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A note on the equivalence of some metric and H-cone metric fixed point theorems for multivalued contractions

Huaping Huang[1], Stojan Radenović[2*] and Dragan Đorić[3]

*Correspondence: fixedpoint50@gmail.com
[2]Faculty of Mathematics and Information Technology, Dong Thap University, Dong Thap, Vietnam
Full list of author information is available at the end of the article

Abstract
In this paper, by using Minkowski functional introduced by Kadelburg *et al.* (Appl. Math. Lett. 24:370-374, 2011) or nonlinear scalarization function introduced by Du (Nonlinear Anal. 72:2259-2261, 2010), we prove some equivalences between vectorial versions of fixed point theorems for *H*-cone metrics in the sense of Arshad and Ahmad and scalar versions of fixed point theorems for (general) Hausdorff-Pompeiu metrics (in usual sense).

MSC: 47H10; 54H25

Keywords: Hausdorff-Pompeiu metric; *H*-cone metric; cone metric space; fixed point; multivalued contraction

1 Introduction

Recently, the investigation of possible equivalence between fixed point results in cone metric spaces (or *tvs*-cone metric spaces) and metric spaces has become a hot topic in many mathematical activities. Namely, by using the properties either of the Minkowski functional q_e or the nonlinear scalarization function ξ_e (in particular their monotonicity), some scholars have made a conclusion that many fixed point results in the setting of cone metric spaces or *tvs*-cone metric spaces can be directly obtained as a consequence of the corresponding results in metric spaces (see [1–12]). However, so far these equivalences have been referred to some fixed results only for single valued mappings, whereas, the ones for multivalued mappings have been seldom involved. The aim of this paper is to consider some fixed point theorem equivalences between H-cone metric fixed point theorems for multivalued or generalized multivalued contractions and (usual) metric fixed point theorems for (general) multivalued mappings. We mainly establish the equivalences between Arshad's and Ahmad's theorem (see [13]) and Nadler's theorem (see [14]), and between Đorić's theorem (see [15]) and Achari's theorem (see [16]), and Ćirić's theorem (see [17]).

Definition 1.1 ([18]) Let E be a real Banach space and θ be its zero element. Suppose that a nonempty closed subset K of E satisfies the following:

(1) $K \neq \{\theta\}$;
(2) $a, b \in \mathbb{R}^+$ and $x, y \in K \Rightarrow ax + by \in K$;
(3) $x, -x \in K \Rightarrow x = \theta$.

Then K is called a cone. If $\operatorname{int} K \neq \emptyset$, then K is called a solid cone, where $\operatorname{int} K$ denotes the interior of K.

Definition 1.2 ([19]) Let K be a cone in a real Banach space $(E, \|\cdot\|)$. The partial orderings $\preceq$, $\prec$, and $\ll$ on E with respect to P are defined as follows, respectively. Let $x, y \in E$. Then
(1) $x \preceq y$ if $y - x \in K$;
(2) $x \prec y$ if $x \preceq y$ and $x \neq y$;
(3) $x \ll y$ if $y - x \in \operatorname{int} K$;
(4) we say that K is normal if there is $M > 0$ such that $\theta \preceq x \preceq y \Rightarrow \|x\| \leq M\|y\|$.

Throughout this paper, unless otherwise specified, we always suppose that E is a real Banach space, K is a solid cone in E, $\preceq$, $\prec$, $\ll$ are partial orderings with respect to K, and Y is a locally convex Hausdorff topological vector space (*tvs* for short) with its zero vector θ.

Definition 1.3 ([20]) Let X be a nonempty set and (Y, K) be an ordered *tvs*. Suppose that a vector-valued function $d : X \times X \to Y$ satisfies:
(i) $\theta \preceq d(x, y)$ for all $x, y \in X$ and $d(x, y) = \theta$ if and only if $x = y$;
(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$;
(iii) $d(x, y) \preceq d(x, z) + d(z, y)$ for all $x, y, z \in X$.
Then d is called a *tvs*-cone metric on X, and (X, d) is called a *tvs*-cone metric space.

Remark 1.4 ([8]) If $Y = E$ in Definition 1.3, then d is said to be a cone metric on X, and (X, d) is said to be a cone metric space. In other words, cone metric space is a special case of *tvs*-cone metric space.

Definition 1.5 ([21]) Let (X, d) be a cone metric space and let $\mathcal{A}$ be a collection of nonempty subsets of X. A map $H : \mathcal{A} \times \mathcal{A} \to E$ is called an H-cone metric in the sense of Wardowski if for any $A, B \in \mathcal{A}$, the following conditions hold:
(H1) $H(A, B) = \theta \Rightarrow A = B$;
(H2) $H(A, B) = H(B, A)$;
(H3) for any $\varepsilon \gg \theta$ and each $x \in A$, there exists $y \in B$ such that $d(x, y) \preceq H(A, B) + \varepsilon$;
(H4) one of the following is satisfied:
 (i) for any $\varepsilon \gg \theta$, there exists $x \in A$ such that for each $y \in B$, $H(A, B) \preceq d(x, y) + \varepsilon$;
 (ii) for any $\varepsilon \gg \theta$, there exists $x \in B$ such that for each $y \in A$, $H(A, B) \preceq d(x, y) + \varepsilon$.

Remark 1.6 If we substitute Y for E, then H is called a *tvs*-H-cone metric (see [20]).

Definition 1.7 ([13]) Let (X, d) be a cone metric space and $\mathcal{A}$ a collection of nonempty subsets of X. A map $H : \mathcal{A} \times \mathcal{A} \to E$ is called an H-cone metric in the sense of Arshad and Ahmad if the following conditions hold:

(H_1) $\theta \preceq H(A, B)$ for all $A, B \in \mathcal{A}$ and $H(A, B) = \theta$ if and only if $A = B$;
(H_2) $H(A, B) = H(B, A)$ for all $A, B \in \mathcal{A}$;
(H_3) $H(A, B) \preceq H(A, C) + H(C, B)$ for all $A, B, C \in \mathcal{A}$;
(H_4) if $A, B \in \mathcal{A}$, $\theta \prec \varepsilon \in E$ with $H(A, B) \prec \varepsilon$, then for each $a \in A$ there exists $b \in B$ such that $d(a, b) \prec \varepsilon$.

Example 1.8 ([13, 21]) Let (X,d) be a metric space and let $\mathcal{A}$ be a family of all nonempty closed bounded subsets of X. Then $H : \mathcal{A} \times \mathcal{A} \to \mathbb{R}^+$ given by the formula

$$H(A,B) = \max\Big\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(y,A)\Big\}, \quad A,B \in \mathcal{A}, \tag{1.1}$$

is an H-cone metric (called a Hausdorff-Pompeiu metric), which satisfies either Definition 1.5 or Definition 1.7.

Remark 1.9 Compared with Definition 1.5, Definition 1.7 minutely modifies Definition 1.5 to make it more comparable with a standard metric. The following example indicates that Definition 1.7 is different from Definition 1.5.

Example 1.10 Let $X = \{a,b,c\}$ and $d : X \times X \to [0,+\infty)$ be defined by

$$d(a,b) = d(b,a) = \frac{1}{2}, \qquad d(a,c) = d(c,a) = d(b,c) = d(c,b) = 1,$$
$$d(a,a) = d(b,b) = d(c,c) = 0.$$

Let $\mathcal{A} = \{\{a\},\{b\},\{c\}\}$, $H : \mathcal{A} \times \mathcal{A} \to [0,+\infty)$ as $H(\{a\},\{b\}) = H(\{b\},\{a\}) = 1$, $H(\{a\},\{c\}) = H(\{c\},\{a\}) = H(\{b\},\{c\}) = H(\{c\},\{b\}) = 2$, $H(\{a\},\{a\}) = H(\{b\},\{b\}) = H(\{c\},\{c\}) = 0$. Then H is an H-cone metric which satisfies Definition 1.7 but not Definition 1.5. In fact, (H4) of Definition 1.5 does not hold.

Recall (see [9] or [22]) that if V is an absolutely convex and absorbing subset of a *tvs* Y, its Minkowski functional is defined by

$$E \ni x \mapsto q_V(x) = \inf\{\lambda > 0 : x \in \lambda V\}.$$

It is a semi-norm on Y and $V \subset W$ implies that $q_W(x) \leq q_V(x)$ for $x \in Y$. If V is an absolutely convex neighborhood of θ in Y, then q_V is continuous and

$$\big\{x \in E : q_V(x) < 1\big\} = \operatorname{int} V \subset V \subset \overline{V} = \big\{x \in Y : q_V(x) \leq 1\big\}.$$

Let (Y,K) be an ordered *tvs* and $e \in \operatorname{int} K$. Then $[-e,e] = (K-e) \cap (e-K) = \{z \in Y : -e \preceq z \preceq e\}$ is an absolutely convex neighborhood of θ; its Minkowski functional $q_{[-e,e]}$ will be denoted by q_e. Clearly, $\operatorname{int}[-e,e] = (\operatorname{int} K - e) \cap (e - \operatorname{int} K)$, $q_e(x) = \inf\{\lambda > 0 : x \prec \lambda e\}$. Moreover, $q_e(x)$ is an increasing function on K. Indeed, if $\theta \preceq x \preceq y$, then $\{\lambda : x \in \lambda[-e,e]\} \supset \{\lambda : y \in \lambda[-e,e]\}$ and it follows that $q_e(x) \leq q_e(y)$.

Lemma 1.11 ([9]) *Let (X,d) be a tvs-cone metric space and let $e \in \operatorname{int} K$. Let q_e be the corresponding Minkowski functional of $[-e,e]$. Then $d_q := q_e \circ d$ is a metric on X.*

Lemma 1.12 ([8]) *Let (X,d) be a tvs-cone metric space and let $e \in \operatorname{int} K$. Let $\xi_e : Y \to \mathbb{R}$ be a nonlinear scalarization function defined by $\xi_e(y) = \inf\{r \in \mathbb{R} : y \in re - K\}$. Then $d_\xi : X \times X \to [0,+\infty)$ defined by $d_\xi := \xi_e \circ d$ is a metric on X.*

For the convenience of the reader, we present some well-known theorems as follows.

Theorem 1.13 (Nadler [14]) *Let (X,d) be a complete metric space and $\mathcal{A}$ be a collection of nonempty, closed, and bounded subsets of X. Suppose that a mapping $T:X\to\mathcal{A}$ is a multivalued contraction, that is, there exists $\lambda\in[0,1)$ such that for all $x,y\in X$,*

$$H(Tx,Ty)\le\lambda d(x,y),$$

where $H(\cdot,\cdot)$ is the Hausdorff-Pompeiu metric (1.1) *induced by d. Then T has a fixed point.*

Theorem 1.14 (Arshad and Ahmad [13]) *Let (X,d) be a complete cone metric space. Let $\mathcal{A}$ be a collection of nonempty closed subsets of X, and let $H:\mathcal{A}\times\mathcal{A}\to E$ be an H-cone metric in the sense of Arshad and Ahmad. If for a map $T:X\to\mathcal{A}$ there exists $\lambda\in[0,1)$ such that for all $x,y\in X$,*

$$H(Tx,Ty)\preceq\lambda d(x,y), \tag{1.2}$$

then T has a fixed point.

Theorem 1.15 (Achari [16]) *Let (X,d) be a complete metric space and let $\mathcal{A}$ be a family of nonempty, closed, and bounded subsets of X. Suppose that $T,S:X\to\mathcal{A}$ are two multivalued mappings and suppose that there exists $\lambda\in[0,1)$ such that for all $x,y\in X$,*

$$H(Tx,Sy)\le\lambda\cdot\max\left\{d(x,y),d(x,Tx),d(y,Sy),\frac{d(x,Sy)+d(y,Tx)}{2}\right\},$$

where $H(\cdot,\cdot)$ is the Hausdorff-Pompeiu metric (1.1) *induced by d. Then T and S have a common fixed point.*

Theorem 1.16 (Đorić [15]) *Let (X,d) be a complete cone metric space. Let $\mathcal{A}$ be a family of nonempty, closed, and bounded subsets of X and let there exist an H-cone metric $H:\mathcal{A}\times\mathcal{A}\to E$ in the sense of Arshad and Ahmad. Suppose that $T,S:X\to\mathcal{A}$ are two multivalued mappings and suppose that there is $\lambda\in[0,1)$ such that, for all $x,y\in X$, at least one of the following conditions holds*:

(C1) $H(Tx,Sy)\preceq\lambda\cdot d(x,y)$;

(C2) $H(Tx,Sy)\preceq\lambda\cdot d(x,u)$ *for each fixed $u\in Tx$*;

(C3) $H(Tx,Sy)\preceq\lambda\cdot d(y,v)$ *for each fixed $v\in Sy$*;

(C4) $H(Tx,Sy)\preceq\lambda\cdot\frac{d(x,v)+d(y,u)}{2}$ *for each fixed $v\in Sy$ and each fixed $u\in Tx$.*

Then T and S have a common fixed point.

Theorem 1.17 (Ćirić [17]) *Let (X,d) be a complete metric space and let $\mathcal{A}$ be a family of nonempty, closed, and bounded subsets of X. Suppose that $T:X\to\mathcal{A}$ is a generalized multivalued contraction, that is, there exists $\lambda\in[0,1)$ such that for all $x,y\in X$,*

$$H(Tx,Ty)\le\lambda\cdot\max\left\{d(x,y),d(x,Tx),d(y,Ty),\frac{d(x,Ty)+d(y,Tx)}{2}\right\},$$

where $H(\cdot,\cdot)$ is the Hausdorff-Pompeiu metric (1.1) *induced by d. Then T has a fixed point.*

Theorem 1.18 (Đorić [15]) *Let (X,d) be a complete cone metric space. Let $\mathcal{A}$ be a family of nonempty, closed, and bounded subsets of X and let there exist an H-cone metric $H:\mathcal{A}\times\mathcal{A}\to E$ in the sense of Arshad and Ahmad. Suppose that $T:X\to\mathcal{A}$ is a cone generalized multivalued contraction, that is, there exists $\lambda\in[0,1)$ such that, for all $x,y\in X$, one of the following conditions holds*:

(D1) $H(Tx,Ty)\preceq\lambda\cdot d(x,y)$;

(D2) $H(Tx,Ty)\preceq\lambda\cdot d(x,u)$ *for each fixed* $u\in Tx$;

(D3) $H(Tx,Ty)\preceq\lambda\cdot d(y,v)$ *for each fixed* $v\in Ty$;

(D4) $H(Tx,Ty)\preceq\lambda\cdot\frac{d(x,v)+d(y,u)}{2}$ *for each fixed* $v\in Ty$ *and each fixed* $u\in Tx$.

Then T has a fixed point.

2 Main results

In what follows, by utilizing the Minkowski functional q_e or the nonlinear scalarization function ξ_e, we present two inequalities. Based on them, we thereupon obtain some equivalences between some well-known theorems for multivalued or generalized multivalued contractions.

Theorem 2.1 *Let (X,d) be a cone metric space and $\mathcal{A}$ a collection of nonempty subsets of X. Let $H:\mathcal{A}\times\mathcal{A}\to E$ be an H-cone metric in the sense of Arshad and Ahmad and let $e\in\operatorname{int}K$ and q_e be the corresponding Minkowski functional of $[-e,e]$. If $H_q=q_e\circ H$ and $d_q=q_e\circ d$, then*

$$H_{d_q}(A,B)\le H_q(A,B)\quad(A,B\in\mathcal{A}),$$

where $H_{d_q}(A,B)$ is the Hausdorff-Pompeiu metric induced by d_q.

Proof On account of (H_1)-(H_3) in Definition 1.7, we conclude that $(\mathcal{A},H)$ is a cone metric space. Using Lemma 1.11, one finds that d_q is a metric on X and H_q is a metric on $\mathcal{A}$. Denote

$$M=\{\lambda>0:H(A,B)\prec\lambda e\}\quad(A,B\in\mathcal{A}),$$
$$N=\{\lambda>0:d(x,y)\prec\lambda e\}\quad(x\in A,y\in B).$$

In view of (H_4), it is not hard to verify that $M\subseteq N$. Thus, $\inf M\ge\inf N$. Further, we have

$$H_q(A,B)=q_e\bigl(H(A,B)\bigr)=\inf\{\lambda>0:H(A,B)\prec\lambda e\}=\inf M.$$

Accordingly, for all $A,B\in\mathcal{A}$, it follows that

$$\begin{aligned}H_{d_q}(A,B)&=\max\Bigl\{\sup_{x\in A}d_q(x,B),\sup_{y\in B}d_q(y,A)\Bigr\}\\&=\max\Bigl\{\sup_{x\in A}\inf_{y\in B}d_q(x,y),\sup_{y\in B}\inf_{x\in A}d_q(y,x)\Bigr\}\\&=\max\Bigl\{\sup_{x\in A}\inf_{y\in B}\inf\{\lambda>0:d(x,y)\prec\lambda e\},\\&\qquad\sup_{y\in B}\inf_{x\in A}\inf\{\lambda>0:d(x,y)\prec\lambda e\}\Bigr\}\end{aligned}$$

$$
\begin{aligned}
&= \max\Bigl\{\sup_{x\in A}\inf_{y\in B}\inf N, \sup_{y\in B}\inf_{x\in A}\inf N\Bigr\} \\
&\le \max\Bigl\{\sup_{x\in A}\inf_{y\in B}\inf M, \sup_{y\in B}\inf_{x\in A}\inf M\Bigr\} \\
&= \max\Bigl\{\sup_{x\in A}\inf_{y\in B} H_q(A,B), \sup_{y\in B}\inf_{x\in A} H_q(A,B)\Bigr\} \\
&= H_q(A,B).
\end{aligned}
$$
□

Theorem 2.2 *Let (X,d) be a cone metric space and $\mathcal{A}$ a collection of nonempty subsets of X. Let $H : \mathcal{A}\times\mathcal{A}\to E$ be an H-cone metric in the sense of Arshad and Ahmad and let $e\in \operatorname{int}K$ and ξ_e be the corresponding nonlinear scalarization function. If $H_\xi = \xi_e\circ H$ and $d_\xi = \xi_e\circ d$, then*

$$H_{d_\xi}(A,B)\le H_\xi(A,B)\quad (A,B\in\mathcal{A}),$$

where $H_{d_\xi}(A,B)$ is the Hausdorff-Pompeiu metric induced by d_ξ.

Proof Similarly as in the proof of Theorem 2.1, by utilizing Lemma 1.12, we obtain the conclusion. □

Theorem 2.3 *Theorem* 1.14 *is equivalent to Theorem* 1.13.

Proof In Theorem 1.14, take $E = \mathbb{R}$, $K = [0,+\infty)$, and H to be the Hausdorff-Pompeiu metric (1.1) introduced by d, and let $\mathcal{A}$ be a collection of nonempty, closed, and bounded subsets of X. Then by Theorem 1.14, we easily get Theorem 1.13. Conversely, let Theorem 1.13 hold. Applying the Minkowski functional q_e to both sides of the inequality (1.2), we establish that

$$q_e\bigl(H(Tx,Ty)\bigr)\le q_e\bigl(\lambda d(x,y)\bigr) = \lambda q_e\bigl(d(x,y)\bigr),$$

that is,

$$H_q(Tx,Ty)\le \lambda d_q(x,y).$$

Here, $H_q = q_e\circ H$ and $d_q = q_e\circ d$ are metrics from Lemma 1.11. By using Theorem 2.1, for all $x,y\in X$, it follows that

$$H_{d_q}(Tx,Ty)\le \lambda d_q(x,y).$$

Hence by Theorem 1.13, T has a fixed point. □

Theorem 2.4 *Theorem* 1.16 *is equivalent to Theorem* 1.15.

Proof In Theorem 1.16, take $E = \mathbb{R}$, $K = [0,+\infty)$, and let H be the Hausdorff-Pompeiu metric (1.1) introduced by d. Then Theorem 1.15 is valid. Indeed, in this case, from (C1)-

(C4) of Theorem 1.16, we conclude that

$$H(Tx,Sy) \leq \lambda \begin{cases} d(x,y), \\ d(x,Tx), \\ d(y,Sy), \\ \frac{1}{2}(d(y,Tx)+d(x,Sy)), \end{cases}$$

where $d(a,B)=\inf_{b\in B} d(a,b)$. Hence,

$$H(Tx,Sy) \leq \lambda \cdot \max\left\{d(x,y), d(x,Tx), d(y,Sy), \frac{d(x,Sy)+d(y,Tx)}{2}\right\}.$$

That is to say, we obtain Theorem 1.15.

Conversely, let Theorem 1.15 hold. Then by (C1)-(C4), it follows that

$$H(Tx,Sy) \leq \lambda \begin{cases} d(x,y), & \\ d(x,u), & u\in Tx, \\ d(y,v), & v\in Sy, \\ \frac{1}{2}(d(y,u)+d(x,v)), & u\in Tx, v\in Sy. \end{cases}$$

Applying the Minkowski functional q_e to both sides of the above inequalities, we establish that

$$H_q(Tx,Sy) \leq \lambda \begin{cases} d_q(x,y), \\ \inf_{u\in Tx} d_q(x,u), \\ \inf_{v\in Sy} d_q(y,v), \\ \frac{1}{2}(\inf_{u\in Tx} d_q(y,u)+\inf_{v\in Sy} d_q(x,v)). \end{cases}$$

That is to say,

$$H_q(Tx,Sy) \leq \lambda \cdot \max\left\{d_q(x,y), d_q(x,Tx), d_q(y,Sy), \frac{d_q(x,Sy)+d_q(y,Tx)}{2}\right\},$$

where $d_q(a,B)=\inf_{b\in B} d_q(a,b)$. $H_q = q_e \circ H$ and $d_q = q_e \circ d$ are metrics from Lemma 1.11. Thus by Theorem 2.1, for all $x, y\in X$, we have

$$H_{d_q}(Tx,Sy) \leq \lambda \cdot \max\left\{d_q(x,y), d_q(x,Tx), d_q(y,Sy), \frac{d_q(x,Sy)+d_q(y,Tx)}{2}\right\}.$$

Therefore, by Theorem 1.15, T and S have a common fixed point. □

Corollary 2.5 *Theorem* 1.18 *is equivalent to Theorem* 1.17.

Proof If one takes $S = T$ in Theorem 1.16 and Theorem 1.15, then by Theorem 2.4, the proof is completed. □

Remark 2.6 According to Theorem 2.3, Theorem 2.4 and Corollary 2.5, we can easily see that the vectorial versions of Nadler's theorem, Achari's theorem and Ćirić's theorem are just equivalent to their scalar versions, respectively. It is worth mentioning that it is possible to obtain the same conclusion using the nonlinear scalarization function ξ_e.

We finally pose the following problems:

Problem 1 Does Definition 1.5 imply Definition 1.7?

Problem 2 Is Theorem 3.1 of [21] equivalent to Theorem 1.13?

Problem 3 Is Theorem 2.4 of [20] equivalent to Theorem 1.13?

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors contributed equally and significantly in writing this paper. All authors read and approved the final manuscript.

Author details
[1]School of Mathematics and Statistics, Hubei Normal University, Huangshi, 435002, China. [2]Faculty of Mathematics and Information Technology, Dong Thap University, Dong Thap, Vietnam. [3]Faculty of Organizational Sciences, University of Belgrade, Jove Ilića 154, Beograd, Serbia.

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