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Common fixed point theorems for hybrid contractive pairs with the (CLR)-property

Afrah AN Abdou*

*Correspondence:
aabdou@kau.edu.sa
Department of Mathematics, King Abdulaziz University, P.O. Box 80203, Jeddah, 21589, Saudi Arabia

Abstract

In this work, we introduce the (CLR)-property for the hybrid pairs of single-valued and multi-valued mappings and give some coincidence and common fixed point theorems for the hybrid pairs of some contractive conditions. Also, we will give some examples to illustrate the main results in this paper. Our results extend and improve some results given by some authors.

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1 Introduction

In 1969, Nadler [1] introduced the notion of a multi-valued (set-valued) contractive mapping in a metric space and also proved Banach's fixed point theorem for a multi-valued mapping in a metric space. Since Nadler, many authors have studied Banach's fixed point theorem for multi-valued mappings in several ways [2–7].

Especially, fixed point theorems for the hybrid contractive pairs of single-valued and multi-valued mappings are always be an interesting area of research due to its majority on only single-valued contractive mappings or only multi-valued contractive mappings in general spaces [8]. Besides, there are many results as regards fixed point theorems for multi-valued mappings in metric spaces with different contractive conditions and applications. For more details, we refer to [9–11] and references therein.

In 1982, Sessa [12] first studied common fixed points results for weakly commuting pair of single-valued mappings in metric spaces. Afterward, Jungck [13] introduced the concept of compatible single-valued mappings in order to generalize the concept of weak commutativity by Sessa [12] and showed that weakly commuting mappings are compatible, but the converse is not true. In 1996, Jungck [14] introduced the concept of weakly compatibility for single-valued mappings. Afterward, Aamri and El Moutawakil [15] introduced the notion of the property $(E.A.)$, which is a special case of the tangential property due to Sastry and Krishna Murthy [16]. In 2011, Sintunaravat and Kumam [17] showed that the notion of the property $(E.A.)$ always requires the completeness (or closedness) of the underlying subspaces for the existence of common fixed points for single-valued mappings. Hence they coined the idea of *common limit in the range* (for brevity, called the (CLR)-property), which relaxes the requirement of completeness (or closedness) of the under-

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lying subspace. They also proved common fixed point results for single-valued mappings via this concept in fuzzy metric spaces. For more details on the (CLR)-property, refer to [18–20] and therein.

Inspired by the notion of the property (CLR)-property, we introduce the (CLR_g)-property for the hybrid pairs of single-valued and multi-valued mappings in metric spaces and give some new coincidence and common fixed point theorems under the hybrid pairs satisfying some contractive conditions. Also, we give some examples to illustrate the main results in this paper. Our results improve, extend, and generalize the corresponding results given by some authors.

2 Preliminaries

Throughout this paper, let (X, d) be a metric space and let $\mathrm{CB}(X)$ denote the class of all nonempty bounded closed subsets of X. Let H be the Hausdorff metric with respect to d, that is,

$$H(A,B) = \Bigl\{\sup_{x\in A} d(x,B), \sup_{x\in B} d(x,A)\Bigr\}$$

for all $A, B \in \mathrm{CB}(X)$, where

$$d(x,A) := \inf\bigl\{d(x,y) : y \in A\bigr\}.$$

In fact, the convergence in the Hausdorff metric H means that, if $\{A_n\}$ is a sequence in $\mathrm{CB}(X)$ and $A \in \mathrm{CB}(X)$, then

$$\lim_{n\to\infty} H(A_n, A) = 0.$$

Note that, if $\lim_{n\to\infty} H(A_n, A) = 0$, then, for any $\varepsilon > 0$, there exists a positive integer N such that

$$A_n \subset N_\varepsilon(A) = \bigl\{x \in X : d(x,A) < \varepsilon\bigr\}$$

for all $n \geq N$. For more details on the convergence in the Hausdorff metric H, refer to [21].

We also denote by $\mathrm{Fix}(T)$ the set of all fixed points of a multi-valued mapping T.

Definition 2.1 ([13]) Let (X, d) be a metric space. Two mappings $f, g : X \to X$ are said to be *compatible* or *asymptotically commuting* if

$$\lim_{n\to\infty} d(gfx_n, fgx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t$$

for some $t \in X$.

In 1989, Kaneko and Sessa [22] introduced the notion of compatible for single-valued and multi-valued mappings as follows.

Definition 2.2 ([22]) Let (X,d) be a metric space. Two mappings $f : X \to X$ and $T : X \to \mathrm{CB}(X)$ are said to be *compatible* if $fTx \in \mathrm{CB}(X)$ for all $x \in X$ and

$$\lim_{n\to\infty} H(Tfx_n, fTx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Tx_n = A$$

for some $A \in \mathrm{CB}(X)$ and

$$\lim_{n\to\infty} fx_n = t \in A$$

for some $t \in X$.

Remark 2.3 Recall that two mappings $f : X \to X$ and $T : X \to \mathrm{CB}(X)$ are *noncompatible* if $fTx \in \mathrm{CB}(X)$ for all $x \in X$ and there exists at least one sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Tx_n = A \in \mathrm{CB}(X)$$

and

$$\lim_{n\to\infty} fx_n = t \in A,$$

but

$$\lim_{n\to\infty} H(Tfx_n, fTx_n) \neq 0$$

or it is nonexistent.

Definition 2.4 ([14]) Let (X,d) be a metric space. Two mappings $f : X \to X$ and $T : X \to \mathrm{CB}(X)$ are said to be *weakly compatible* if they commute at their coincidence points, *i.e.*, if $fTx = Tfx$ whenever $fx \in Tx$.

It is easy to see that two compatible mappings are weakly compatible, but the converse is not true.

Definition 2.5 ([17]) Let (X,d) be a metric space. Two mappings $f,g : X \to X$ are said to satisfy the *common limit in the range of f* with respect to g (for brevity, the (CLR_f)-property w.r.t. g) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = fu$$

for some $u \in X$.

Example 2.6 Let $X = [1,\infty)$ with usual metric. Define two single-valued mappings $f,g : X \to X$ by

$$fx = \frac{x}{2}, \qquad gx = 2x$$

for all $x \in X$. Consider the sequence $\{x_n\}$ defined by $x_n = \frac{1}{n}$. Then we have

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = f(0).$$

Therefore, f and g satisfy the property (CLR_f) w.r.t. g.

3 Main results

Now, we define the (CLR_f)-property for a hybrid pairs of single-valued and multi-valued mappings in metric spaces.

Definition 3.1 Let (X,d) be a metric space. Two mappings $f : X \to X$ and $T : X \to \mathrm{CB}(X)$ are said to satisfy the *common limit in the range of the f* w.r.t. g (for brevity, the (CLR_f)-property w.r.t. T) if there exist a sequence $\{x_n\}$ in X and $A \in \mathrm{CB}(X)$ such that

$$\lim_{n\to\infty} fx_n = f(u) \in A = \lim_{n\to\infty} Tx_n$$

for some $u \in X$.

Remark 3.2 Note that, if $f(X)$ is closed, then a noncompatible hybrid pair (f, T) satisfies the (CLR_f) w.r.t. T.

Now, we give an example for two mappings satisfying the (CLR_f)-property w.r.t. T.

Example 3.3 Let $X = [1,\infty)$ with the usual metric. Define two mappings $f : X \to X$ and $T : X \to \mathrm{CB}(X)$ by

$$fx = x + 2, \qquad Tx = [1, x + 2]$$

for all $x \in X$. Consider the sequence $\{x_n\}$ in X defined by $x_n = \frac{1}{n}$. Clearly, we have

$$\lim_{n\to\infty} fx_n = 2 = f(0) \in [1,2] = \lim_{n\to\infty} Tx_n.$$

Therefore, f and T satisfy the (CLR_f) w.r.t. T.

Here, we state and prove the main result in this paper.

Theorem 3.4 *Let (X,d) be a metric space and let $f : X \to X$, $T : X \to \mathrm{CB}(X)$ be two mappings satisfying the following conditions*:

(1) *f and T satisfy the (CLR_f)-property w.r.t. T*;

(2) *for all $x, y \in X$*,

$$H^p(Tx,Ty) \le \varphi\left(\max\left\{d^p(fx,fy), \frac{d^p(fx,Tx)d^p(fy,Ty)}{1+d^p(fx,fy)}, \frac{d^p(fx,Ty)d^p(fy,Tx)}{1+d^p(fx,fy)}\right\}\right),$$

where $p \ge 1$ and $\varphi : [0,\infty) \to [0,\infty)$ is a continuous monotone increasing function such that $\varphi(0) = 0$ and $\varphi(t) < t$ for all $t > 0$.

Then f and T have a coincidence point in X.

Proof Since f and T satisfy the (CLR_f)-property w.r.t. T, there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = f(u) \in A = \lim_{n\to\infty} Tx_n$$

for some $u \in X$ and $A \in \mathrm{CB}(X)$.

Now, we show that $fu \in Tu$. In fact, suppose that $fu \notin Tu$. Then, using the condition (2) with $x = x_n$ and $y = u$, we have

$$H^p(Tx_n, Tu) \le \varphi\left(\max\left\{d^p(fx_n,fu), \frac{d^p(fx_n, Tx_n)d^p(fu, Tu)}{1 + d^p(fx_n,fu)}, \frac{d^p(fx_n, Tu)d^p(fu, Tx_n)}{1 + d^p(fx_n,fu)}\right\}\right)$$

for all $n \in \mathbb{N}$. Letting $n \to \infty$, we have $H^p(A, Tu) = 0$. Since $fu \in A$, it follows from the definition of Hausdorff metric that

$$d^p(fu, Tu) \le H^p(A, Tu) \le 0,$$

which implies that $d^p(fu, Tu) = 0$, that is, $fu \in Tu$. This implies that u is a coincidence point of f and T. This completes the proof. □

From Remark 3.2, we have the following result.

Corollary 3.5 *Let (X, d) be a metric space and let $f : X \to X$ and $T : X \to \mathrm{CB}(X)$ be two mappings such that $f(X)$ is a closed subset of X and f, T satisfying the following conditions*:

(1) *f and T are noncompatible*;

(2) *for all $x, y \in X$,*

$$H^p(Tx, Ty) \le \varphi\left(\max\left\{d^p(fx,fy), \frac{d^p(fx, Tx)d^p(fy, Ty)}{1 + d^p(fx,fy)}, \frac{d^p(fx, Ty)d^p(fy, Tx)}{1 + d^p(fx,fy)}\right\}\right),$$

where $p \ge 1$ and $\varphi : [0,\infty) \to [0,\infty)$ is a continuous monotone increasing function such that $\varphi(0) = 0$ and $\varphi(t) < t$ for all $t > 0$.

Then f and T have a coincidence point in X.

Theorem 3.6 *Let (X, d) be a metric space and let $f : X \to X$ and $T : X \to \mathrm{CB}(X)$ are two mappings satisfying the conditions* (1), (2) *of Theorem* 3.4. *If f and T are weakly compatible at a and $ffa = fa$ for some $a \in C(f, T) \neq \emptyset$, then f and T have a common fixed point in X.*

Proof From Theorem 3.4, there exists $u \in X$ such hat $fu \in Tu$, that is, $C(f, T) \neq \emptyset$. By the assumption, we have $ffa = fa$. Since f and T are weakly compatible, we have $Tfa = fTa$. Now, letting $t := fa$. Then we obtain

$$t = ft = ffa \in fTa = Tfa = Tt,$$

that is, t is a common fixed point of f and T. This completes the proof. □

Corollary 3.7 *Let* (X,d) *be a metric space and let* $f : X \to X$, $T : X \to \mathrm{CB}(X)$ *be two mappings satisfying the conditions* (1), (2) *of Theorem* 3.4. *If* f *and* T *are weakly compatible at* a *and* $ffa = fa$ *for all* $a \in C(f,T) \neq \emptyset$, *then* f *and* T *have a common fixed point*.

Next, we give one interesting example to illustrate Theorems 3.4 and 3.6, but Corollary 3.7 is not applicable.

Example 3.8 Let $X = [1,\infty)$ with usual metric. Define two mappings $f : X \to X$ and $T : X \to \mathrm{CB}(X)$ by

$$fx = x^2, \qquad Tx = [1, x+1]$$

for all $x \in X$. Then f and T satisfy the (CLR_f) w.r.t. T for the sequence $\{x_n\}$ defined by $x_n = 1 + \frac{1}{n}$. Indeed, we have

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^2 = 1 = f(1) \in [1,2] = \lim_{n\to\infty} Tx_n.$$

Now, we show that f and T satisfy the condition (2) in Theorem 3.4 with $p = 1$ and $\varphi(t) = \frac{1}{2}t$. For all $x, y \in [1,\infty)$, we have

$$\begin{aligned} H(Tx,Ty) &= |x-y| \leq \frac{|x+y|}{2}|x-y| = \frac{1}{2}\left|x^2-y^2\right| \\ &\leq \frac{1}{2}\max\left\{d(fx,fy), \frac{d(fx,Tx)d(fy,Ty)}{1+d(fx,fy)}, \frac{d(fx,Ty)d(fy,Tx)}{1+d(fx,fy)}\right\} \\ &= \varphi\left(\max\left\{d(fx,fy), \frac{d(fx,Tx)d(fy,Ty)}{1+d(fx,fy)}, \frac{d(fx,Ty)d(fy,Tx)}{1+d(fx,fy)}\right\}\right). \end{aligned}$$

This means that f and T satisfy the condition (2) in Theorem 3.4 with $p = 1$ and $\varphi(t) = \frac{1}{2}t$. Thus all the conditions in Theorem 3.4 are satisfied. Then f and T have a coincidence point in X. It is easy to see that f and T have infinitely coincidence point in X. Indeed, $C(f,T) = [1, \frac{1+\sqrt{5}}{2}]$.

Next, we claim that f and T have a common fixed point in X by using Theorem 3.6. Also, we can see that f and T are weakly compatible at a point a and $ffa = fa$ for $a = 1 \in C(f,T)$. So, all the conditions of Theorem 3.6 are satisfied. Therefore, f and T have a common fixed point in X. In this case, the point 1 is a unique common fixed point of f and T.

Remark 3.9 From Example 3.8, we can see that f and T are not weakly compatible at a point a with $a \in C(f,T) = [1, \frac{1+\sqrt{5}}{2}]$. Also, $ffa \neq fa$ for all $a \in C(f,T) = [1, \frac{1+\sqrt{5}}{2}]$. Therefore, Corollary 3.7 cannot be applicable in this case.

If we take $p = 1$ in Theorem 3.6, then we have the following result.

Corollary 3.10 *Let* (X,d) *be a metric space and let* $f : X \to X$, $T : X \to \mathrm{CB}(X)$ *be two mappings satisfying the following conditions*:

(1) f *and* T *satisfy the* (CLR_f)*-property w.r.t.* T;

(2) *for all* $x, y \in X$,

$$H(Tx,Ty) \leq \varphi\left(\max\left\{d(fx,fy), \frac{d(fx,Tx)d(fy,Ty)}{1+d(fx,fy)}, \frac{d(fx,Ty)d(fy,Tx)}{1+d(fx,fy)}\right\}\right),$$

where $p \geq 1$ *and* $\varphi : [0,\infty) \to [0,\infty)$ *is a continuous monotone increasing function such that* $\varphi(0) = 0$ *and* $\varphi(t) < t$ *for all* $t > 0$.

If f *and* T *are weakly compatible at a point* a *and* $ffa = fa$ *for some* $a \in C(f,T) \neq \emptyset$, *then* f *and* T *have a common fixed point in* X.

Corollary 3.11 *Let* (X,d) *be a metric space and let* $f : X \to X$, $T : X \to \mathrm{CB}(X)$ *be two mappings satisfying the following conditions*:

(1) f *and* T *satisfy the* (CLR_f)*-property w.r.t.* T;

(2) *for all* $x, y \in X$,

$$H(Tx,Ty) \leq \varphi\left(\max\left\{d(fx,fy), \frac{d(fx,Tx)d(fy,Ty)}{1+d(fx,fy)}, \frac{d(fx,Ty)d(fy,Tx)}{1+d(fx,fy)}\right\}\right),$$

where $p \geq 1$ *and* $\varphi : [0,\infty) \to [0,\infty)$ *is a continuous monotone increasing function such that* $\varphi(0) = 0$ *and* $\varphi(t) < t$ *for all* $t > 0$.

If f *and* T *are weakly compatible at a point* a *and* $ffa = fa$ *for all* $a \in C(f,T) \neq \emptyset$, *then* f *and* T *have a common fixed point in* X.

Corollary 3.12 *Let* (X,d) *be a metric space and let* $f : X \to X$, $T : X \to \mathrm{CB}(X)$ *be two mappings satisfying the following conditions*:

(1) f *and* T *satisfy the* (CLR_f)*-property w.r.t.* T;

(2) *there exists* $k \in [0,1)$ *such that*

$$H(Tx,Ty) \leq k \max\left\{d(fx,fy), \frac{d(fx,Tx)d(fy,Ty)}{1+d(fx,fy)}, \frac{d(fx,Ty)d(fy,Tx)}{1+d(fx,fy)}\right\}$$

for all $x, y \in X$.

If f *and* T *are weakly compatible at a point* a *and* $ffa = fa$ *for some* $a \in C(f,T) \neq \emptyset$, *then* f *and* T *have a common fixed point in* X.

Proof Take $\varphi(t) = kt$ in Corollary 3.10. Then we have the conclusion. □

Corollary 3.13 *Let* (X,d) *be a metric space and let* $f : X \to X$, $T : X \to \mathrm{CB}(X)$ *be two mappings satisfying the following conditions*:

(1) f *and* T *satisfy the* (CLR_f)*-property w.r.t.* T;

(2) *there exists* $k \in [0,1)$ *such that*

$$H(Tx,Ty) \leq k \max\left\{d(fx,fy), \frac{d(fx,Tx)d(fy,Ty)}{1+d(fx,fy)}, \frac{d(fx,Ty)d(fy,Tx)}{1+d(fx,fy)}\right\}$$

for all $x, y \in X$.

If f *and* T *are weakly compatible at* a *and* $ffa = fa$ *for all* $a \in C(f,T) \neq \emptyset$, *then* f *and* T *have a common fixed point in* X.

If we take $f = I$ (the identity mapping in X) in Theorem 3.6, then we have the following result.

Corollary 3.14 *Let (X,d) be a metric space and let $T : X \to \mathrm{CB}(X)$ be a mapping satisfying the following conditions*:

(1) *there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Tx_n = A$$

for some $A \in \mathrm{CB}(X)$ and

$$\lim_{n\to\infty} x_n = u \in A$$

for some $u \in X$;

(2) *for all $x, y \in X$,*

$$H^p(Tx, Ty) \leq \varphi\left(\max\left\{d^p(x,y), \frac{d^p(x,Tx)d^p(y,Ty)}{1+d^p(x,y)}, \frac{d^p(x,Ty)d^p(y,Tx)}{1+d^p(x,y)}\right\}\right),$$

where $p \geq 1$ and $\varphi : [0,\infty) \to [0,\infty)$ is a monotone increasing function such that $\varphi(0) = 0$ and $\varphi(t) < t$ for all $t > 0$.

Then T has a fixed point in X.

Corollary 3.15 *Let (X,d) be a metric space and let $T : X \to \mathrm{CB}(X)$ be a mapping satisfying the following conditions*:

(1) *there exists a sequence $\{x_n\}$ in X such that*

$$\lim_{n\to\infty} Tx_n = A$$

for some $A \in \mathrm{CB}(X)$ and

$$\lim_{n\to\infty} x_n = u \in A$$

for some $u \in X$;

(2) *there exists $k \in [0,1)$ such that*

$$H(Tx, Ty) \leq k \max\left\{d(x,y), \frac{d(x,Tx)d(y,Ty)}{1+d(x,y)}, \frac{d(x,Ty)d(y,Tx)}{1+d(x,y)}\right\}$$

for all $x, y \in X$.

Then T has a unique fixed point in X.

Proof Take $p = 1$ and $\varphi(t) = kt$ in Corollary 3.14. Then we have the conclusion. □

4 Conclusion

Recently, some authors have required some conditions, that is, the completeness of X, the closedness or the convexity of some suitable subset of X, the continuity of one mapping or more mappings, and the containment of the range of the given mappings, to prove some common fixed point results for single-valued and multi-valued mappings in a metric space X, but, as in our results, if we use the (CLR)-property for single-valued and multi-valued mappings, then we do not need the conditions mentioned above.

Competing interests
The author declares to have no competing interests.

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References

1. Nadler, NB Jr.: Multi-valued contraction mappings. Pac. J. Math. **30**, 475-488 (1969)
2. Eldred, AA, Anuradha, J, Veeramani, P: On equivalence of generalized multi-valued contractions and Nadler's fixed point theorem. J. Math. Anal. Appl. **336**, 751-757 (2007)
3. Chaipunya, P, Mongkolkeha, C, Sintunavarat, W, Kumam, P: Fixed point theorems for multi-valued mappings in modular metric spaces. Abstr. Appl. Anal. **2012**, Article ID 503504 (2012)
4. Eshaghi Gordji, M, Baghani, H, Khodaei, H, Ramezant, M: A generalization of Nadler's fixed point theorem. J. Nonlinear Sci. Appl. **3**, 148-151 (2010)
5. Pathak, HK, Agarwal, RP, Cho, YJ: Coincidence and fixed points for multi-valued mappings and its application to nonconvex integral inclusions. J. Comput. Appl. Math. **283**, 201-217 (2015)
6. Sintunavarat, W, Kumam, P: Gregus-type common fixed point theorems for tangential multi-valued mappings of integral type in metric spaces. Int. J. Math. Math. Sci. **2011**, Article ID 923458 (2011)
7. Sintunavarat, W, Lee, DM, Cho, YJ: Mizoguchi-Takahashi's type common fixed point theorems without T-weakly commuting condition and invariant approximations. Fixed Point Theory Appl. **2014**, 112 (2014)
8. Kamran, T: Coincidence and fixed points for hybrid strict contractions. J. Math. Anal. Appl. **299**, 235-241 (2004)
9. Imdad, M, Ahmed, MA: Some common fixed point theorems for hybrid pairs of maps without the completeness assumption. Math. Slovaca **62**, 301-314 (2012)
10. Liu, Y, Wu, J, Li, Z: Common fixed points of single-valued and multi-valued maps. Int. J. Math. Math. Sci. **19**, 3045-3055 (2005)
11. Pant, BD, Samet, B, Chauhan, S: Coincidence and common fixed point theorems for single-valued and set-valued mappings. Commun. Korean Math. Soc. **27**, 733-743 (2012)
12. Sessa, S: On a weak commutativity condition in fixed point considerations. Publ. Inst. Math. (Belgr.) **34**(46), 149-153 (1982)
13. Jungck, G: Compatible mappings and common fixed points. Int. J. Math. Math. Sci. **9**, 771-779 (1986)
14. Jungck, G, Rhoades, BE: Fixed points for set-valued functions without continuity. Indian J. Pure Appl. Math. **29**, 227-238 (1998)
15. Aamri, M, El Moutawakil, D: Some new common fixed point theorems under strict contractive conditions. J. Math. Anal. Appl. **270**, 181-188 (2002)
16. Sastry, KPR, Krishna Murthy, ISR: Common fixed points of two partially commuting tangential selfmaps on a metric space. J. Math. Anal. Appl. **250**, 731-734 (2000)
17. Sintunavarat, W, Kumam, P: Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces. J. Appl. Math. **2011**, Article ID 637958 (2011)
18. Chauhan, S, Sintunavarat, W, Kumam, P: Common fixed point theorems for weakly compatible mappings in fuzzy metric spaces using the $(JCLR)$-property. Appl. Math. **3**(9), 976-982 (2012)
19. Roldán, A, Sintunavarat, W: Common fixed point theorems in fuzzy metric spaces using the (CLR_g)-property. Fuzzy Sets Syst. (in press)
20. Wairojjana, N, Sintunavarat, W, Kumam, P: Common tripled fixed point theorems for W-compatible mappings along with the CLR_g-property in abstract metric spaces. J. Inequal. Appl. **2014**, 133 (2014)
21. Geletu, A: Introduction to Topological Spaces and Set-valued Maps. Lecture Notes, Institute of Mathematics, Ilmenau University of Technology (2006)
22. Kaneko, H, Sessa, S: Fixed point theorems for compatible multi-valued and single-valued mappings. Int. J. Math. Math. Sci. **12**, 257-262 (1989)