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Cyclic contractions and fixed point theorems on various generating spaces

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Abstract
In this paper, we prove some fixed point theorems using various cyclic contractions in weaker forms of generating spaces.

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Keywords: generating space of b-dislocated quasi-metric family; generating space of b-quasi-metric family; generating space of quasi-metric family; generating space of b-dislocated metric family; generating space of dislocated quasi-metric family; generating space of dislocated metric family; cyclic contraction; G_{bdq}-cyclic-Banach contraction; G_{bd}-cyclic-Kannan contraction; G_{bdq}-cyclic-Ciric contraction

1 Introduction

In 1922, Banach [1] established a remarkable fixed point theorem known as the 'Banach contraction principle'.

Theorem 1.1 (Banach contraction principle [1]) *Let (X, d) be a complete metric space and $T : X \to X$ be a mapping, there exists a number t, $0 \leq t < 1$, such that, for each $x, y \in X$,*

$$d(Tx, Ty) \leq td(x, y).$$

Then T has a fixed point.

This theorem assures the existence and uniqueness of fixed points of certain self-maps of metric spaces, and it gives a constructive method to find those fixed points. In recent years, a number of generalizations of the above Banach contraction principle have appeared. Of all these, the following generalization of Kannan [2] and Ciric [3] stands at the top.

In 1969, Kannan [2] initiated a new type of contraction mapping, which is called the Kannan-contraction type. He also established a fixed point result for such a type.

Theorem 1.2 (Kannan fixed point theorem [2]) *Let (X, d) be a complete metric space and $T : X \to X$ be a mapping; there exists a number t, $0 < t < \frac{1}{2}$, such that, for each $x, y \in X$,*

$$d(Tx, Ty) \leq t\big[d(x, Tx) + d(y, Ty)\big].$$

Then T has a fixed point.

It is interesting that Kannan's fixed point theorem is very predominant because Subrahmanyam [4] proved that Kannan's theorem describes the completeness of the metric. In other words, a metric space X is complete if and only if every Kannan mapping on X has a fixed point. Recently, many authors generalized the Kannan fixed point theorem in some class of spaces (see [5–9]).

In 1974, Ciric [3] established a fixed point theorem known as the 'Ciric quasi-contraction principle'.

Theorem 1.3 (Ciric quasi-contraction principle [3]) *Let (X, d) be a complete metric space. Assume there is a map $T : X \to X$ such that there exists a constant h, $0 \leq h < 1$, and for each $x, y \in X$,*

$$d(Tx, Ty) \leq h \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}.$$

Then T has a fixed point.

The above-mentioned theorems play a very important role not only in analysis but also in other areas of science involving mathematics, especially in graph theory, dynamical systems, and programming languages. These renowned theorems have several mathematical and real world illustrations.

Since then, many authors proved and generalized the above important theorems in various generalized metric spaces.

In 1989, Bakhtin [10] introduced the very interesting concept of a 'b-metric space' as an analog of metric space. He proved the contraction mapping principle in b-metric space that generalizes the famous Banach contraction principle in metric spaces. Since then many mathematicians have done work involving fixed points for single-valued and multivalued operators in b-metric spaces (see for example [11–21]).

In 1997, Chang et al. [22] introduced the definition of a generating space of a quasi-metric family and established some interesting fixed point theorems and coincidence point theorems in the generating space of a quasi-metric family. In 1999, Gue Myung Lee et al. [23] defined a family of weak quasi-metrics in a generating space of quasi-metric family. He proved a Takahashi-type minimization theorem, a generalized Ekeland variational principle, and a general Caristi-type fixed point theorem for set-valued maps in complete generating spaces of a quasi-metric family by using a family of weak quasi-metrics. Also without considering lower semi-continuity, he proved fixed point theorem for set-valued maps in complete generating spaces of a quasi-metric family.

In 2003, Kirk et al. [24] introduced cyclic contractions in metric spaces and investigated the existence of proximity points and fixed points for cyclic contraction mappings. Since then many results appeared in this field (see [22–33]). Sankar Raj and Veeramani [26] proved the existence of best proximity points for relatively non-expansive maps. Later, Karapinar and Erhan [31] proved the existence of fixed points for various types of cyclic contractions in a metric space.

In this paper, motivated by the above facts, we first introduce the notions of 'generating space of b-dislocated quasi-metric family' (abbreviated 'G_{bdq}-family'), 'generating space of b-dislocated metric family' (abbreviated 'G_{bd}-family'), 'generating space of b-quasi-metric family' (abbreviated G_{bq}-family), 'generating space of dislocated quasi-metric family' (abbreviated G_{dq}-family) and 'generating space of dislocated metric family' (abbreviated G_d-family). By following the approaches of Kirk et al. [24], we introduce the *G_{bdq}-cyclic-Banach contraction*, the *G_{bd}-cyclic-Kannan contraction*, and the *G_{bq}-cyclic-Ciric contraction*. Then we prove the existence of fixed point theorems in weaker forms of generating spaces by using the above mentioned contractive conditions. From our results in various generating spaces, we obtain the corresponding theorems for cyclic maps in weaker forms of metric spaces. Moreover, some examples are provided to illustrate the usability of the obtained results.

2 Preliminaries

Throughout this paper, we assume that $\mathbb{R}^+ = [0,\infty)$, $\mathbb{N}$ denotes the set of all positive integers.

Definition 2.1 ([10]) Let X be a non-empty set, let $d : X \times X \longrightarrow [0,\infty)$ and let $s \in \mathbb{R}^+$. Then (X,d) is said to be b-metric space if the following conditions are satisfied.

(i) $d(x,y) = 0$ if and only if $x = y$ for all $x,y \in X$.
(ii) $d(x,y) = d(y,x)$ for all $x,y \in X$.
(iii) There exists a real number $s \geq 1$ such that $d(x,y) \leq s[d(x,z) + d(z,y)]$ for all $x,y,z \in X$.

Example 2.2 Let $X = \mathbb{R}^+$, define $d : X \times X \to \mathbb{R}^+$ by $d(x,y) = (x-y)^2$. Then (X,d) is a b-metric space with coefficient $s = 2$.

The concept of a b-metric space is very important as the class of b-metric spaces is larger than that of metric spaces, since a b-metric space is a metric space when $s = 1$.

In [34], it was proved that each b-metric on X generates a topology $\Im$ whose base is the family of open balls $\mathcal{B}_d(x,\epsilon) = \{y \in X/d(x,y) < \epsilon\}$.

Recently, Kumari [35] discussed some topological aspects of b-dislocated quasi-metric spaces which is more general than of b-metric spaces and also derived some fixed point theorems in such a space.

Definition 2.3 ([35]) A b-dislocated quasi(simply b_{dq}) metric on a non-empty set X is a function $b_{dq} : X \times X \to \mathbb{R}^+$ such that for all x, y, z in X and $s \in \mathbb{R}^+$, the following conditions hold.

(i) $b_{dq}(x,y) \geq 0$.
(ii) $b_{dq}(x,y) = 0 = b_{dq}(y,x) \Rightarrow x = y$.
(iii) There exists a real number $s \geq 1$ such that $b_{dq}(x,y) \leq s[b_{dq}(x,z) + b_{dq}(z,y)]$.

Then the pair (X,b_{dq}) is called a b_{dq}-metric space.

In [36], we note that if b_{dq} satisfies an extra condition i.e. $b_{dq}(x,y) = b_{dq}(y,x)$ then (X,b_{dq}) is called a b-dislocated metric space (or simply a b_d-metric space).

Example 2.4 Let $X = \{0,1,2\}$, and let $b_{dq}(0,0) = \frac{1}{2}$, $b_{dq}(0,1) = 0$, $b_{dq}(0,2) = 1$, $b_{dq}(1,0) = \frac{1}{2}$, $b_{dq}(1,1) = 0$, $b_{dq}(1,2) = 2$, $b_{dq}(2,0) = \frac{1}{2}$, $b_{dq}(2,1) = 1$ and $b_{dq}(2,2) = 2$ then (X,b_{dq}) is a b_{dq} metric space with coefficient $s = 2$, but since $b_{dq}(0,1) \neq b_{dq}(1,0)$, it is not a b_d metric space.

Obviously (X,b_{dq}) is not a dislocated quasi-metric space as the triangle inequality does not hold; $b_{dq}(2,1) \nleq b_{dq}(2,0) + b_{dq}(0,1)$.

Definition 2.5 ([22]) Let X be a non-empty set and $\{d_\alpha : \alpha \in (0,1]\}$ a family of mappings d_α of $X \times X$ into $\mathbb{R}^+$. Then (X, d_α) is called a generating space of a quasi-metric family if it satisfies the following conditions for all $x, y, z \in X$:

(i) $d_\alpha(x,y) = 0$ if and only if $x = y$.
(ii) $d_\alpha(x,y) = d_\alpha(y,x)$.
(iii) For any $\alpha \in (0,1]$ there exists $\beta \in (0,\alpha]$ such that $d_\alpha(x,z) \leq d_\beta(x,y) + d_\beta(y,z)$.
(iv) For any $x, y \in X$, $d_\alpha(x,y)$ is non-increasing and left continuous in α.

In [37], it was proved that each generating space of a quasi-metric family generates a topology $\Im_{d_\alpha}$ whose base is the family of open balls.

Example 2.6 Let (X,d) be a metric space. If we take $d_\alpha(x,y) = d(x,y)$ for all $\alpha \in (0,1]$ and $x, y \in X$, then (X,d) is a generating space of a quasi-metric family.

For more examples, the reader can refer to [22].

Now, we first introducing following concepts:

Definition 2.7 Let X be a non-empty set and $\{d_\alpha : \alpha \in (0,1]\}$ a family of mapping d_α of $X \times X$ into $\mathbb{R}^+$. Then (X, d_α) is called a generating space of a b-quasi-metric family if it satisfies the following conditions for any $x, y, z \in X$ and $s \geq 1$.

(i) $d_\alpha(x,y) = 0$ if and only if $x = y$.
(ii) $d_\alpha(x,y) = d_\alpha(y,x)$.
(iii) For any $\alpha \in (0,1]$ there exists $\beta \in (0,\alpha]$ such that $d_\alpha(x,z) \leq s[d_\beta(x,y) + d_\beta(y,z)]$.
(iv) For any $x, y \in X$, $d_\alpha(x,y)$ is non-increasing and left continuous in α.

Definition 2.8

1. Let (X, d_α) be a G_{bq}-family and $\{x_n\}$ be a sequence in X. We say that $\{x_n\}$ G_{bq}-*converges* to x in (X, d_α) if $\lim_{n\to\infty} d_\alpha(x_n, x) = 0$ for all $\alpha \in (0,1]$. In this case we write $x_n \to x$.
2. Let (X, d_α) be a G_{bq}-family and let $A \subseteq X$, $x \in X$. We say that x is a G_{bq}-*limit point* of A if there exists a sequence $\{x_n\}$ in $A - \{x\}$ such that $\lim_{n\to\infty} x_n = x$.
3. A sequence $\{x_n\}$ in a G_{bq}-family is called a G_{bq}-Cauchy sequence if, given $\epsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n, m \geq n_0$, we have $d_\alpha(x_n, x_m) < \epsilon$ or $\lim_{n,m\to\infty} d_\alpha(x_n, x_m) = 0$ for all $\alpha \in (0,1]$.
4. A G_{bq}-family (X, d_α) is called complete if every G_{bq}-Cauchy sequence in X is G_{bq}-convergent.

Remark 2.9 Every G_{bq}-convergent sequence in a G_{bq}-family is G_{bq}-Cauchy.

Now we introducing the concepts of a generating space of a b-dislocated quasi-metric family (simply 'G_{bdq}-family') and a generating space of a b-dislocated metric family (simply 'G_{bd}-family').

Definition 2.10 Let X be a non-empty set and $\{d_\alpha : \alpha \in (0,1]\}$ a family of mapping d_α of $X \times X$ into $\mathbb{R}^+$. Then (X, d_α) is called a generating space of b-dislocated metric family if it satisfied the following conditions for any $x, y, z \in X$ and $s \geq 1$.

(i) $d_\alpha(x,y) = 0$ implies $x = y$.

(ii) $d_\alpha(x,y) = d_\alpha(y,x)$.
(iii) For any $\alpha \in (0,1]$ there exists $\beta \in (0,\alpha]$ such that $d_\alpha(x,z) \leq s[d_\beta(x,y) + d_\beta(y,z)]$.
(iv) For any $x,y \in X$, $d_\alpha(x,y)$ is non-increasing and left continuous in α.

Generating space of b-dislocated metric family is a generalization of b-dislocated metric space and generating space of quasi-metric family.

Now we introduce the following according to Kumari [38–41].

Definition 2.11

1. Let (X,d_α) be a G_{bd}-family and $\{x_n\}$ be a sequence in X. We say that $\{x_n\}$ G_{bd}-*converges* to x in (X,d_α) if $\lim_{n\to\infty} d_\alpha(x_n,x) = 0$ for all $\alpha \in (0,1]$. In this case we write $x_n \to x$.
2. Let (X,d_α) be a G_{bd}-family and let $A \subseteq X$, $x \in X$. We say that x is a G_{bd}-*limit point* of A if there exists a sequence $\{x_n\}$ in $A - \{x\}$ such that $\lim_{n\to\infty} x_n = x$.
3. A sequence $\{x_n\}$ in a G_{bd}-family is called a G_{bd}-Cauchy sequence if, given $\epsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n,m \geq n_0$, we have $d_\alpha(x_n,x_m) < \epsilon$ or $\lim_{n,m\to\infty} d_\alpha(x_n,x_m) = 0$ for all $\alpha \in (0,1]$.
4. A G_{bd}-family (X,d_α) is called complete if every G_{bd}-Cauchy sequence in X is G_{bd}-convergent.

Remark 2.12 Every G_{bd}-convergent sequence in a G_{bd}-family is G_{bd}-Cauchy.

The pair (X,d_α) is called 'generating space of dislocated metric family' if we take $s = 1$ in Definition 2.10.

Definition 2.13 Let X be a non-empty set and $\{d_\alpha : \alpha \in (0,1] : X \times X \to \mathbb{R}^+\}$. (X,d_α) is a generating space of b-dislocated quasi-metric family (simply 'G_{bdq}-family') if the following conditions hold for any $x,y,z \in X$ and $s \geq 1$.

(i) $d_\alpha(x,y) = d_\alpha(y,x) = 0 \Rightarrow x = y$.
(ii) For any $\alpha \in (0,1]$ there exists $\beta \in (0,\alpha]$ such that $d_\alpha(x,z) \leq s[d_\beta(x,y) + d_\beta(y,z)]$.
(iii) For any $x,y \in X$, $d_\alpha(x,y)$ is non-increasing and left continuous in α.

The generating space of a b-dislocated quasi-metric family is a generalization of a b-dislocated quasi-metric space and the generating space of a quasi-metric family.

We introducing the concepts of a G_{bdq}-convergent sequence, a G_{bdq}-Cauchy sequence, and a G_{bdq}-complete space according to Kumari [42].

Definition 2.14

1. A sequence $\{x_n\}$ in a G_{bdq}-family (X,d_α) is called G_{bdq}-converges to $x \in X$ if

$$\lim_{n\to\infty} d_\alpha(x_n,x) = 0 = \lim_{n\to\infty} d_\alpha(x,x_n) \quad \text{for all } \alpha \in (0,1].$$

   In this case x is called a G_{bdq}-limit of $\{x_n\}$, and we write $x_n \to x$.
2. A sequence $\{x_n\}$ in a G_{bdq}-family (X,d_α) is called G_{bdq}-Cauchy if

$$\lim_{n,m\to\infty} d_\alpha(x_n,x_m) = 0 = \lim_{n,m\to\infty} d_\alpha(x_m,x_n) \quad \text{for all } \alpha \in (0,1].$$

3. A G_{bdq}-family (X, d_α) is complete if every Cauchy sequence in it is G_{bdq}-convergent in X.

The pair (X, d_α) is called 'generating space of dislocated quasi-metric family' if we take $s = 1$ in Definition 2.13.

Now, we recall the definition of cyclic map in a metric space.

Definition 2.15 ([24]) Let A and B be non-empty subsets of a metric space (X, d) and $T : A \cup B \to A \cup B$. T is called a cyclic map iff $T(A) \in B$ and $T(B) \in A$.

Definition 2.16 ([24]) Let A and B be non-empty subsets of a metric space (X, d). A cyclic map $T : A \cup B \to A \cup B$ is said to be a cyclic contraction if there exists $k \in [0, 1)$ such that $d(Tx, Ty) \leq k(d(x, y))$ for all $x \in A$ and $y \in B$.

3 Fixed point theorems on various generating spaces

Definition 3.1 Let A and B be non-empty subsets of a G_{bdq}-family (X, d_α). A cyclic map $T : A \cup B \to A \cup B$ is said to be a *G_{bdq}-cyclic-Banach contraction* if for all $\alpha \in (0, 1]$ such that

$$d_\alpha(Tx, Ty) \leq kd_\alpha(x, y)$$

for all $x \in A$, $y \in B$ and $k \in [0, \frac{1}{s})$.

Remark Cyclic contraction is continuous in a G_{bdq}-family.

Theorem 3.2 *Let A and B be non-empty closed subsets of a complete G_{bdq}-family (X, d_α). Let T be a cyclic mapping that satisfies the condition of a G_{bdq}*-cyclic-Banach contraction. *Then T has a unique fixed point in $A \cap B$.*

Proof Let $x \in A$ (fix) and using the contractive condition of the theorem, we have

$$d_\alpha\left(T^2x, Tx\right) = d_\alpha\left(T(Tx), Tx\right) \leq kd_\alpha(Tx, x) \quad \text{and}$$
$$d_\alpha\left(Tx, T^2x\right) = d_\alpha\left(Tx, T(Tx)\right) \leq kd_\alpha(x, Tx).$$

So,

$$d_\alpha\left(T^2x, Tx\right) \leq k\delta \tag{1}$$

and

$$d_\alpha\left(Tx, T^2x\right) \leq k\delta, \tag{2}$$

where $\delta = \max\{d_\alpha(Tx, x), d_\alpha(x, Tx)\}$.

From (1) and (2), we have $d_\alpha(T^3x, T^2x) \leq k^2\delta$ and $d_\alpha(T^2x, T^3x) \leq k^2\delta$.

For all $n \in \mathbb{N}$, we get

$$d_\alpha\left(T^{n+1}x, T^nx\right) \leq k^n\delta$$

and

$$d_\alpha\left(T^n x, T^{n+1}x\right) \leq k^n \delta.$$

Let $n, m \in \mathbb{N}$ with $m > n$, by using the definition of G_{bdq}-family, we have

$$\begin{aligned}
&d_\alpha\left(T^m x, T^n x\right) \\
&\quad \leq s\left[d_\beta\left(T^m x, T^{m-1}x\right) + d_\beta\left(T^{m-1}x, T^n x\right)\right] \\
&\quad \leq sd_\beta\left(T^m x, T^{m-1}x\right) + sd_\beta\left(T^{m-1}x, T^n x\right) \\
&\quad \leq sd_\beta\left(T^m x, T^{m-1}x\right) + s^2 d_\beta\left(T^{m-1}x, T^{m-2}x\right) + s^2 d_\beta\left(T^{m-2}x, T^n x\right) \\
&\quad \leq sd_\beta\left(T^m x, T^{m-1}x\right) + s^2 d_\beta\left(T^{m-1}x, T^{m-2}x\right) + s^3 d_\beta\left(T^{m-2}x, T^{m-3}x\right) \\
&\qquad + s^3 d_\beta\left(T^{m-3}x, T^n x\right).
\end{aligned}$$

By repeating this process, we get

$$\begin{aligned}
d_\alpha\left(T^m x, T^n x\right) &\leq sd_\beta\left(T^m x, T^{m-1}x\right) + s^2 d_\beta\left(T^{m-1}x, T^{m-2}x\right) + s^3 d_\beta\left(T^{m-2}x, T^{m-3}x\right) + \cdots \\
&\vdots \\
&\leq sk^{m-1}\delta + s^2 k^{m-2}\delta + s^3 k^{m-3}\delta + \cdots \\
&= sk^{m-1}\delta\left(1 + \frac{s}{k} + \frac{s^2}{k^2} + \cdots\right) \\
&\leq \left(\frac{sk^{m-1}}{1 - \frac{s}{k}}\right)\delta.
\end{aligned}$$

Taking the limits as $m \to \infty$, we get $d_\alpha(T^m x, T^n x) \to 0$ for all $\alpha \in [0,1)$ as $sk < 1$.

Similarly, let $n, m \in \mathbb{N}$ with $m < n$, we have

$$\begin{aligned}
d_\alpha\left(T^n x, T^m x\right) &\leq sd_\beta\left(T^n x, T^{n+1}x\right) + s^2 d_\beta\left(T^{n+1}x, T^{n+2}x\right) + s^3 d_\beta\left(T^{n+2}x, T^{n+3}x\right) + \cdots \\
&\leq sk^n\delta + s^2 k^{n+1}\delta + s^3 k^{n+2}\delta + \cdots \\
&= sk^n\delta\left(1 + sk + (sk)^2 + \cdots\right) \\
&\leq \left(\frac{sk^n}{1 - sk}\right)\delta.
\end{aligned}$$

Letting $n \to \infty$, we get $d_\alpha(T^n x, T^m x) \to 0$ for all $\alpha \in [0,1)$ as $sk < 1$.

Thus $\{T^n x\}$ is a G_{bdq}-Cauchy sequence.

Since (X, d_α) is G_{bdq}-complete, we see that $\{T^n x\}$ G_{bdq}-converges to some $u \in X$ for all $\alpha \in (0,1]$.

We note that $\{T^{2n}x\}$ is a sequence in A and $\{T^{2n-1}x\}$ is a sequence in B in such a way that both sequences tend to the same limit u.

Since A and B are closed, we have $u \in A \cap B$, and then $A \cap B \neq \emptyset$.

Now, we will show that $Tu = u$.

Consider

$$d_\alpha\left(T^n x, Tu\right) \leq s\left[d_\beta\left(T^n x, T^{n+1}x\right) + d_\beta\left(T^{n+1}x, Tu\right)\right]. \tag{3}$$

Since T is continuous, $\lim_{n\to\infty} d_\beta(T^{n+1}x, Tu) = 0$ and $d_\beta(T^n x, T^{n+1}x) \le k^n\delta$.

This implies that $sd_\beta(T^n x, T^{n+1}x) \le sk^n\delta \le (sk)^n\delta$.

Letting $n \to \infty$ in the above inequality (3), we get

$$d_\alpha(u, Tu) = 0, \tag{4}$$

since $sk < 1$. Similarly,

$$d_\alpha(Tu, u) = 0. \tag{5}$$

From (4) and (5), it follows that $u = Tu$.

Hence u is a fixed point of T.

Finally, to obtain the uniqueness of a fixed point, let $v \in X$ be another fixed point of T such that $Tv = v$.

Then we have

$$d_\alpha(u, v) = d_\alpha(Tu, Tv) \le kd_\alpha(u, v). \tag{6}$$

Similarly,

$$d_\alpha(v, u) \le kd_\alpha(v, u). \tag{7}$$

From (6) and (7), we obtain $d_\alpha(v, u) = d_\alpha(u, v) = 0$, this implies that $u = v$.

Thus u is a unique fixed point of T. This completes the proof. □

If we take $s = 1$ in the above theorem, we obtain the following theorem in generating space of dislocated quasi-metric family.

Theorem 3.3 *Let A and B be nonempty closed subsets of a complete G_{dq}-family (X, d_α). Let T be a cyclic mapping that satisfies the condition of a G_{dq}*-cyclic-Banach contraction. *Then T has a unique fixed point in $A \cap B$.*

If we take $d_\alpha = d$ in Theorem 3.2, we obtain the following corollary in b-dislocated quasi-metric space.

Corollary 3.4 *Let A and B be non-empty closed subsets of a complete b-dislocated quasi-metric space (X, d). Let $T : A \cup B \to A \cup B$ be a cyclic mapping that satisfies the condition $d(Tx, Ty) \le kd(x, y)$, where $k \in [0, \frac{1}{s})$ for all $x \in A$, $y \in B$ and $s \ge 1$ then T has a unique fixed point in $A \cap B$.*

Example 3.5 Let $X = \{-1, 0, 1\}$ and $T : A \cup B \to A \cup B$ defined by $Tx = -x$. Suppose that $A = \{-1, 0\}$, $B = \{0, 1\}$. Define the function $d : X \times X \to [0, \infty)$ by $d(0, 0) = 0$, $d(0, 1) = 2$, $d(0, -1) = 0$, $d(1, 1) = 2$, $d(1, 0) = 0$, $d(1, -1) = 0$, $d(-1, -1) = 1$, $d(-1, 1) = 1$ and $d(-1, 0) = 2$.

Clearly (X, d) is a complete b-dislocated quasi-metric with the coefficient $s = 2$ but not a dislocated quasi-metric, since the triangle inequality does not hold, that is, $d(-1, 0) \nleq d(-1, 1) + d(1, 0)$.

Now let $x \in A$ then $Tx \in B$. Further let $x \in B$ then $Tx \in A$. Hence the map T is cyclic on X because $T(A) \subset B$ and $T(B) \subset A$.

Now, we have the following cases:

- *Case* 1 For $x = 0 \in A$, $y = 0 \in B$.
  Then $d(Tx, Ty) = d(T0, T0) = d(0,0) = 0$; $kd(x,y) = kd(0,0) = 0$.
  Thus $d(Tx, Ty) \leq kd(x,y)$.
- *Case* 2 For $x = 0 \in A$, $y = 1 \in B$.
  Then $d(Tx, Ty) = d(T0, T1) = d(0,-1) = 0$; $kd(x,y) = kd(0,1) = 2k$.
  Thus $d(Tx, Ty) \leq kd(x,y)$, for $k \in [0, \frac{1}{2})$.
- *Case* 3 For $x = -1 \in A$, $y = 0 \in B$.
  Then $d(Tx, Ty) = d(T(-1), T0) = d(1,0) = 0$; $kd(x,y) = kd(-1,0) = 2k$.
  Thus $d(Tx, Ty) \leq kd(x,y)$, for $k \in [0, \frac{1}{2})$.
- *Case* 4 For $x = -1 \in A$, $y = 1 \in B$.
  Then $d(Tx, Ty) = d(T(-1), T1) = d(1,-1) = 0$; $kd(x,y) = kd(-1,1) = 2k$.
  Thus $d(Tx, Ty) \leq kd(x,y)$, for $k \in [0, \frac{1}{2})$.

Thus for all $x \in A$, $y \in B$ and $k \in [0, \frac{1}{2})$, T satisfies all the conditions of the above corollary. Hence T has a unique fixed point in $A \cap B$. In fact $0 \in A \cap B$ is the unique fixed point.

If we take the parameter $s = 1$ and $d_\alpha = d$ in Theorem 3.2, we obtain the following corollary in dislocated quasi-metric space.

Corollary 3.6 *Let A and B be non-empty closed subsets of a complete dislocated quasi-metric space (X,d). Let $T : A \cup B \to A \cup B$ be a cyclic mapping that satisfies the condition $d(Tx, Ty) \leq kd(x,y)$, where $k \in [0,1)$ for all $x \in A$, $y \in B$. Then T has a unique fixed point in $A \cap B$.*

We now give an example to illustrate the above corollary.

Example 3.7 Let $X = [-1,1]$ and $T : X \to X$ defined by $Tx = -\frac{x}{2}$. Suppose that $A = [-1,0]$ and $B = [0,1]$. Define the function $d : X \times X \to [0,\infty)$ by $d(x,y) = |x|$.

Hence d is complete dislocated quasi-metric on X. Further $T(A) \subset B$ and $T(B) \subset A$.

Thus T is cyclic on X.

Now consider

$$\begin{aligned} d(Tx, Ty) &= d\left(-\frac{x}{2}, -\frac{y}{2}\right) \\ &= \left|-\frac{x}{2}\right| \\ &= \frac{x}{2} \\ &\leq kd(x,y) \quad \text{for } \frac{1}{2} \leq k < 1. \end{aligned} \tag{8}$$

Thus T satisfies all the conditions of above corollary and '0' is a unique fixed point.

Example 3.8 Let $X = [-1,1]$ and $T : X \to X$ defined by $Tx = \frac{-x}{2}$. Suppose that $A = [-1,0]$ and $B = [0,1]$. Define the function $d : X \times X \to [0,\infty)$ by $d(x,y) = |x| + y^2$;

$$\begin{aligned} d(Tx, Ty) &= d\left(-\frac{x}{2}, -\frac{y}{2}\right) \\ &= \left|-\frac{x}{2}\right| + \frac{y^2}{4} \\ &= \frac{x}{2} + \frac{y^2}{4} \\ &= \frac{1}{2}\left(|x| + \frac{y^2}{2}\right) \\ &\le \frac{1}{2}\left(|x| + \frac{y^2}{2}\right) \\ &\le kd(x,y), \quad \text{where } \frac{1}{2} \le k < 1. \end{aligned} \tag{9}$$

Thus T satisfies all the conditions of above corollary and '0' is a unique fixed point of T.

Definition 3.9 ([2]) Let (X,d) be a complete metric space. $T : X \to X$ is called a Kannan contraction if there exists a number t, $0 < t < \frac{1}{2}$, such that, for each $x, y \in X$,

$$d(Tx, Ty) \le t\big[d(x, Tx) + d(y, Ty)\big].$$

Inspired by the above definition, we introduce the G_{bd}*-cyclic-Kannan contraction* in the setting of generating space of a b-dislocated metric family.

Definition 3.10 Let A and B be non-empty subsets of a G_{bd}-family (X, d_α). A cyclic map $T : A \cup B \to A \cup B$ is said to be a G_{bd}*-cyclic-Kannan contraction* if there exists $t \in [0, \frac{1}{4s}]$ where $s \ge 1$ and for any $\alpha \in (0,1]$ such that

$$d_\alpha(Tx, Ty) \le t\big[d_\alpha(x, Tx) + d_\alpha(y, Ty)\big] \tag{10}$$

for all $x \in A$, $y \in B$ and $st < \frac{1}{4}$.

Now we state and prove the fixed point theorem for a *cyclic-Kannan contraction* in a generating space of b-dislocated metric family.

Theorem 3.11 *Let A and B be non-empty closed subsets of a complete generating space of b-dislocated metric family (X, d_α). Let T be a continuous cyclic mapping that satisfies the condition of a G_{bd}*-cyclic-Kannan contraction. *Then T has a unique fixed point in $A \cap B$.*

Proof Let $x \in A$ (fix) and, using the definition of a G_{bd}*-cyclic-Kannan contraction*, we have

$$\begin{aligned} d_\alpha\big(Tx, T^2x\big) &= d_\alpha\big(Tx, T(Tx)\big) \\ &\le td_\alpha(x, Tx) + td_\alpha\big(Tx, T^2x\big), \end{aligned}$$

so

$$d_\alpha\big(Tx, T^2x\big) \le \left(\frac{t}{1-t}\right) d_\alpha(x, Tx). \tag{11}$$

By using (10), we have

$$\begin{aligned}
d_\alpha\big(T^2x, T^3x\big) &\le d_\alpha\big(T(Tx), T\big(T^2x\big)\big) \\
&\le t\big[d_\alpha\big(Tx, T^2x\big) + d_\alpha\big(T^2x, T^3x\big)\big] \\
&\le \left(\frac{t}{1-t}\right) d_\alpha\big(Tx, T^2x\big) \\
&\le \left(\frac{t}{1-t}\right)^2 d_\alpha(x, Tx).
\end{aligned}$$

For all $n \in \mathbb{N}$, we have

$$d_\alpha\big(T^nx, T^{n+1}x\big) \le \left(\frac{t}{1-t}\right)^n d_\alpha(x, Tx) \quad \text{for all } \alpha \in [0,1).$$

Let $n, m \in \mathbb{N}$ with $m < n$, by using the definition of G_{bd}-family, we get

$$\begin{aligned}
&d_\alpha\big(T^mx, T^nx\big) \\
&\quad \le s\big[d_\alpha\big(T^mx, T^{m+1}x\big) + d_\alpha\big(T^{m+1}x, T^nx\big)\big] \\
&\quad \le sd_\alpha\big(T^mx, T^{m+1}x\big) + s^2 d_\alpha\big(T^{m+1}x, T^{m+2}x\big) + s^3 d_\alpha\big(T^{m+2}x, T^{m+3}x\big) + \cdots \\
&\quad \le s\left(\frac{t}{1-t}\right)^m d_\alpha(x, Tx) + s^2\left(\frac{t}{1-t}\right)^{m+1} d_\alpha(x, Tx) + s^3\left(\frac{t}{1-t}\right)^{m+2} d_\alpha(x, Tx) + \cdots \\
&\quad = s\left(\frac{t}{1-t}\right)^m d_\alpha(x, Tx)\left(1 + s\left(\frac{t}{1-t}\right) + s^2\left(\frac{t}{1-t}\right)^2 + \cdots\right) \\
&\quad \le \left(\frac{st^m}{1-t}\right) d_\alpha(x, Tx)\left(\frac{1}{1-\frac{st}{1-t}}\right) \le \left(\frac{st^m}{1-t-st}\right) d_\alpha(x, Tx).
\end{aligned}$$

Since $st < 1$, $d_\alpha(T^mx, T^nx) \to 0$ as $m, n \to \infty$.

Hence $\{T^nx\}$ is a G_{bd}-Cauchy sequence.

Since (X, d_α) is complete generating space of b-dislocated metric family, $\{T^nx\}$ G_{bd}-converges to some $u \in X$, i.e. $x_n \to u$.

We note that $\{T^{2n}x\}$ is a sequence in A and $\{T^{2n-1}x\}$ is a sequence in B in a way that both sequences tend to the same limit u.

Since A and B are closed, we have $u \in A \cap B$, and then $A \cap B \neq \emptyset$.

Now, we prove that $Tu = u$.

Consider $Tu = T(\lim_{n\to\infty} x_n) = \lim_{n\to\infty} T(x_n) = \lim_{n\to\infty} x_{n+1} = u$.

Now we prove that u is the unique fixed point.

Suppose u^* is another fixed point of T such that $Tu^* = u^*$.

Then from the contractive condition, we get

$$d_\alpha\big(u, u^*\big) = d_\alpha\big(Tu, Tu^*\big)$$

$$\begin{aligned}
&\le t\big[d_\alpha(u,Tu)+d_\alpha\big(u^*,Tu^*\big)\big]\\
&\le t\big[d_\alpha(u,u)+d_\alpha\big(u^*,u^*\big)\big]\\
&\le t\big[s\big[d_\beta\big(u,u^*\big)+d_\beta\big(u^*,u\big)\big]+s\big[d_\beta\big(u^*,u\big)+d_\beta\big(u,u^*\big)\big]\big]\\
&\le 4tsd_\alpha\big(u,u^*\big),
\end{aligned}$$

which yields $(1-4ts)d_\alpha(u,u^*)\le 0$.

Since $1-4ts>0$, $d_\alpha(u,u^*)=0$.

Thus $u=u^*$. This completes the proof. □

If we take $s=1$ in the above theorem, we obtain the following theorem in generating space of dislocated metric family.

Theorem 3.12 *Let A and B be nonempty closed subsets of a complete generating space of dislocated metric family (X,d_α). Let T be a continuous cyclic mapping that satisfies the condition of a G_d*-cyclic-Kannan contraction. *Then T has a unique fixed point in $A\cap B$.*

If we take $d_\alpha=d$ in Theorem 3.11, we obtain the following corollary in b-dislocated metric space.

Corollary 3.13 *Let A and B be non-empty closed subsets of a complete b-dislocated metric space (X,d). Let T be a continuous cyclic mapping that satisfies the condition*

$$d(Tx,Ty)\le t\big[d(x,Tx)+d(y,Ty)\big],$$

where $t\in(0,\frac{1}{4s})$. Then T has a unique fixed point in $A\cap B$.

If we take the parameter $s=1$ and $d_\alpha=d$ in Theorem 3.11, we obtain the following corollary in complete dislocated metric space.

Corollary 3.14 *Let A and B be non-empty closed subsets of a complete dislocated metric space (X,d). Let T be a continuous cyclic mapping that satisfies the condition*

$$d(Tx,Ty)\le t\big[d(x,Tx)+d(y,Ty)\big],$$

where $t\in(0,\frac{1}{4})$. Then T has a unique fixed point in $A\cap B$.

Example 3.15 Let $X=[0,1]$ and $T:A\cup B\to A\cup B$ defined by $Tx=\frac{x}{9}$. Suppose that $A,B=[0,1]$. Define the function $d:X\times X\to[0,\infty)$ by $d(x,y)=\max\{x,y\}$.

Clearly d is a complete dislocated metric space.

- *Case* (i) If $x>y$.

$$\begin{aligned}
d(Tx,Ty)&=d\left(\frac{x}{9},\frac{y}{9}\right)=\max\left\{\frac{x}{9},\frac{y}{9}\right\}\\
&=\frac{x}{9}=\frac{1}{9}(x)\\
&\le\frac{1}{9}(x+y)
\end{aligned}$$

$$= \frac{1}{9}\left(\max\left\{x, \frac{x}{9}\right\} + \max\left\{y, \frac{y}{9}\right\}\right)$$
$$= \frac{1}{9}\left(d\left(x, \frac{x}{9}\right) + d\left(y, \frac{y}{9}\right)\right)$$
$$\leq t\left[d(x, Tx) + d(y, Ty)\right]$$

so $\frac{1}{9} \leq t < \frac{1}{4}$.

- *Case* (ii) If $x < y$, similar proof as above. Hence omitted.

Thus T satisfies all the conditions of the above corollary.

Hence T has a unique fixed point. In fact 0 is the unique fixed point of T.

In 1974, Ciric [3] introduced the concept of a quasi-contraction and extended some results concerning generalized contractions of [43] and [44] to quasi-contractions. He proved fixed point theorems for single-valued quasi-contractions and provided an illustrative example for his extensions. Some generalizations on quasi-contractions can be found in [45–47].

Definition 3.16 ([5]) Let (X, d) be a complete metric space. A map $T : X \to X$ such that there exists a constant h, $0 \leq h < 1$, and for each $x, y \in X$,

$$d(Tx, Ty) \leq h \max\left\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\right\}$$

is called a Ciric quasi-contraction.

Based on above definition, we introduce the following definition in the setting of generating space of b-quasi-metric family.

Definition 3.17 Let A and B be non-empty subsets of a G_{bq}-family (X, d_α). A cyclic map $T : A \cup B \to A \cup B$ is said to be a *G_{bq}-cyclic-Ciric contraction* if there exists $h \in [0, \frac{s}{2})$ where $s \geq 1$ and for any $\alpha \in (0, 1]$ such that

$$s^2 d_\alpha(Tx, Ty) \leq h \max\left\{d_\alpha(x, y), d_\alpha(x, Tx), d_\alpha(y, Ty), d_\alpha(x, Ty), d_\alpha(y, Tx)\right\}$$

for all $x \in A$, $y \in B$.

Now, we derive the existence of fixed point theorems for *G_{bq}-cyclic-Ciric contraction* in G_{bq}-family which is larger than the generating space of a quasi-metric family and metric space.

Theorem 3.18 *Let A and B be non-empty closed subsets of a complete G_{bq}-family (X, d_α). Let T be a continuous cyclic mapping that satisfies the condition of a G_{bq}*-cyclic-Ciric contraction. *Then T has a unique fixed point in $A \cap B$.*

Proof Let $x \in A$(Fix).

By using *G_{bq}-cyclic-Ciric contraction*, we have

$$s^2 d_\alpha(Tx, Ty) \leq h \max\left\{d_\alpha(x, y), d_\alpha(x, Tx), d_\alpha(y, Ty), d_\alpha(x, Ty), d_\alpha(y, Tx)\right\}.$$

Consider

$$\begin{aligned} & s^2 d_\alpha\left(Tx, T^2x\right) \\ & \quad \le h \max\left\{d_\alpha(x, Tx), d_\alpha(x, Tx), d_\alpha\left(Tx, T^2x\right), d_\alpha\left(x, T^2x\right), d_\alpha(Tx, Tx)\right\} \\ & \quad \le h \max\left\{d_\alpha(x, Tx), d_\alpha\left(Tx, T^2x\right), s\left[d_\beta(x, Tx) + d_\beta\left(Tx, T^2x\right)\right]\right\} \\ & \quad \le hsd_\beta(x, Tx) + hsd_\beta\left(Tx, T^2x\right), \end{aligned}$$

which yields

$$d_\alpha\left(Tx, T^2x\right) \le \frac{h}{s-h} d_\alpha(x, Tx).$$

Thus

$$d_\alpha\left(Tx, T^2x\right) \le kd_\alpha(x, Tx),$$

where $k = \frac{h}{s-h}$, which implies $0 \le k < 1$.

Choose $sk \le 1$. Repeating the same process for all $n \in \mathbb{N}$, we get

$$d_\alpha\left(T^n x, T^{n+1}x\right) \le k^n d_\alpha(x, Tx).$$

Now we shall show that $\{T^n x\}$ is a G_{bq}-Cauchy sequence.

Let $n, m \in \mathbb{N}$ with $m > n$, we have

$$\begin{aligned} & d_\alpha\left(T^n x, T^m x\right) \\ & \quad \le s\left[d_\beta\left(T^n x, T^{n+1}x\right) + d_\beta\left(T^{n+1}x, T^m x\right)\right] \\ & \quad \le sd_\beta\left(T^n x, T^{n+1}x\right) + s^2 d_\beta\left(T^{n+1}x, T^{n+2}x\right) + s^3 d_\beta\left(T^{n+2}x, T^{n+3}x\right) + \cdots \\ & \quad \vdots \\ & \quad \le sk^n d_\beta(x, Tx) + s^2 k^{n+1} d_\beta(x, Tx) + s^3 k^{n+2} d_\beta(x, Tx) + \cdots \\ & \quad \le sk^n\left[1 + sk + (sk)^2 + \cdots\right] d_\alpha(x, Tx) \\ & \quad \le \frac{sk^n}{1 - sk} d_\alpha(x, Tx). \end{aligned}$$

Letting $n \to \infty$, we get $d_\alpha(T^n x, T^m x) \to 0$ for all $\alpha \in (0, 1]$ as $sk \le 1$.

Hence $\{T^n x\}$ is a G_{bq}-Cauchy sequence.

Since X is G_{bq}-complete, the sequence $\{T^n x\}$ converges to some point $z \in X$. We note that $\{T^{2n}x\}$ is a sequence in A and $\{T^{2n-1}x\}$ is a sequence in B in such a way that both sequences tend to the same limit z.

Since A and B are closed, we have $z \in A \cap B$, and then $A \cap B \neq \emptyset$.

Now, we will show that $Tz = z$.

Consider

$$d_\alpha(z, Tz) \le s\left[d_\beta\left(z, T^{n+1}x\right) + d_\beta\left(T^{n+1}x, Tz\right)\right].$$

Letting $n \to \infty$, we get $d_\alpha(z, Tz) = 0$.

This implies $z = Tz$.

Now we prove that z is the unique fixed point.

In order to do this, let $z^* \in X$ be another fixed point of T such that $Tz^* = z^*$.

Then we have

$$\begin{aligned} s^2 d_\alpha(z, z^*) &= s^2 d_\alpha(Tz, Tz^*) \\ &\leq h \max\{d_\alpha(z, z^*), d_\alpha(z, Tz), d_\alpha(z^*, Tz^*), d_\alpha(z, Tz^*), d_\alpha(z^*, Tz)\}, \end{aligned}$$

which implies that

$$d_\alpha(z, z^*) \leq \left(\frac{h}{s^2}\right) d_\alpha(z, z^*)$$

since $\frac{h}{s^2} < 1$, $d_\alpha(z, z^*) = 0$. Thus $z = z^*$. This completes the proof. □

If we take $s = 1$ in the above theorem, we obtain the following theorem in generating space of quasi-metric family.

Theorem 3.19 *Let A and B be nonempty closed subsets of a complete generating space of quasi-metric family (X, d_α). Let T be a continuous cyclic mapping that satisfies the condition of a G_q*-cyclic-Kannan contraction. *Then T has a unique fixed point in $A \cap B$.*

If we take $d_\alpha = d$ in Theorem 3.18, we obtain the following corollary in b-metric space.

Corollary 3.20 *Let A and B be non-empty closed subsets of a complete b-metric space. Let $T : A \cup B \to A \cup B$ be a continuous cyclic mapping that satisfies the condition $s^2 d(Tx, Ty) \leq h \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$, where $h \in [0, \frac{s}{2})$ for all $x \in A$, $y \in B$ and $s \geq 1$. Then T has a unique fixed point in $A \cap B$.*

Example 3.21 Let $X = [0, 1]$ and $T : A \cup B \to A \cup B$ defined by $Tx = \frac{x}{6}$. Suppose that $A, B = [0, 1]$. Define the function $d : X \times X \to [0, \infty)$ by $d(x, y) = |x - y|^2$ and $s = 2$.

Clearly d is complete b-metric on X. Further $T(A) \subset B$ and $T(B) \subset A$. Hence T is continuous cyclic map on X. Now, we consider

$$\begin{aligned} s^2 d(Tx, Ty) &= 4d\left(\frac{x}{6}, \frac{y}{6}\right) \\ &= 4\left|\frac{x-y}{6}\right|^2 \\ &= \frac{4}{36}|x - y|^2 \\ &= \frac{1}{9} d(x, y) \\ &\leq h \max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}, \end{aligned}$$

where $\frac{1}{9} \leq h < 1$. Thus T satisfies all the conditions of the above corollary and 0 is the unique fixed point in $A \cap B$.

If we take the parameter $s = 1$ and $d_\alpha = d$ in Theorem 3.18, we obtain the following corollary in a complete metric space.

Corollary 3.22 *Let A and B be non-empty closed subsets of a complete metric space. Let $T : A \cup B \to A \cup B$ be a continuous cyclic mapping that satisfies the condition $d(Tx, Ty) \leq h\max\{d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\}$, where $h \in [0, \frac{1}{2})$ for all $x \in A$, $y \in B$ and $s \geq 1$. Then T has a unique fixed point in $A \cap B$.*

Example 3.23 Let $X = [-1,1]$ and $T : A \cup B \to A \cup B$ defined by $Tx = -\frac{x}{5}$. Suppose that $A = [-1,0]$ and $B = [0,1]$. Define the function $d : X \times X \to [0,\infty)$ by $d(x,y) = |x-y|$.

Clearly d is complete metric on X. Further $T(A) \subset B$ and $T(B) \subset A$. Hence T is continuous cyclic map on X. Now, we consider

$$\begin{aligned} d(Tx,Ty) &= d\left(-\frac{x}{5}, -\frac{y}{5}\right) \\ &= \left|-\frac{x}{5} + \frac{y}{5}\right| \\ &= \left|\frac{x}{5} - \frac{y}{5}\right| \\ &= \frac{1}{5}|x-y| \\ &= \frac{1}{5}d(x,y) \\ &\leq h\max\bigl\{d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\bigr\}, \end{aligned}$$

where $\frac{1}{5} \leq h < 1$. Thus T satisfies all the conditions of the above corollary and 0 is the unique fixed point in $A \cap B$.

Competing interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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