q_2\bigl(gx^*, F(x,y,z)\bigr) + q_2\bigl(gy^*, F(y,z,x)\bigr) + q_2\bigl(gz^*, F(z,x,y)\bigr)\bigr] \\
&= k_1\bigl[q_2\bigl(gx, gx^*\bigr) + q_2\bigl(gy, gy^*\bigr) + q_2\bigl(gz, gz^*\bigr)\bigr] \\
&\quad + k_2\bigl[q_2(gx, gx) + q_2(gy, gy) + q_2(gz, gz)\bigr] \\
&\quad + k_3\bigl[q_2\bigl(gx^*, gx^*\bigr) + q_2\bigl(gy^*, gy^*\bigr) + q_2\bigl(gz^*, gz^*\bigr)\bigr] \\
&\quad + k_4\bigl[q_2\bigl(gx, gx^*\bigr) + q_2\bigl(gy, gy^*\bigr) + q_2\bigl(gz, gz^*\bigr)\bigr] \\
&\quad + k_5\bigl[q_2\bigl(gx^*, gx\bigr) + q_2\bigl(gy^*, gy\bigr) + q_2\bigl(gz^*, gz\bigr)\bigr] \\
&\le (k_1 + k_4)\bigl[q_1\bigl(gx, gx^*\bigr) + q_1\bigl(gy, gy^*\bigr) + q_1\bigl(gz, gz^*\bigr)\bigr] \\
&\quad + k_2\bigl[q_1(gx, gx) + q_1(gy, gy) + q_1(gz, gz)\bigr] \\
&\quad + k_3\bigl[q_1\bigl(gx^*, gx^*\bigr) + q_1\bigl(gy^*, gy^*\bigr) + q_1\bigl(gz^*, gz^*\bigr)\bigr] \\
&\quad + k_5\bigl[q_1\bigl(gx^*, gx\bigr) + q_1\bigl(gy^*, gy\bigr) + q_1\bigl(gz^*, gz\bigr)\bigr] \\
&\le (k_1 + k_3 + k_4)\bigl[q_1\bigl(gx, gx^*\bigr) + q_1\bigl(gy, gy^*\bigr) + q_1\bigl(gz, gz^*\bigr)\bigr] \\
&\quad + k_5\bigl[q_1\bigl(gx^*, gx\bigr) + q_1\bigl(gy^*, gy\bigr) + q_1\bigl(gz^*, gz\bigr)\bigr].
\end{aligned}
$$

This implies that

$$
\begin{aligned}
&q_1\bigl(gx, gx^*\bigr) + q_1\bigl(gy, gy^*\bigr) + q_1\bigl(gz, gz^*\bigr) \\
&\quad \le \frac{k_5}{1 - k_1 - k_3 - k_4} \cdot \bigl[q_1\bigl(gx^*, gx\bigr) + q_1\bigl(gy^*, gy\bigr) + q_1\bigl(gz^*, gz\bigr)\bigr].
\end{aligned} \tag{2.19}
$$

Similarly, we have

$$
\begin{aligned}
&q_1\bigl(gx^*, gx\bigr) + q_1\bigl(gy^*, gy\bigr) + q_1\bigl(gz^*, gz\bigr) \\
&\quad \le \frac{k_5}{1 - k_1 - k_3 - k_4} \cdot \bigl[q_1\bigl(gx, gx^*\bigr) + q_1\bigl(gy, gy^*\bigr) + q_1\bigl(gz, gz^*\bigr)\bigr].
\end{aligned} \tag{2.20}
$$

Substituting (2.20) into (2.19), we obtain

$$
\begin{aligned}
&q_1\bigl(gx, gx^*\bigr) + q_1\bigl(gy, gy^*\bigr) + q_1\bigl(gz, gz^*\bigr) \\
&\quad \le \left(\frac{k_5}{1 - k_1 - k_3 - k_4}\right)^2 \cdot \bigl[q_1\bigl(gx, gx^*\bigr) + q_1\bigl(gy, gy^*\bigr) + q_1\bigl(gz, gz^*\bigr)\bigr].
\end{aligned} \tag{2.21}
$$

Since $\frac{k_5}{1-k_1-k_3-k_4} < 1$, from (2.21), we must have $q_1(gx, gx^*) = q_1(gy, gy^*) = q_1(gz, gz^*) = 0$. By Lemma 1.1, we get $gx = gx^*$, $gy = gy^*$, and $gz = gz^*$, which implies that the uniqueness of the tripled point of coincidence of F and g, that is, (gx, gy, gz).

Next, we will show that $gx = gy = gz$. In fact, from (2.2), (2.12)-(2.14) we have

$$
\begin{aligned}
&q_1(gx, gy) + q_1(gy, gz) + q_1(gz, gx) \\
&\quad = q_1\bigl(F(x,y,z), F(y,z,x)\bigr) + q_1\bigl(F(y,z,x), F(z,x,y)\bigr) + q_1\bigl(F(z,x,y), F(x,y,z)\bigr) \\
&\quad \le k_1\bigl[q_2(gx, gy) + q_2(gy, gz) + q_2(gz, gx)\bigr]
\end{aligned}
$$

$$
\begin{aligned}
&+ k_2\big[q_2\big(gx, F(x,y,z)\big) + q_2\big(gy, F(y,z,x)\big) + q_2\big(gz, F(z,x,y)\big)\big] \\
&+ k_3\big[q_2\big(gy, F(y,z,x)\big) + q_2\big(gz, F(z,x,y)\big) + q_2\big(gx, F(x,y,z)\big)\big] \\
&+ k_4\big[q_2\big(gx, F(y,z,x)\big) + q_2\big(gy, F(z,x,y)\big) + q_2\big(gz, F(x,y,z)\big)\big] \\
&+ k_5\big[q_2\big(gy, F(x,y,z)\big) + q_2\big(gz, F(y,z,x)\big) + q_2\big(gx, F(z,x,y)\big)\big] \\
= {}& k_1\big[q_2(gx,gy) + q_2(gy,gz) + q_2(gz,gx)\big] \\
&+ k_2\big[q_2(gx,gx) + q_2(gy,gy) + q_2(gz,gz)\big] \\
&+ k_3\big[q_2(gy,gy) + q_2(gz,gz) + q_2(gx,gx)\big] \\
&+ k_4\big[q_2(gx,gy) + q_2(gy,gz) + q_2(gz,gx)\big] \\
&+ k_5\big[q_2(gy,gx) + q_2(gz,gy) + q_2(gx,gz)\big] \\
\le {}& k_1\big[q_1(gx,gy) + q_1(gy,gz) + q_1(gz,gx)\big] \\
&+ k_2\big[q_1(gx,gx) + q_1(gy,gy) + q_1(gz,gz)\big] \\
&+ k_3\big[q_1(gy,gy) + q_1(gz,gz) + q_1(gx,gx)\big] \\
&+ k_4\big[q_1(gx,gy) + q_1(gy,gz) + q_1(gz,gx)\big] \\
&+ k_5\big[q_1(gy,gx) + q_1(gz,gy) + q_1(gx,gz)\big] \\
= {}& (k_1 + k_4)\big[q_1(gx,gy) + q_1(gy,gz) + q_1(gz,gx)\big] \\
&+ k_5\big[q_1(gy,gx) + q_1(gz,gy) + q_1(gx,gz)\big].
\end{aligned}
$$

This implies that

$$
\begin{aligned}
&q_1(gx,gy) + q_1(gy,gz) + q_1(gz,gx) \\
&\quad \le \frac{k_5}{1-k_1-k_4} \cdot \big[q_1(gy,gx) + q_1(gz,gy) + q_1(gx,gz)\big].
\end{aligned} \tag{2.22}
$$

By similar arguments as above, we can show that

$$
\begin{aligned}
&q_1(gy,gx) + q_1(gz,gy) + q_1(gx,gz) \\
&\quad \le \frac{k_5}{1-k_1-k_4} \cdot \big[q_1(gx,gy) + q_1(gy,gz) + q_1(gz,gx)\big].
\end{aligned} \tag{2.23}
$$

Substituting (2.23) into (2.22), we have

$$
\begin{aligned}
&q_1(gx,gy) + q_1(gy,gz) + q_1(gz,gx) \\
&\quad \le \left(\frac{k_5}{1-k_1-k_4}\right)^2 \cdot \big[q_1(gx,gy) + q_1(gy,gz) + q_1(gz,gx)\big].
\end{aligned} \tag{2.24}
$$

Since $\frac{k_5}{1-k_1-k_4} < 1$, from (2.24), we must have $q_1(gx,gy) = q_1(gy,gz) = q_1(gz,gx) = 0$. By Lemma 1.1, we get $gx = gy = gz$.

Finally, assume that F and g are w-compatible. Let $u = gx$, then we have $u = gx = F(x,y,z) = gy = F(y,z,x) = gz = F(z,x,y)$, and so that

$$
gu = ggx = g\big(F(x,y,z)\big) = F(gx,gy,gz) = F(u,u,u). \tag{2.25}
$$

Consequently, (u,u,u) is a tripled coincidence point of F and g, and so (gu,gu,gu) is a tripled point of coincidence of F and g, and by its uniqueness, we get $gu = gx$. Thus, we obtain $F(u,u,u) = gu = u$. Therefore, (u,u,u) is the unique common tripled fixed point of F and g. This completes the proof of Theorem 2.1. □

Remark 2.1 Theorem 2.1 improves and extends Theorem 2.1 of Shatanawi and Pitea [42] in the following aspects:

(1) The single quasi-partial metric extends to two quasi-partial metrics.
(2) The coupled fixed point extends to a tripled fixed point.
(3) The contractive condition defined by (1.1) is replaced by the new contractive condition defined by (2.2).

In Theorem 2.1, if we take $q_1(x,y) = q_2(x,y)$ for all $x,y \in X$, then we get the following.

Corollary 2.1 *Let (X,q) be a quasi-partial metric space, $F : X^3 \to X$ and $g : X \to X$ be two mappings. Suppose that there exist k_1, k_2, k_3, k_4, and k_5 in $[0,1)$ with $k_1 + k_2 + k_3 + 2k_4 + k_5 < 1$ such that the condition*

$$\begin{aligned} &q\bigl(F(x,y,z),F(u,v,w)\bigr) + q\bigl(F(y,z,x),F(v,w,u)\bigr) + q\bigl(F(z,x,y),F(w,u,v)\bigr) \\ &\quad \le k_1\bigl[q(gx,gu) + q(gy,gv) + q(z,w)\bigr] \\ &\qquad + k_2\bigl[q\bigl(gx,F(x,y,z)\bigr) + q\bigl(gy,F(y,z,x)\bigr) + q\bigl(gz,F(z,x,y)\bigr)\bigr] \\ &\qquad + k_3\bigl[q\bigl(gu,F(u,v,w)\bigr) + q\bigl(gv,F(v,w,u)\bigr) + q\bigl(gw,F(w,u,v)\bigr)\bigr] \\ &\qquad + k_4\bigl[q\bigl(gx,F(u,v,w)\bigr) + q\bigl(gy,F(v,w,u)\bigr) + q\bigl(gz,F(w,u,v)\bigr)\bigr] \\ &\qquad + k_5\bigl[q\bigl(gu,F(x,y,z)\bigr) + q\bigl(gv,F(y,z,x)\bigr) + q\bigl(gw,F(z,x,y)\bigr)\bigr] \end{aligned} \tag{2.26}$$

holds for all $x,y,z,u,v,w \in X$. Also, suppose we have the following hypotheses:

(i) *$F(X^3) \subset g(X)$.*
(ii) *$g(X)$ is a complete subspace of X.*

Then the mappings F and g have a tripled coincidence point (x,y,z) satisfying

$$gx = F(x,y,z) = gy = F(y,z,x) = F(z,x,y) = gz.$$

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u,u,u).

Remark 2.2 Corollary 2.1 improves and extends Corollary 2.2 of Aydi and Abbas [41] to quasi-partial metric spaces.

Corollary 2.2 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \le q_1(x,y)$, for all $x,y \in X$, and $F : X^3 \to X$, $g : X \to X$ be two mappings. Suppose that there exist $a_i \in [0,1)$ $(i = 1,2,3,\ldots,15)$ with*

$$\left(\sum_{i=1}^{9} a_i\right) + 2\left(\sum_{i=10}^{12} a_i\right) + \left(\sum_{i=13}^{15} a_i\right) < 1 \tag{2.27}$$

such that the condition

$$\begin{aligned} &q_1\big(F(x,y,z),F(u,v,w)\big)\\ &\quad\le a_1q_2(gx,gu)+a_2q_2(gy,gv)+a_3q_2(gz,gw)\\ &\qquad+a_4q_2\big(gx,F(x,y,z)\big)+a_5q_2\big(gy,F(y,z,x)\big)+a_6q_2\big(gz,F(z,x,y)\big)\\ &\qquad+a_7q_2\big(gu,F(u,v,w)\big)+a_8q_2\big(gv,F(v,w,u)\big)+a_9q_2\big(gw,F(w,u,v)\big)\\ &\qquad+a_{10}q_2\big(gx,F(u,v,w)\big)+a_{11}q_2\big(gy,F(v,w,u)\big)+a_{12}q_2\big(gz,F(w,u,v)\big)\\ &\qquad+a_{13}q_2\big(gu,F(x,y,z)\big)+a_{14}q_2\big(gv,F(y,z,x)\big)+a_{15}q_2\big(gw,F(z,x,y)\big) \end{aligned}\tag{2.28}$$

holds for all $x,y,z,u,v,w\in X$. Also, suppose we have the following hypotheses:

(i) $F(X^3)\subset g(X)$.

(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.*

Then the mappings F and g have a tripled coincidence point (x,y,z) satisfying

$$gx=F(x,y,z)=gy=F(y,z,x)=gz=F(z,x,y).$$

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u,u,u).

Proof Given $x,y,z,u,v,w\in X$. It follows from (2.29) that

$$\begin{aligned} &q_1\big(F(x,y,z),F(u,v,w)\big)\\ &\quad\le a_1q_2(gx,gu)+a_2q_2(gy,gv)+a_3q_2(gz,gw)\\ &\qquad+a_4q_2\big(gx,F(x,y,z)\big)+a_5q_2\big(gy,F(y,z,x)\big)+a_6q_2\big(gz,F(z,x,y)\big)\\ &\qquad+a_7q_2\big(gu,F(u,v,w)\big)+a_8q_2\big(gv,F(v,w,u)\big)+a_9q_2\big(gw,F(w,u,v)\big)\\ &\qquad+a_{10}q_2\big(gx,F(u,v,w)\big)+a_{11}q_2\big(gy,F(v,w,u)\big)+a_{12}q_2\big(gz,F(w,u,v)\big)\\ &\qquad+a_{13}q_2\big(gu,F(x,y,z)\big)+a_{14}q_2\big(gv,F(y,z,x)\big)+a_{15}q_2\big(gw,F(z,x,y)\big), \end{aligned}\tag{2.29}$$

$$\begin{aligned} &q_1\big(F(y,z,x),F(v,w,u)\big)\\ &\quad\le a_1q_2(gy,gv)+a_2q_2(gz,gw)+a_3q_2(gx,gu)\\ &\qquad+a_4q_2\big(gy,F(y,z,x)\big)+a_5q_2\big(gz,F(z,x,y)\big)+a_6q_2\big(gx,F(x,y,z)\big)\\ &\qquad+a_7q_2\big(gv,F(v,w,u)\big)+a_8q_2\big(gw,F(w,u,v)\big)+a_9q_2\big(gu,F(u,v,w)\big)\\ &\qquad+a_{10}q_2\big(gy,F(v,w,u)\big)+a_{11}q_2\big(gz,F(w,u,v)\big)+a_{12}q_2\big(gx,F(u,v,w)\big)\\ &\qquad+a_{13}q_2\big(gv,F(y,z,x)\big)+a_{14}q_2\big(gw,F(z,x,y)\big)+a_{15}q_2\big(gu,F(x,y,z)\big), \end{aligned}\tag{2.30}$$

and

$$\begin{aligned} &q_1\big(F(z,x,y),F(w,u,v)\big)\\ &\quad\le a_1q_2(gz,gw)+a_2q_2(gx,gu)+a_3q_2(gy,gv) \end{aligned}$$

$$\begin{aligned}&+ a_4 q_2\big(gz, F(z,x,y)\big) + a_5 q_2\big(gx, F(x,y,z)\big) + a_6 q_2\big(gy, F(y,z,x)\big)\\&+ a_7 q_2\big(gw, F(w,u,v)\big) + a_8 q_2\big(gu, F(u,v,w)\big) + a_9 q_2\big(gv, F(v,w,u)\big)\\&+ a_{10} q_2\big(gz, F(w,u,v)\big) + a_{11} q_2\big(gx, F(u,v,w)\big) + a_{12} q_2\big(gy, F(v,w,u)\big)\\&+ a_{13} q_2\big(gw, F(z,x,y)\big) + a_{14} q_2\big(gu, F(x,y,z)\big) + a_{15} q_2\big(gv, F(y,z,x)\big).\end{aligned} \tag{2.31}$$

Adding inequality (2.29) and (2.30) to inequality (2.31), we get

$$\begin{aligned}&q_1\big(q_1\big(F(x,y,z), F(u,v,w)\big) + F(y,z,x), F(v,w,u)\big) + q_1\big(F(z,x,y), F(w,u,v)\big)\\&\quad\le (a_1+a_2+a_3)\big[q_2(gx,gu) + q_2(gy,gv) + q_2(gz,gw)\big]\\&\qquad+ (a_4+a_5+a_6)\big[q_2\big(gx,F(x,y,z)\big) + q_2\big(gy,F(y,z,x)\big) + q_2\big(gz,F(z,x,y)\big)\big]\\&\qquad+ (a_7+a_8+a_9)\big[q_2\big(gu,F(u,v,w)\big) + q_2\big(gv,F(v,w,u)\big) + q_2\big(gw,F(w,u,v)\big)\big]\\&\qquad+ (a_{10}+a_{11}+a_{12})\big[q_2\big(gx,F(u,v,w)\big) + q_2\big(gy,F(v,w,u)\big) + q_2\big(gz,F(w,u,v)\big)\big]\\&\qquad+ (a_{13}+a_{14}+a_{15})\big[q_2\big(gu,F(x,y,z)\big) + q_2\big(gv,F(y,z,x)\big) + q_2\big(gw,F(z,x,y)\big)\big].\end{aligned} \tag{2.32}$$

Therefore, the result follows from Theorem 2.1. □

Remark 2.3 If we take $q_1(x,y) = q_2(x,y) = p(x,y)$ for all $x, y \in X$, where p is a partial metric on X. Then Corollary 2.2 is reduced to Theorems 2.1 and 2.4 of Aydi and Abbas [41]. Corollary 2.2 also improves and extends Corollary 2.1 of Shatanawi and Pitea [35].

Corollary 2.3 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \le q_1(x,y)$, for all $x, y \in X$, and $F : X^3 \to X$, $g : X \to X$ be two mappings. Suppose that there exists $k \in [0,1)$ such that the condition*

$$\begin{aligned}&q_1\big(F(x,y,z), F(u,v,w)\big) + q_1\big(F(y,z,x), F(v,w,u)\big) + q_1\big(F(z,x,y), F(w,u,v)\big)\\&\quad\le k\big[q_2(gx,gu) + q_2(gy,gv) + q_2(gz,gw)\big]\end{aligned} \tag{2.33}$$

holds for all $x, y, z, u, v, w \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^3) \subset g(X)$.
(ii) *$g(X)$ is a complete subspace of X with respect to the quasi-partial metric q_1.*

Then the mappings F and g have a tripled coincidence point (x, y, z) satisfying

$$gx = F(x,y,z) = gy = F(y,z,x) = gz = F(z,x,y).$$

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u, u, u).

Corollary 2.4 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \le q_1(x,y)$, for all $x, y \in X$, and $F : X^3 \to X$, $g : X \to X$ be two mappings. Suppose that there exists $k \in [0,1)$ such that the condition*

$$\begin{aligned}&q_1\big(F(x,y,z), F(u,v,w)\big) + q_1\big(F(y,z,x), F(v,w,u)\big) + q_1\big(F(z,x,y), F(w,u,v)\big)\\&\quad\le k\big[q_2\big(gx, F(x,y,z)\big) + q_2\big(gy, F(y,z,x)\big) + q_2\big(F\big(gz, F(z,x,y)\big)\big)\big]\end{aligned} \tag{2.34}$$

holds for all $x, y, z, u, v, w \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^3) \subset g(X)$.

(ii) $g(X)$ *is a complete subspace of X with respect to the quasi-partial metric q_1*.

Then the mappings F and g have a tripled coincidence point (x, y, z) satisfying

$$gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y).$$

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u, u, u).

Corollary 2.5 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x, y) \leq q_1(x, y)$, for all $x, y \in X$, and $F : X^3 \to X$, $g : X \to X$ be two mappings. Suppose that there exists $k \in [0, 1)$ such that the condition*

$$\begin{aligned} &q_1\bigl(F(x, y, z), F(u, v.w)\bigr) + q_1\bigl(F(y, z, x), F(v, w, u)\bigr) + q_1\bigl(F(z, x, y), F(w, u, v)\bigr) \\ &\quad \leq k\bigl[q_2\bigl(gu, F(u, v, w)\bigr) + q_2\bigl(gv, F(v, w, u)\bigr) + q_2\bigl(gw, F(w, u, v)\bigr)\bigr] \end{aligned} \tag{2.35}$$

holds for all $x, y, z, u, v, w \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^3) \subset g(X)$.

(ii) $g(X)$ *is a complete subspace of X with respect to the quasi-partial metric q_1*.

Then the mappings F and g have a tripled coincidence point (x, y, z) satisfying

$$gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y).$$

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u, u, u).

Remark 2.4 Corollaries 2.3-2.5 improve and extend Corollaries 2.2-2.4 of Shatanawi and Pitea [42] in the following aspects:

(1) The single quasi-partial metric extends to two quasi-partial metrics.

(2) The coupled fixed point extends to a tripled fixed point.

Corollary 2.6 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x, y) \leq q_1(x, y)$, for all $x, y \in X$, and $F : X^3 \to X$, $g : X \to X$ be two mappings. Suppose that there exists $k \in [0, \frac{1}{2})$ such that the condition*

$$\begin{aligned} &q_1\bigl(F(x, y, z), F(u, v, w)\bigr) + q\bigl(F(y, z, x), F(v, w, u)\bigr) + q_1\bigl(F(z, x, y), F(w, u, v)\bigr) \\ &\quad \leq k\bigl[q_2\bigl(gx, F(u, v, w)\bigr) + q_2\bigl(gy, F(v, w, u)\bigr) + q_2\bigl(gz, F(w, u, v)\bigr)\bigr] \end{aligned} \tag{2.36}$$

holds for all $x, y, z, u, v, w \in X$. Also, suppose we have the following hypotheses:

(i) $F(X^3) \subset g(X)$.
(ii) $g(X)$ *is a complete subspace of* X *with respect to the quasi-partial metric* q_1.

Then the mappings F and g have a tripled coincidence point (x, y, z) satisfying

$$gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y).$$

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u, u, u).

Corollary 2.7 *Let* q_1 *and* q_2 *be two quasi-metrics on* X *such that* $q_2(x, y) \leq q_1(x, y)$, *for all* $x, y \in X$, *and* $F : X^3 \to X$, $g : X \to X$ *be two mappings. Suppose that there exists* $k \in [0, 1)$ *such that the condition*

$$\begin{aligned} &q_1\big(F(x, y, z), F(u, v, w)\big) + q\big(F(y, z, x), F(v, w, u)\big) + q_1\big(F(z, x, y), F(w, u, v)\big) \\ &\quad \leq k\big[q_2\big(gu, F(x, y, z)\big) + q_2\big(gv, F(y, z, x)\big) + q_2\big(gw, F(z, x, y)\big)\big] \end{aligned} \tag{2.37}$$

holds for all $x, y, z, u, v, w \in X$. *Also, suppose we have the following hypotheses*:
(i) $F(X^3) \subset g(X)$.
(ii) $g(X)$ *is a complete subspace of* X *with respect to the quasi-partial metric* q_1.

Then the mappings F and g have a tripled coincidence point (x, y, z) satisfying

$$gx = F(x, y, z) = gy = F(y, z, x) = gz = F(z, x, y).$$

Moreover, if F and g are w-compatible, then F and g have a unique common tripled fixed point of the form (u, u, u).

Let $g = I_X$ (the identity mapping) in Theorem 2.1 and Corollaries 2.1-2.7. Then we have the following results.

Corollary 2.8 *Let* q_1 *and* q_2 *be two quasi-metrics on* X *such that* $q_2(x, y) \leq q_1(x, y)$, *for all* $x, y \in X$, *and* $F : X^3 \to X$ *be a mapping. Suppose that there exist* k_1, k_2, k_3, k_4, *and* k_5 *in* $[0, 1)$ *with* $k_1 + k_2 + k_3 + 2k_4 + k_5 < 1$ *such that the condition*

$$\begin{aligned} &q_1\big(F(x, y, z), F(u, v, w)\big) + q_1\big(F(y, z, x), F(v, w, u)\big) + q_1\big(F(z, x, y), F(w, u, v)\big) \\ &\quad \leq k_1\big[q_2(x, u) + q_2(y, v) + q_2(z, w)\big] \\ &\qquad + k_2\big[q_2\big(x, F(x, y, z)\big) + q_2\big(y, F(y, z, x)\big) + q_2\big(z, F(z, x, y)\big)\big] \\ &\qquad + k_3\big[q_2\big(u, F(u, v, w)\big) + q_2\big(v, F(v, w, u)\big) + q_2\big(w, F(w, u, v)\big)\big] \\ &\qquad + k_4\big[q_2\big(x, F(u, v, w)\big) + q_2\big(y, F(v, w, u)\big) + q_1\big(z, F(w, u, v)\big)\big] \\ &\qquad + k_5\big[q_2\big(u, F(x, y, z)\big) + q_2\big(v, F(y, z, x)\big) + q_2\big(w, F(z, x, y)\big)\big] \end{aligned} \tag{2.38}$$

holds for all $x, y, z, u, v, w \in X$. *If* (X, q_1) *is a complete quasi-partial metric space, then the mapping* F *has a unique tripled fixed point of the form* (u, u, u).

Corollary 2.9 *Let (X,q) be a complete quasi-partial metric space, $F: X^3 \to X$ be a mapping. Suppose that there exist k_1, k_2, k_3, k_4, and k_5 in $[0,1)$ with $k_1+k_2+k_3+2k_4+k_5<1$ such that the condition*

$$\begin{aligned} &q\bigl(F(x,y,z),F(u,v,w)\bigr)+q\bigl(F(y,z,x),F(v,w,u)\bigr)+q\bigl(F(z,x,y),F(w,u,v)\bigr)\\ &\quad\le k_1\bigl[q(x,u)+q(y,v)+q(z,w)\bigr]\\ &\qquad+k_2\bigl[q\bigl(x,F(x,y,z)\bigr)+q\bigl(y,F(y,z,x)\bigr)+q\bigl(z,F(z,x,y)\bigr)\bigr]\\ &\qquad+k_3\bigl[q\bigl(u,F(u,v,w)\bigr)+q\bigl(v,F(v,w,u)\bigr)+q\bigl(w,F(w,u,v)\bigr)\bigr]\\ &\qquad+k_4\bigl[q\bigl(x,F(u,v,w)\bigr)+q\bigl(y,F(v,w,u)\bigr)+q\bigl(z,F(w,u,v)\bigr)\bigr]\\ &\qquad+k_5\bigl[q\bigl(u,F(x,y,z)\bigr)+q\bigl(v,F(y,z,x)\bigr)+q\bigl(w,F(z,x,y)\bigr)\bigr] \end{aligned} \tag{2.39}$$

holds for all $x,y,z,u,v,w\in X$. Then F has a unique tripled fixed point of the form (u,u,u).

Remark 2.5 Corollary 2.9 improves and extends Corollary 2.5 of Shatanawi and Pitea [42], the contractive condition is replaced by the new contractive condition defined by (2.39).

Corollary 2.10 *Let (X,p) be a complete partial metric space, $F: X^3 \to X$ be a mapping. Suppose that there exist k_1, k_2, k_3, k_4, and k_5 in $[0,1)$ with $k_1+k_2+k_3+2k_4+k_5<1$ such that the condition*

$$\begin{aligned} &p\bigl(F(x,y,z),F(u,v,w)\bigr)\\ &\quad\le k_1\bigl[p(x,u)+p(y,v)+p(z,w)\bigr]\\ &\qquad+k_2\bigl[p\bigl(x,F(x,y,z)\bigr)+p\bigl(y,F(y,z,x)\bigr)+p\bigl(z,F(z,x,y)\bigr)\bigr]\\ &\qquad+k_3\bigl[p\bigl(u,F(u,v,w)\bigr)+p\bigl(v,F(v,w,u)\bigr)+p\bigl(w,F(w,u,v)\bigr)\bigr]\\ &\qquad+k_4\bigl[p\bigl(x,F(u,v,w)\bigr)+p\bigl(y,F(v,w,u)\bigr)+p\bigl(z,F(w,u,v)\bigr)\bigr]\\ &\qquad+k_5\bigl[p\bigl(u,F(x,y,z)\bigr)+p\bigl(v,F(y,z,x)\bigr)+p\bigl(w,F(z,x,y)\bigr)\bigr] \end{aligned} \tag{2.40}$$

holds for all $x,y,z,u,v,w\in X$. Then the mapping F has a unique tripled fixed point of the form (u,u,u).

Corollary 2.11 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y)\le q_1(x,y)$, for all $x,y\in X$, and $F: X^3\to X$ be a mapping. Suppose that there exist $a_i\in[0,1)$ $(i=1,2,3,\dots,15)$ with*

$$\left(\sum_{i=1}^{9}a_i\right)+2\left(\sum_{i=10}^{12}a_i\right)+\left(\sum_{i=13}^{15}a_i\right)<1 \tag{2.41}$$

such that the condition

$$\begin{aligned} &q_1\bigl(F(x,y,z),F(u,v,w)\bigr)\\ &\quad\le a_1q_2(x,u)+a_2q_2(y,v)+a_3q_2(z,w) \end{aligned}$$

$$
\begin{aligned}
&+ a_4 q_2\bigl(x, F(x,y,z)\bigr) + a_5 q_2\bigl(y, F(y,z,x)\bigr) + a_6 q_2\bigl(z, F(z,x,y)\bigr) \\
&+ a_7 q_2\bigl(u, F(u,v,w)\bigr) + a_8 q_2\bigl(v, F(v,w,u)\bigr) + a_9 q_2\bigl(w, F(w,u,v)\bigr) \\
&+ a_{10} q_2\bigl(x, F(u,v,w)\bigr) + a_{11} q_2\bigl(y, F(v,w,u)\bigr) + a_{12} q_2\bigl(z, F(w,u,v)\bigr) \\
&+ a_{13} q_2\bigl(u, F(x,y,z)\bigr) + a_{14} q_2\bigl(v, F(y,z,x)\bigr) + a_{15} q_2\bigl(w, F(z,x,y)\bigr)
\end{aligned}
\tag{2.42}
$$

holds for all $x,y,z,u,v,w \in X$. If (X,q_1) is a complete quasi-partial metric space. Then the mapping F has a unique coupled fixed point of the form (u,u,u).

Remark 2.6 Corollary 2.11 improves and extends Corollary 2.6 of Shatanawi and Pitea [42] in the following aspects:

(1) The single quasi-partial metric extends to two quasi-partial metrics.
(2) The coupled fixed point extends to a tripled fixed point.
(3) The contractive condition is replaced by the new contractive condition defined by (2.42).

Corollary 2.12 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \le q_1(x,y)$, for all $x,y \in X$, and $F : X^3 \to X$ be a mapping. Suppose that there exists $k \in [0,1)$ such that the condition*

$$
\begin{aligned}
&q_1\bigl(F(x,y,z), F(u,v,w)\bigr) + q_1\bigl(F(y,z,x), F(v,w,u)\bigr) + q_1\bigl(F(z,x,y), F(w,u,v)\bigr) \\
&\quad \le k\bigl[q_2(x,u) + q_2(y,v) + q_2(z,w)\bigr]
\end{aligned}
\tag{2.43}
$$

holds for all $x,y,z,u,v,w \in X$. If (X,q_1) is a complete quasi-partial metric space. Then the mapping F has a unique tripled fixed point of the form (u,u,u).

Corollary 2.13 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \le q_1(x,y)$, for all $x,y \in X$, and $F : X^3 \to X$ be a mapping. Suppose that there exists $k \in [0,1)$ such that the condition*

$$
\begin{aligned}
&q_1\bigl(F(x,y,z), F(u,v,w)\bigr) + q_1\bigl(F(y,z,x), F(v,w,u)\bigr) + q_1\bigl(F(z,x,y), F(w,u,v)\bigr) \\
&\quad \le k\bigl[q_2\bigl(x, F(x,y,z)\bigr) + q_2\bigl(y, F(y,z,x)\bigr) + q_2\bigl(z, F(z,x,y)\bigr)\bigr]
\end{aligned}
\tag{2.44}
$$

holds for all $x,y,z,u,v,w \in X$. If (X,q_1) is a complete quasi-partial metric space. Then the mapping F has a unique tripled fixed point of the form (u,u,u).

Corollary 2.14 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \le q_1(x,y)$, for all $x,y \in X$, and $F : X^3 \to X$ be a mapping. Suppose that there exists $k \in [0,1)$ such that the condition*

$$
\begin{aligned}
&q_1\bigl(F(x,y,z), F(u,v,w)\bigr) + q\bigl(F(y,z,x), F(v,w,u)\bigr) + q_1\bigl(F(z,x,y), F(w,u,v)\bigr) \\
&\quad \le k\bigl[q_2\bigl(u, F(u,v,w)\bigr) + q_2\bigl(v, F(v,w,u)\bigr) + q_2\bigl(w, F(w,u,v)\bigr)\bigr]
\end{aligned}
\tag{2.45}
$$

holds for all $x,y,z,u,v,w \in X$. If (X,q_1) is a complete quasi-partial metric space. Then the mapping F has a unique tripled fixed point of the form (u,u,u).

Remark 2.7 Corollaries 2.12-2.14 improve and extend Corollaries 2.7-2.9 of Shatanawi and Pitea [42] in the following aspects:

(1) The single quasi-partial metric extends to two quasi-partial metrics.
(2) The coupled fixed point extends to a tripled fixed point.

Corollary 2.15 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \leq q_1(x,y)$, for all $x, y \in X$, and $F : X^3 \to X$ be a mapping. Suppose that there exists $k \in [0, \frac{1}{2})$ such that the condition*

$$\begin{aligned} &q_1\bigl(F(x,y,z),F(u,v,w)\bigr) + q\bigl(F(y,z,x),F(v,w,u)\bigr) + q_1\bigl(F(z,x,y),F(w,u,v)\bigr) \\ &\quad \leq k\bigl[q_2\bigl(x,F(u,v,w)\bigr) + q_2\bigl(y,F(v,w,u)\bigr) + q_2\bigl(z,F(w,u,v)\bigr)\bigr] \end{aligned}$$

holds for all $x, y, z, u, v, w \in X$. If (X, q_1) is a complete quasi-partial metric space. Then the mapping F has a unique tripled fixed point of the form (u, u, u).

Corollary 2.16 *Let q_1 and q_2 be two quasi-metrics on X such that $q_2(x,y) \leq q_1(x,y)$, for all $x, y \in X$, and $F : X^3 \to X$ be a mapping. Suppose that there exists $k \in [0, 1)$ such that the condition*

$$\begin{aligned} &q_1\bigl(F(x,y,z),F(u,v,w)\bigr) + q_1\bigl(F(y,z,x),F(v,w,u)\bigr) + q_1\bigl(F(z,x,y),F(w,u,v)\bigr) \\ &\quad \leq k\bigl[q_2\bigl(u,F(x,y,z)\bigr) + q_2\bigl(v,F(y,z,x)\bigr) + q_2\bigl(w,F(z,x,y)\bigr)\bigr] \end{aligned}$$

holds for all $x, y, z, u, v, w \in X$. If (X, q_1) is a complete quasi-partial metric space. Then the mapping F has a unique tripled fixed point of the form (u, u, u).

Now, we introduce an example to support our results.

Example 2.1 Let $X = [0, 1]$, and two quasi-partial metrics q_1, q_2 on X be given as

$$q_1(x,y) = |x-y| + x \quad \text{and} \quad q_2(x,y) = \frac{1}{2}\bigl[|x-y| + x\bigr]$$

for all $x, y \in X$. Also, define $F : X^3 \to X$ and $g : X \to X$ as

$$F(x,y,z) = \frac{x+y+z}{27} \quad \text{and} \quad gx = \frac{x}{3}$$

for all $x, y, z \in X$. Then

(1) (X, q_1) is a complete quasi-partial metric space.
(2) $F(X^3) \subset X$.
(3) F and g are w-compatible.
(4) For any $x, y, z, u, v, w \in X$, we have

$$\begin{aligned} &q_1\bigl(F(x,y,z),F(u,v,w)\bigr) + q_1\bigl(F(y,z,x) + F(v,w,u)\bigr) + q_1\bigl(F(z,x,y),F(w,u,v)\bigr) \\ &\quad \leq \frac{1}{3}\bigl(q_2(gx,gu) + q_2(gy,gv) + q_2(gz,gw)\bigr). \end{aligned}$$

Proof The proofs of (1), (2), and (3) are clear. Next we show that (4). In fact, for $x, y, z, u, v, w \in X$, we have

$$\begin{aligned}
& q_1\bigl(F(x,y,z), F(u,v,w)\bigr) + q_1\bigl(F(y,z,x) + F(v,w,u)\bigr) + q_1\bigl(F(z,x,y), F(w,u,v)\bigr) \\
&\quad = q_1\left(\frac{x+y+z}{27}, \frac{u+v+w}{27}\right) + q_1\left(\frac{y+z+x}{27}, \frac{v+w+u}{27}\right) + q_1\left(\frac{z+x+y}{27}, \frac{w+u+v}{27}\right) \\
&\quad = \frac{1}{9}\bigl(\bigl|x+y+z-(u+v+w)\bigr| + (x+y+z)\bigr) \\
&\quad \le \frac{1}{9}\bigl(|x-u| + |y-v| + |z-w| + x+y+z\bigr) \\
&\quad = \frac{1}{3}\left(\left|\frac{1}{3}x - \frac{1}{3}u\right| + \frac{1}{3}x + \left|\frac{1}{3}y - \frac{1}{3}v\right| + \frac{1}{3}y + \left|\frac{1}{3}z - \frac{1}{3}w\right| + \frac{1}{3}z\right) \\
&\quad = \frac{1}{3}\bigl(q_2(gx,gu) + q_2(gy,gv) + q_2(gz,gw)\bigr).
\end{aligned}$$

Thus, F and g satisfy all the hypotheses of Corollary 2.3. So, F and g have a unique common coupled fixed point. Here $(0,0,0)$ is the unique common tripled fixed point of F and g. □

Competing interests
The author declares that they have no competing interests.

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