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Fixed point results for generalized F-contractions in modular metric and fuzzy metric spaces

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Abstract

The notion of modular metric space, being a natural generalization of classical modulars over linear spaces, was recently introduced. In this paper, we introduce a generalized F-contraction in modular metric space and investigate the existence of fixed points for such contractions. As applications, we derive some new fixed point theorems in partially ordered modular metric spaces, Suzuki type fixed point theorems in modular metric spaces and fixed point theorems for contractions involving integral inequalities. Moreover, we deduce new fixed point results in triangular fuzzy metric spaces and provide some examples to illustrate the usability of the obtained results.

Keywords: modular metric space; fixed point; α-GF-contraction; α-η-continuous function; orbitally continuous function

1 Introduction and preliminaries

In 1950 [1], Nakano initiated the theory of modular spaces and further it was generalized and redefined by Musielak and Orlicz [2, 3] in 1959. In 2010, Chistyakov introduced the notion of modular metric spaces in [4, 5]. The main idea behind this new concept is the physical interpretation of the modular. A metric on a set represents nonnegative finite distances between any two points of the set, a modular on a set attributes a nonnegative (possibly, infinite valued) 'field of (generalized) velocities': to each time $\lambda > 0$ the absolute value of an average velocity $\omega_\lambda(x, y)$ is associated in such a way that in order to cover the distance between points $x, y \in X$ it takes time λ to move from x to y with velocity $\omega_\lambda(x, y)$. In recent years, there was a strong interest to study the fixed point property in modular function spaces after the first paper [6] was published in 1990. Many mathematicians studied fixed point results for modular metric spaces; see [7–12]. Recently, Wardowski [13] introduced and studied a new contraction, called F-contraction, to prove a fixed point result as a generalization of the Banach contraction principle. Hussain and Salimi [14] further generalized the concept of F-contraction and introduced an α-GF-contraction with respect to a general family of functions G (see also [15–17] and references therein). In this paper, we obtain fixed point results for α-η-GF-contractions in modular metric and partially ordered modular metric spaces. As an application of our results we deduce Suzuki type results for GF-contractions. We derive fixed and periodic point results for orbitally

continuous generalized F-contractions in modular metric spaces. In the last section, we prove new fixed point results in triangular fuzzy metric spaces.

We begin with some basic definitions and results which will be used in the sequel. Throughout this article, $\mathbb{N}$, $\mathbb{R}+$, $\mathbb{R}$ denote the set of all natural numbers, the set of all positive real numbers, and the set of all real numbers, respectively.

Let X be a nonempty set. Throughout this paper for a function $\omega : (0,\infty) \times X \times X \to [0,\infty]$, we will write

$$\omega_\lambda(x,y) = \omega(\lambda,x,y), \tag{1.1}$$

for all $\lambda > 0$ and $x,y \in X$.

Definition 1.1 A function $\omega : (0,\infty) \times X \times X \to [0,\infty]$ is said to be modular metric on X if it satisfies the following axioms:

(1) $x = y$ if and only if $\omega_\lambda(x,y) = 0$, for all $\lambda > 0$;

(2) $\omega_\lambda(x,y) = \omega_\lambda(y,x)$, for all $\lambda > 0$ and $x,y \in X$;

(3) $\omega_\lambda(x,y) = \omega_\lambda(x,z) + \omega_\lambda(z,y)$, for all $\lambda > 0$ and $x,y,z \in X$.

Definition 1.2 [18] Let T be a self-mapping on X and $\alpha : X \times X \to [0,+\infty)$ be a function. We say that T is an α-admissible mapping if

$$x,y \in X, \quad \alpha(x,y) \geq 1 \quad \Longrightarrow \quad \alpha(Tx,Ty) \geq 1.$$

Definition 1.3 [19] Let T be a self-mapping on X and $\alpha,\eta : X \times X \to [0,+\infty)$ be two functions. We say that T is an α-admissible mapping with respect to η if

$$x,y \in X, \quad \alpha(x,y) \geq \eta(x,y) \quad \Longrightarrow \quad \alpha(Tx,Ty) \geq \eta(Tx,Ty).$$

Note that if we take $\eta(x,y) = 1$ then this definition reduces to Definition 1.2. Also, if we take $\alpha(x,y) = 1$ then we say that T is an η-subadmissible mapping.

Definition 1.4 [20] Let (X,d) be a metric space. Let $\alpha,\eta : X \times X \to [0,\infty)$ be functions. We say $T : X \to X$ is an α-η-continuous mapping on (X,d), if, for given $x \in X$ and sequence $\{x_n\}$ with

$$x_n \to x \quad \text{as } n \to \infty, \qquad \alpha(x_n,x_{n+1}) \geq \eta(x_n,x_{n+1}) \quad \text{for all } n \in \mathbb{N} \quad \Longrightarrow \quad Tx_n \to Tx.$$

A mapping $T : X \to X$ is called orbitally continuous at $p \in X$ if $\lim_{n\to\infty} T^n x = p$ implies that $\lim_{n\to\infty} TT^n x = Tp$. The mapping T is orbitally continuous on X if T is orbitally continuous for all $p \in X$. A metric space X is called T-orbitally complete if every Cauchy sequence of the form $\{T^{n_i}x\}_{i=1}^{\infty}$, $x \in X$, converges in X.

Remark 1.1 [20] Let $T : X \to X$ be a self-mapping on an T-orbitally complete metric space X. Define, $\alpha,\eta : X \times X \to [0,+\infty)$ by

$$\alpha(x,y) = \begin{cases} 3, & \text{if } x,y \in O(w), \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \eta(x,y) = 1,$$

where $O(w) = \{T^k w : k \geq 0\}$ is an orbit of a point $w \in X$. If $T : X \to X$ is an orbitally continuous map on (X, d), then T is α-η-continuous on (X, d).

2 Fixed point results for α-*GF*-contractions

Consistent with Wardowski [13], we denote by Δ_F the set of all functions $F : \mathbb{R}^+ \to \mathbb{R}$ satisfying the following conditions:

(F$_1$) F is strictly increasing;
(F$_2$) for all sequence $\{\alpha_n\} \subseteq \mathbb{R}^+$, $\lim_{n \to \infty} \alpha_n = 0$ if and only if $\lim_{n \to \infty} F(\alpha_n) = -\infty$;
(F$_3$) there exist $0 < k < 1$ such that $\lim_{n \to 0^+} \alpha^k F(\alpha) = 0$.

Now we introduce the following family of new functions (see also [21]).
Let Δ_G denotes the set of all functions $G : \mathbb{R}^{+^4} \to \mathbb{R}^+$ satisfying:
(G) for all $t_1, t_t, t_3, t_4 \in \mathbb{R}^+$ with $t_1 t_2 t_3 t_4 = 0$ there exists $\tau > 0$ such that $G(t_1, t_2, t_3, t_4) = \tau$.

Example 2.1 If $G(t_1, t_2, t_3, t_4) = L \min\{t_1, t_2, t_3, t_4\} + \tau$ where $L \in \mathbb{R}^+$ and $\tau > 0$, then $G \in \Delta_G$.

Example 2.2 If $G(t_1, t_2, t_3, t_4) = \tau e^{L \min\{t_1, t_2, t_3, t_4\}}$ where $L \in \mathbb{R}^+$ and $\tau > 0$, then $G \in \Delta_G$.

Example 2.3 If $G(t_1, t_2, t_3, t_4) = L \ln(\min\{t_1, t_2, t_3, t_4\} + 1) + \tau$ where $L \in \mathbb{R}^+$ and $\tau > 0$, then $G \in \Delta_G$.

Definition 2.1 Let X_ω be a modular metric space and T be a self-mapping on X_ω. Suppose that $\alpha, \eta : X_\omega \times X_\omega \to [0, \infty)$ be two functions. We say T is an α-η-GF-contraction if for $x, y \in X_\omega$ with $\eta(x, Tx) \leq \alpha(x, y)$, $\omega_{\lambda/l}(Tx, Ty) > 0$, and $\lambda > 0$ we have

$$G\big(\omega_{\lambda/l}(x, Tx), \omega_{\lambda/l}(y, Ty), \omega_{\lambda/l}(x, Ty), \omega_{\lambda/l}(y, Tx)\big) + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big), \quad (2.1)$$

where $G \in \Delta_G$ and $F \in \Delta_F$.

Now we state and prove the main result of this section.

Theorem 2.1 *Let X_ω be a complete modular metric space. Let $T : X_\omega \to X_\omega$ be a self-mapping satisfying the following assertions*:
   (i) *T is an α-admissible mapping with respect to η;*
  (ii) *there exists $x_0 \in X_\omega$ such that $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$;*
 (iii) *T is an α-η-continuous function;*
 (iv) *T is an α-η-GF-contraction.*
Then T has a fixed point. Moreover, T has a unique fixed point when $\alpha(x, y) \geq \eta(x, x)$ for all $x, y \in \mathrm{Fix}(T)$.

Proof Let $x_0 \in X_\omega$ such that $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$. For $x_0 \in X_\omega$, we define the sequence $\{x_n\}$ by $x_n = T^n x_0 = Tx_{n-1}$. Now since T is an α-admissible mapping with respect to η, $\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0) = \eta(x_0, x_1)$. By continuing this process we have

$$\eta(x_{n-1}, Tx_{n-1}) = \eta(x_{n-1}, x_n) \leq \alpha(x_{n-1}, x_n)$$

for all $n \in \mathbb{N}$. Also, let there exists $n_0 \in X$ such that $x_{n_0} = x_{n_0+1}$. Then x_{n_0} is fixed point of T and we have nothing to prove. Hence, we assume, $x_n \neq x_{n+1}$ or $\omega_{\lambda/l}(Tx_{n-1}, Tx_n) > 0$ for all

$n \in \mathbb{N} \cup \{0\}$. Since T is an α-η-GF-contraction, by taking $x = x_{n-1}$ and $y = x_n$ in (2.1), we get

$$G\big(\omega_{\lambda/l}(x_{n-1}, Tx_{n-1}), \omega_{\lambda/l}(x_n, Tx_n), \omega_{\lambda/l}(x_{n-1}, Tx_n), \omega_{\lambda/l}(x_n, Tx_{n-1})\big) + F\big(\omega_{\lambda/l}(Tx_{n-1}, Tx_n)\big)$$
$$\leq F\big(\omega_{\lambda/l}(x_{n-1}, x_n)\big),$$

which implies

$$G\big(\omega_{\lambda/l}(x_{n-1}, x_n), \omega_{\lambda/l}(x_n, x_{n+1}), \omega_{\lambda/l}(x_{n-1}, x_{n+1}), \omega_{\lambda/l}(x_n, x_n)\big) + F\big(\omega_{\lambda/l}(x_n, x_{n+1})\big)$$
$$\leq F\big(\omega_{\lambda/l}(x_{n-1}, x_n)\big). \tag{2.2}$$

So,

$$G\big(\omega_{\lambda/l}(x_{n-1}, x_n), \omega_{\lambda/l}(x_n, x_{n+1}), \omega_{\lambda/l}(x_{n-1}, x_{n+1}), 0\big) + F\big(\omega_{\lambda/l}(x_n, x_{n+1})\big) \leq F\big(\omega_{\lambda/l}(x_{n-1}, x_n)\big).$$

Now since $\omega_{\lambda/l}(x_{n-1}, x_n).\omega_\lambda\lambda/l(x_n, x_{n+1}).\omega_{\lambda/l}(x_{n-1}, x_{n+1}).0 = 0$, so from (G) there exists $\tau > 0$ such that

$$G\big(\omega_{\lambda/l}(x_{n-1}, x_n), \omega_{\lambda/l}(x_n, x_{n+1}), \omega_{\lambda/l}(x_{n-1}, x_{n+1}), 0\big) = \tau.$$

From (2.2) we deduce that

$$F\big(\omega_{\lambda/l}(x_n, x_{n+1})\big) \leq F\big(\omega_{\lambda/l}(x_{n-1}, x_n)\big) - \tau.$$

Therefore,

$$F\big(\omega_{\lambda/l}(x_n, x_{n+1})\big) \leq F\big(\omega_{\lambda/l}(x_{n-1}, x_n)\big) - \tau \leq F\big(\omega_{\lambda/l}(x_{n-2}, x_{n-1})\big) - 2\tau \leq \cdots$$
$$\leq F\big(\omega_{\lambda/l}(x_0, x_1)\big) - n\tau.$$

So,

$$F\big(\omega_{\lambda/l}(x_n, x_{n+1})\big) \leq F\big(\omega_\lambda(x_0, x_1)\big) - n\tau. \tag{2.3}$$

By taking the limit as $n \to \infty$ in (2.3) we have $F(\omega_\lambda(x_n, x_{n+1})) \to -\infty$ and since $F \in \Delta_F$ we obtain

$$\lim_{n \to \infty} \omega_{\lambda/l}(x_n, x_{n+1}) = 0. \tag{2.4}$$

So, for each $\lambda > 0$, we see, for all $\epsilon > 0$, that there exists $n_0 \in \mathbb{N}$ such that

$$\omega_{\lambda/l}(x_n, x_{n+1}) < \epsilon,$$

for all $n \in \mathbb{N}$ with $n \geq n_0$. Without loss of generality, suppose $m, n \in \mathbb{N}$ and $m > n$. Observe that, for $\frac{\lambda}{l(m-n)} > 0$, there exists $n_{\frac{\lambda}{(m-n)}} \in \mathbb{N}$ such that

$$\omega_{\frac{\lambda}{l(m-n)}}(x_n, x_{n+1}) < \frac{\epsilon}{l(m-n)},$$

for all $n \geq n_{\frac{\lambda}{(m-n)}}$. Now, we have

$$\omega_{\lambda/l}(x_n, x_m) \leq \omega_{\frac{\lambda}{l(m-n)}}(x_n, x_{n+1}) + \omega_{\frac{\lambda}{l(m-n)}}(x_{n+1}, x_{n+2}) + \cdots + \omega_{\frac{\lambda}{l(m-n)}}(x_{m-1}, x_m)$$

$$< \frac{\epsilon}{l(m-n)} + \frac{\epsilon}{l(m-n)} + \cdots + \frac{\epsilon}{l(m-n)}$$

$$= \epsilon/l,$$

for all $m, n \geq n_{\frac{\lambda}{(m-n)}}$. This shows that $\{x_n\}$ is a Cauchy sequence. Since X_ω is complete, there exists $x^* \in X_\omega$ such that $x_n \to x^*$ as $n \to \infty$. Now since T is an α-η-continuous and $\eta(x_{n-1}, x_n) \leq \alpha(x_{n-1}, x_n)$, $x_{n+1} = Tx_n \to Tx^*$ as $n \to \infty$. Therefore,

$$Tx^* = \lim_{n \to \infty} Tx_n = \lim_{n \to \infty} x_{n+1} = x^*.$$

Thus T has a fixed point. Now let $x, y \in \text{Fix}(T)$ where $x \neq y$. Then

$$G\big(\omega_{\lambda/l}(x, Tx), \omega_{\lambda/l}(y, Ty), \omega_{\lambda/l}(x, Ty), \omega_{\lambda/l}(y, Tx)\big) + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big),$$

$$G\big(0, 0, \omega_{\lambda/l}(x, Ty), \omega_{\lambda/l}(y, Tx)\big) + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big),$$

so there exists $\tau > 0$ such that

$$G\big(0, 0, \omega_{\lambda/l}(x, Ty), \omega_{\lambda/l}(y, Tx)\big) + F\big(\omega_{\lambda/l}(Tx, Ty)\big) = \tau.$$

We get

$$\tau + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big),$$

which is a contradiction. Hence, $x = y$. Therefore, T has a unique fixed point. $\square$

Combining Theorem 2.1 and Example 2.1 we deduce the following corollary.

Corollary 2.1 *Let X_ω be a complete modular metric space and $T : X_\omega \to X_\omega$ be a self-mapping satisfying the following assertions*:

  (i) *T is an α-admissible mapping with respect to η*;

 (ii) *if for $x, y \in X$ with $\eta(x, Tx) \leq \alpha(x, y)$ and $\omega_{\lambda/l}(Tx, Ty) > 0$ we have*

$$\tau + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big), \tag{2.5}$$

  where $\tau > 0$ and $F \in \Delta_F$;

(iii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$*;

 (iv) *T is an α-η-continuous function.*

Then T has a fixed point. Moreover, T has a unique fixed point when $\alpha(x, y) \geq \eta(x, y)$ for all $x, y \in \text{Fix}(T)$.

Example 2.4 Let $X = [0, 1]$ and $\omega_\lambda(x, y) = (1/\lambda)|x - y|$. Define, $T : X_\omega \to X_\omega$, $\alpha, \eta : X_\omega \times X_\omega \to [0, \infty)$, $G : \mathbb{R}^{+^4} \to \mathbb{R}^+$ and $F : \mathbb{R}^+ \to \mathbb{R}$ by

$$Tx = \begin{cases} \frac{1}{2}e^{-\tau}x, & \text{if } x \text{ is rational,} \\ \frac{1}{3}e^{-\tau}x & \text{if } x \text{ is irrational,} \end{cases}$$

$\alpha(x,y) = x + y$ and $\eta(x,y) = \frac{x+y}{2}$, $G(t_1, t_2, t_3, t_4) = \tau$ where $\tau > 0$ and $F(r) = \ln r$.

Let $\alpha(x,y) \geq \eta(x,y)$, then $x, y \in [0,1]$. On the other hand, $Tw \in [0,1]$ for all $w \in [0,1]$. Then $\alpha(Tx, Ty) \geq \eta(Tx, Ty)$. That is, T is an α-admissible mapping with respect to η.

Case I: x and y both are rational.

Let $\alpha(x,y) \geq \eta(x, Tx)$, then

$$\omega_{\lambda/l}(Tx, Ty) = \frac{l}{2\lambda}e^{-\tau}|x - y| \leq \frac{l}{\lambda}e^{-\tau}|x - y| = e^{-\tau}\omega_{\lambda/l}(x,y),$$

which implies

$$\begin{aligned} F\big(\omega_{\lambda/l}(Tx, Ty)\big) &= \ln \omega_{\lambda/l}(Tx, Ty) \\ &\leq \ln e^{-\tau}\omega_{\lambda/l}(x,y) \\ &= -\tau + \ln \omega_{\lambda/l}(x,y) \\ &= -\tau + F\big(\omega_{\lambda/l}(x,y)\big). \end{aligned}$$

Hence

$$\tau + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x,y)\big).$$

Hence, T is an α-η-*GF*-contraction mapping.

Case II: x and y both are irrational.

Let $\alpha(x,y) \geq \eta(x, Tx)$, then

$$\omega_{\lambda/l}(Tx, Ty) = \frac{l}{3\lambda}e^{-\tau}|x - y| \leq \frac{l}{\lambda}e^{-\tau}|x - y| = e^{-\tau}\omega_{\lambda/l}(x,y),$$

which implies

$$\begin{aligned} F\big(\omega_{\lambda/l}(Tx, Ty)\big) &= \ln \omega_{\lambda/l}(Tx, Ty) \\ &\leq \ln e^{-\tau}\omega_{\lambda/l}(x,y) \\ &= -\tau + \ln \omega_{\lambda/l}(x,y) \\ &= -\tau + F\big(\omega_{\lambda/l}(x,y)\big). \end{aligned}$$

Hence

$$\tau + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x,y)\big).$$

Hence, T is an α-η-*GF*-contraction mapping.

Case III: x is rational and y is irrational.

Let $\alpha(x,y) \geq \eta(x, Tx)$, then

$$\omega_{\lambda/l}(Tx, Ty) = \frac{l}{\lambda}e^{-\tau}\left|\frac{x}{2} - \frac{y}{3}\right| \leq \frac{l}{\lambda}e^{-\tau}|x - y| = e^{-\tau}\omega_{\lambda/l}(x,y),$$

which implies

$$F\big(\omega_{\lambda/l}(Tx, Ty)\big) = \ln \omega_{\lambda/l}(Tx, Ty)$$
$$\leq \ln e^{-\tau} \omega_{\lambda/l}(x, y)$$
$$= -\tau + \ln \omega_{\lambda/l}(x, y)$$
$$= -\tau + F\big(\omega_{\lambda/l}(x, y)\big).$$

Hence

$$\tau + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big).$$

Hence, T is an α-η-GF-contraction mapping. Thus all conditions of Theorem 2.1 hold and note that 0 is a fixed point of T.

Theorem 2.2 *Let X_ω be a complete modular metric space. Let $T : X_\omega \to X_\omega$ be a self-mapping satisfying the following assertions*:

(i) *T is an α-admissible mapping with respect to η;*

(ii) *there exists $x_0 \in X_\omega$ such that $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$;*

(iii) *if $\{x_n\}$ is a sequence in X_ω such that $\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$ with $x_n \to x$ as $n \to \infty$, then either*

$$\eta\big(Tx_n, T^2 x_n\big) \leq \alpha(Tx_n, x) \quad or \quad \eta\big(T^2 x_n, T^3 x_n\big) \leq \alpha\big(T^2 x_n, x\big)$$

holds for all $n \in \mathbb{N}$;

(iv) *T is an α-η-GF-contraction.*

Then T has a fixed point. Moreover, T has a unique fixed point whenever $\alpha(x, y) \geq \eta(x, x)$ for all $x, y \in \mathrm{Fix}(T)$.

Proof Let $x_0 \in X_\omega$ such that $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$. As in proof of Theorem 2.1 we can deduce a sequence $\{x_n\}$ such that $x_{n+1} = Tx_n$ with

$$\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$$

and there exists $x^* \in X_\omega$ such that $x_n \to x^*$ as $n \to \infty$. So, from (iii), either

$$\eta\big(Tx_n, T^2 x_n\big) \leq \alpha\big(Tx_n, x^*\big) \quad or \quad \eta\big(T^2 x_n, T^3 x_n\big) \leq \alpha\big(T^2 x_n, x^*\big)$$

holds for all $n \in \mathbb{N}$. This implies

$$\eta(x_{n+1}, x_{n+2}) \leq \alpha(x_{n+1}, x) \quad or \quad \eta(x_{n+2}, x_{n+3}) \leq \alpha(x_{n+2}, x)$$

holds for all $n \in \mathbb{N}$. Let $\eta(x_{n+1}, x_{n+2}) \leq \alpha(x_{n+1}, x)$, equivalently, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that

$$\eta(x_{n_k}, Tx_{n_k}) = \eta(x_{n_k}, x_{n_k+1}) \leq \alpha\big(x_{n_k}, x^*\big)$$

and so from (2.1) for $x = x_{n_k}$ and $y = x^*$, we deduce that

$$G\big(\omega_{\lambda/l}(x_{n_k}, Tx_{n_k}), \omega_{\lambda/l}(x^*, Tx^*), \omega_{\lambda/l}(x_{n_k}, Tx^*), \omega_{\lambda/l}(x^*, Tx_{n_k})\big) + F\big(\omega_{\lambda/l}(Tx_{n_k}, Tx^*)\big)$$
$$\leq F\big(\omega_{\lambda/l}(x_{n_k}, x^*)\big),$$

which implies

$$F\big(\omega_{\lambda/l}(Tx_{n_k}, Tx^*)\big) \leq F\big(\omega_{\lambda/l}(x_{n_k}, x^*)\big). \tag{2.6}$$

From (F1) we have

$$\omega_{\lambda/l}(x_{n_k+1}, Tx^*) < \omega_{\lambda/l}(x_{n_k}, x^*).$$

By taking the limit as $k \to \infty$ in the above inequality we get

$$\omega_{\lambda/l}(x^*, Tx^*) = \omega_{\lambda/l}(x^*, x^*),$$

which is possible only when

$$\omega_{\lambda/l}(x^*, Tx^*) = 0,$$

i.e., $x^* = Tx^*$. Similarly we can deduce that $x^* = Tx^*$ when $\eta(x_{n+2}, x_{n+3}) \leq \alpha(x_{n+2}, x)$. Uniqueness follows similarly to Theorem 2.1. □

If in Theorem 2.2 we take $\alpha(x, y) = \eta(x, y) = 1$ for all $x, y \in X$, then we deduce the main result of Wardowski [13], Theorem 2.1, in the set up of modular metric spaces.

Corollary 2.2 *Let X_ω be a complete modular metric space and $T : X_\omega \to X_\omega$ be a self-mapping satisfying for $x, y \in X$ with $\omega_{\lambda/l}(Tx, Ty) > 0$ we have*

$$\tau + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big), \tag{2.7}$$

where $\tau > 0$ and $F \in \Delta_F$. Then T has a unique fixed point.

Recall that a self-mapping T is said to have the property P if $\mathrm{Fix}(T^n) = F(T)$ for every $n \in \mathbb{N}$.

Theorem 2.3 *Let X_ω be a complete modular metric space and $T : X_\omega \to X_\omega$ be an α-continuous self-mapping. Assume that there exists $\tau > 0$ such that*

$$\tau + F\big(\omega_{\lambda/l}(Tx, T^2x)\big) \leq F\big(\omega_{\lambda/l}(x, Tx)\big) \tag{2.8}$$

holds for all $x \in X_\omega$ with $\omega_{\lambda/l}(Tx, T^2x) > 0$ where $F \in \Delta_F$. If T is an α-admissible mapping and there exists $x_0 \in X_\omega$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has the property P.

Proof Let $x_0 \in X_\omega$ such that $\alpha(x_0, Tx_0) \geq 1$. For $x_0 \in X_\omega$, we define the sequence $\{x_n\}$ by $x_n = T^n x_0 = Tx_n$. Now since T is an α-admissible mapping, so $\alpha(x_1, x_2) = \alpha(Tx_0, Tx_1) \geq 1$. By continuing this process we have

$$\alpha(x_{n-1}, x_n) \geq 1$$

for all $n \in \mathbb{N}$. If there exists $n_0 \in \mathbb{N}$ such that $x_{n_0} = x_{n_0+1} = Tx_{n_0}$, then x_{n_0} is fixed point of T and we have nothing to prove. Hence, we assume, $x_n \neq x_{n+1}$ or $\omega_{\lambda/l}(Tx_{n-1}, T^2 x_{n-1}) > 0$ for all $n \in \mathbb{N} \cup \{0\}$. From (2.8) we have

$$\tau + F\big(\omega_{\lambda/l}\big(Tx_{n-1}, T^2 x_{n-1}\big)\big) \leq F\big(\omega_{\lambda/l}(x_{n-1}, Tx_{n-1})\big),$$

which implies

$$\tau + F\big(\omega_{\lambda/l}(x_n, x_{n+1})\big) \leq F\big(\omega_{\lambda/l}(x_{n-1}, x_n)\big) \tag{2.9}$$

and so

$$F\big(\omega_{\lambda/l}(x_n, x_{n+1})\big) \leq F\big(\omega_{\lambda/l}(x_{n-1}, x_n)\big) - \tau.$$

Therefore,

$$F\big(\omega_{\lambda/l}(x_n, x_{n+1})\big) \leq F\big(\omega_{\lambda/l}(x_{n-1}, x_n)\big) - \tau \leq F\big(\omega_{\lambda/l}(x_{n-2}, x_{n-1})\big) - 2\tau \leq \cdots$$
$$\leq F\big(\omega_{\lambda/l}(x_0, x_1)\big) - n\tau. \tag{2.10}$$

By taking the limit as $n \to \infty$ in (2.10) we have $\lim_{n \to \infty} F(\omega_{\lambda/l}(x_n, x_{n+1})) = -\infty$, and since $F \in \Delta_F$ we obtain

$$\lim_{n \to \infty} \omega_{\lambda/l}(x_n, x_{n+1}) = 0. \tag{2.11}$$

So, for each $\lambda > 0$, we see, for all $\epsilon > 0$, that there exists $n_0 \in \mathbb{N}$ such that

$$\omega_{\lambda/l}(x_n, x_{n+1}) < \epsilon,$$

for all $n \in \mathbb{N}$ with $n \geq n_0$. Without loss of generality, suppose $m, n \in \mathbb{N}$ and $m > n$. Observe that for $\frac{\lambda}{l(m-n)} > 0$, there exists $n_{\frac{\lambda}{(m-n)}} \in \mathbb{N}$ such that

$$\omega_{\frac{\lambda}{l(m-n)}}(x_n, x_{n+1}) < \frac{\epsilon}{l(m-n)},$$

for all $n \geq n_{\frac{\lambda}{m-n}}$. Now, we have

$$\omega_{\lambda/l}(x_n, x_m) \leq \omega_{\frac{\lambda}{l(m-n)}}(x_n, x_{n+1}) + \omega_{\frac{\lambda}{l(m-n)}}(x_{n+1}, x_{n+2}) + \cdots + \omega_{\frac{\lambda}{l(m-n)}}(x_{m-1}, x_m)$$
$$< \frac{\epsilon}{l(m-n)} + \frac{\epsilon}{l(m-n)} + \cdots + \frac{\epsilon}{l(m-n)}$$
$$= \epsilon/l,$$

for all $m, n \geq n_{\frac{\lambda}{m-n}}$. Thus we proved that $\{x_n\}$ is a Cauchy sequence. Completeness of X_ω ensures that there exists $x^* \in X_\omega$ such that $x_n \to x^*$ as $n \to \infty$. Now since T is α-continuous and $\alpha(x_{n-1}, x_n) \geq 1$, $x_{n+1} = Tx_n \to Tx^*$ as $n \to \infty$. That is, $x^* = Tx^*$. Thus T has a fixed point and $F(T^n) = F(T)$ for $n = 1$. Let $n > 1$. Assume contrarily that $z \in F(T^n)$ and $z \notin F(T)$. Then $\omega_{\lambda/l}(z, Tz) > 0$. Now we have

$$
\begin{aligned}
F\big(\omega_{\lambda/l}(z, Tz)\big) &= F\big(\omega_{\lambda/l}\big(T\big(T^{n-1}z\big)\big), T^2\big(T^{n-1}z\big)\big) \\
&\leq F\big(\omega_{\lambda/l}\big(T^{n-1}z\big), T^n z\big) - \tau \\
&\leq F\big(\omega_{\lambda/l}\big(T^{n-2}z\big), T^{n-1}z\big) - 2\tau \leq \cdots \\
&\leq \omega_{\lambda/l}(z, Tz) - n\tau.
\end{aligned}
$$

By taking the limit as $n \to \infty$ in the above inequality we have $F(\omega_{\lambda/l}(z, Tz)) = -\infty$. Hence, by (F2) we get $\omega_{\lambda/l}(z, Tz) = 0$, which is a contradiction. Therefore, $F(T^n) = F(T)$ for all $n \in \mathbb{N}$. □

Let $(X_\omega, \preceq)$ be a partially ordered modular metric space. Recall that $T : X_\omega \to X\omega$ is nondecreasing if $\forall x, y \in X$, $x \preceq y \Rightarrow T(x) \preceq T(y)$. Fixed point theorems for monotone operators in ordered metric spaces are widely investigated and have found various applications in differential and integral equations (see [20, 22–26] and references therein). From Theorems 2.1-2.3, we derive the following new results in partially ordered modular metric spaces.

Theorem 2.4 *Let $(X_\omega, \preceq)$ be a complete partially ordered modular metric space. Assume that the following assertions hold true:*

(i) *T is nondecreasing and ordered GF-contraction;*

(ii) *there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;*

(iii) *either for a given $x \in X$ and sequence $\{x_n\}$*

$$x_n \to x \quad as\ n \to \infty \quad and \quad x_n \preceq x_{n+1} \quad for\ all\ n \in \mathbb{N}\ we\ have\ Tx_n \to Tx$$

or if $\{x_n\}$ is a sequence such that $x_n \preceq x_{n+1}$ with $x_n \to x$ as $n \to \infty$, then either

$$Tx_n \preceq x \quad or \quad T^2 x_n \preceq x$$

holds for all $n \in \mathbb{N}$.
Then T has a fixed point.

Theorem 2.5 *Let $(X_\omega, \preceq)$ be a complete partially ordered modular metric space. Assume that the following assertions hold true:*

(i) *T is nondecreasing and satisfies (2.8) for all $x \in X$ with $\omega_{\lambda/l}(Tx, T^2x) > 0$ where $F \in \Delta_F$ and $\tau > 0$;*

(ii) *there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;*

(iii) *T is continuous.*
Then T has a property P.

3 Suzuki-Wardowski type fixed point results

In this section, as an application of our results proved above, we deduce certain Suzuki-Wardowski type fixed point theorems. For related work of Suzuki we refer to [27, 28].

Theorem 3.1 *Let X_ω be a complete modular metric space and T be a continuous self-mapping on X_ω. Assume for $x, y \in X$ with $\omega_{\lambda/l}(x, Tx) \leq \omega_{\lambda/l}(x, y)$ and $\omega_{\lambda/l}(Tx, Ty) > 0$ we have*

$$G\big(\omega_{\lambda/l}(x, Tx), \omega_{\lambda/l}(y, Ty), \omega_{\lambda/l}(x, Ty), \omega_{\lambda/l}(y, Tx)\big) + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big), \quad (3.1)$$

where $G \in \Delta_G$ and $F \in \Delta_F$. Then T has a unique fixed point.

Proof Define, $\alpha, \eta : X \times X \to [0, \infty)$ by

$$\alpha(x, y) = \omega_{\lambda/l}(x, y) \quad \text{and} \quad \eta(x, y) = \omega_{\lambda/l}(x, y)$$

for all $x, y \in X$. Clearly, $\eta(x, y) \leq \alpha(x, y)$ for all $x, y \in X$. That is, conditions (i) and (iii) of Theorem 2.1 hold true. Since T is continuous, T is α-η-continuous. Let $\eta(x, Tx) \leq \alpha(x, y)$ with $\omega_{\lambda/l}(Tx, Ty) > 0$. Equivalently, if $\omega_{\lambda/l}(x, Tx) \leq \omega_{\lambda/l}(x, y)$ with $\omega_{\lambda/l}(Tx, Ty) > 0$, then, from (3.1), we have

$$G\big(\omega_{\lambda/l}(x, Tx), \omega_{\lambda/l}(y, Ty), \omega_{\lambda/l}(x, Ty), \omega_{\lambda/l}(y, Tx)\big) + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big).$$

That is, T is an α-η-GF-contraction mapping. Hence, all conditions of Theorem 2.1 hold and T has a unique fixed point. □

Corollary 3.1 *Let X_ω be a complete modular metric space and T be a continuous self-mapping on X. If for $x, y \in X$ with $\omega_{\lambda/l}(x, Tx) \leq \omega_{\lambda/l}(x, y)$ and $\omega_{\lambda/l}(Tx, Ty) > 0$ we have*

$$\tau + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big),$$

where $\tau > 0$ and $F \in \Delta_F$, then T has a unique fixed point.

Corollary 3.2 *Let X_ω be a complete metric space and T be a continuous self-mapping on X. Assume for $x, y \in X$ with $\omega_{\lambda/l}(x, Tx) \leq \omega_{\lambda/l}(x, y)$ and $\omega_{\lambda/l}(Tx, Ty) > 0$ we have*

$$\tau e^{L \min\{\omega_{\lambda/l}(x, Tx), \omega_{\lambda/l}(y, Ty), \omega_{\lambda/l}(x, Ty), \omega_{\lambda/l}(y, Tx)\}} + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big),$$

where $\tau > 0$, $L \geq 0$, and $F \in \Delta_F$. Then T has a unique fixed point.

Theorem 3.2 *Let X_ω be a complete modular metric space and T be a self-mapping on X. Assume that there exists $\tau > 0$ such that*

$$\frac{1}{2(1 + \tau)} \omega_{\lambda/l}(x, Tx) \leq \omega_{\lambda/2l}(x, y) \quad \text{implies} \quad \tau + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big) \quad (3.2)$$

for $x, y \in X$ with $\omega_{\lambda/l}(Tx, Ty) > 0$ where $F \in \Delta_F$. Then T has a unique fixed point.

Proof Define, $\alpha, \eta : X \times X \to [0, \infty)$ by

$$\alpha(x, y) = \omega_{\lambda/2l}(x, y) \quad \text{and} \quad \eta(x, y) = \frac{1}{2(1+\tau)}\omega_{\lambda/l}(x, y)$$

for all $x, y \in X$ where $\tau > 0$. Now, since $\frac{1}{2(1+\tau)}\omega_{\lambda/l}(x, y) \leq \omega_{\lambda/2l}(x, y)$ for all $x, y \in X$, so $\eta(x, y) \leq \alpha(x, y)$ for all $x, y \in X$. That is, conditions (i) and (iii) of Theorem 2.2 hold true. Let $\{x_n\}$ be a sequence with $x_n \to x$ as $n \to \infty$. Assume that $\omega_{\lambda/l}(Tx_n, T^2x_n) = 0$ for some n. Then $Tx_n = T^2x_n$. That is, Tx_n is a fixed point of T and we have nothing to prove. Hence we assume $Tx_n \neq T^2x_n$ for all $n \in \mathbb{N}$. Since $\frac{1}{2(1+\tau)}\omega_{\lambda/l}(Tx_n, T^2x_n) \leq \omega_{\lambda/l}(Tx_n, T^2x_n)$ for all $n \in \mathbb{N}$, from (3.2) we get

$$F\big(\omega_{\lambda/l}\big(T^2x_n, T^3x_n\big)\big) \leq \tau + F\big(\omega_{\lambda/l}\big(T^2x_n, T^3x_n\big)\big) \leq F\big(\omega_{\lambda/l}\big(Tx_n, T^2x_n\big)\big)$$

and so from (F1) we get

$$\omega_{\lambda/l}\big(T^2x_n, T^3x_n\big) \leq \omega_{\lambda/l}\big(Tx_n, T^2x_n\big). \tag{3.3}$$

Suppose that there exists $n_0 \in \mathbb{N}$ such that

$$\eta\big(Tx_{n_0}, T^2x_{n_0}\big) > \alpha(Tx_{n_0}, x) \quad \text{and} \quad \eta\big(T^2x_{n_0}, T^3x_{n_0}\big) > \alpha\big(T^2x_{n_0}, x\big),$$

then

$$\frac{1}{2(1+\tau)}\omega_{\lambda/l}\big(Tx_{n_0}, T^2x_{n_0}\big) > \omega_{\lambda/2l}(Tx_{n_0}, x) \quad \text{and}$$

$$\frac{1}{2(1+\tau)}\omega_{\lambda/l}\big(T^2x_{n_0}, T^3x_{n_0}\big) > \omega_{\lambda/2l}\big(T^2x_{n_0}, x\big),$$

so by (3.3) we have

$$\begin{aligned}
\omega_{\lambda/l}\big(Tx_{n_0}, T^2x_{n_0}\big) &\leq \omega_{\lambda/2l}(Tx_{n_0}, x) + \omega_{\lambda/2l}\big(T^2x_{n_0}, x\big) \\
&< \frac{1}{2(1+\tau)}\omega_{\lambda/l}\big(Tx_{n_0}, T^2x_{n_0}\big) + \frac{1}{2(1+\tau)}\omega_{\lambda/l}\big(T^2x_{n_0}, T^3x_{n_0}\big) \\
&\leq \frac{1}{2(1+\tau)}\omega_{\lambda/l}\big(Tx_{n_0}, T^2x_{n_0}\big) + \frac{1}{2(1+\tau)}\omega_{\lambda/l}\big(Tx_{n_0}, T^2x_{n_0}\big) \\
&= \frac{2}{2(1+\tau)}\omega_{\lambda/l}\big(Tx_{n_0}, T^2x_{n_0}\big) \leq \omega_{\lambda/l}\big(Tx_{n_0}, T^2x_{n_0}\big),
\end{aligned}$$

which is a contradiction. Hence, either

$$\eta\big(Tx_n, T^2x_n\big) \leq \alpha(Tx_n, x) \quad \text{or} \quad \eta\big(T^2x_n, T^3x_n\big) \leq \alpha\big(T^2x_n, x\big)$$

holds for all $n \in \mathbb{N}$. That is, condition (iv) of Theorem 2.2 holds. Let $\eta(x, Tx) \leq \alpha(x, y)$. So, $\frac{1}{2(1+\tau)}\omega_{\lambda/l}(x, Tx) \leq \omega_{\lambda/2l}(x, y)$. Then from (3.2) we get $\tau + F(\omega_{\lambda/l}(Tx, Ty)) \leq F(\omega_{\lambda/l}(x, y))$. Hence, all conditions of Theorem 2.2 hold and T has a unique fixed point. $\qquad\square$

4 Applications to orbitally continuous mappings

Theorem 4.1 *Let X_ω be a complete modular metric space and $T : X_\omega \to X_\omega$ be a self-mapping satisfying the following assertions*:

(i) *for $x, y \in O(z)$ with $\omega_{\lambda/l}(Tx, Ty) > 0$ we have*

$$G\big(\omega_{\lambda/l}(x, Tx), \omega_{\lambda/l}(y, Ty), \omega_{\lambda/l}(x, Ty), \omega_{\lambda/l}(y, Tx)\big) + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big),$$

where $G \in \Delta_G$ and $F \in \Delta_F$;

(ii) *T is an orbitally continuous function.*

Then T has a fixed point. Moreover, T has a unique fixed point when $\mathrm{Fix}(T) \subseteq O(z)$.

Proof Define, $\alpha, \eta : X \times X \to [0, +\infty)$ by

$$\alpha(x, y) = \begin{cases} 3, & \text{if } x, y \in O(z), \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \eta(x, y) = 1,$$

where $O(z)$ is an orbit of a point $z \in X$. From Remark 1.1 we know that T is an α-η-continuous mapping. Let $\alpha(x, y) \geq \eta(x, y)$, then $x, y \in O(z)$. So $Tx, Ty \in O(z)$. That is, $\alpha(Tx, Ty) \geq \eta(Tx, Ty)$. Therefore, T is an α-admissible mapping with respect to η. Since $z, Tz \in O(z)$, $\alpha(z, Tz) \geq \eta(z, Tz)$. Let $\alpha(x, y) \geq \eta(x, Tx)$ and $\omega_{\lambda/l}(Tx, Ty) > 0$. Then $x, y \in O(z)$ and $\omega_{\lambda/l}(Tx, Ty) > 0$. Therefore from (i) we have

$$G\big(\omega_{\lambda/l}(x, Tx), \omega_{\lambda/l}(y, Ty), \omega_{\lambda/l}(x, Ty), \omega_{\lambda/l}(y, Tx)\big) + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big),$$

which implies that T is an α-η-GF-contraction mapping. Hence, all conditions of Theorem 2.1 hold true and T has a fixed point. If $\mathrm{Fix}(T) \subseteq O(z)$, then $\alpha(x, y) \geq \eta(x, y)$ for all $x, y \in \mathrm{Fix}(T)$ and so from Theorem 2.1 T has a unique fixed point. □

Corollary 4.1 *Let X_ω be a complete modular metric space and $T : X_\omega \to X_\omega$ be a self-mapping satisfying the following assertions*:

(i) *for $x, y \in O(z)$ with $\omega_{\lambda/l}(Tx, Ty) > 0$ we have*

$$\tau + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big),$$

where $\tau > 0$ and $F \in \Delta_F$;

(ii) *T is orbitally continuous.*

Then T has a fixed point. Moreover, T has a unique fixed point when $\mathrm{Fix}(T) \subseteq O(z)$.

Corollary 4.2 *Let X_ω be a complete modular metric space and $T : X_\omega \to X_\omega$ be a self-mapping satisfying the following assertions*:

(i) *for $x, y \in O(z)$ with $\omega_{\lambda/l}(Tx, Ty) > 0$ we have*

$$\tau e^{L \min\{\omega_{\lambda/l}(x, Tx), \omega_{\lambda/l}(y, Ty), \omega_{\lambda/l}(x, Ty), d(y, Tx)\}} + F\big(\omega_{\lambda/l}(Tx, Ty)\big) \leq F\big(\omega_{\lambda/l}(x, y)\big),$$

where $\tau > 0$, $L \geq 0$, and $F \in \Delta_F$;

(ii) *T is orbitally continuous.*

Then T has a fixed point. Moreover, T has a unique fixed point when $\mathrm{Fix}(T) \subseteq O(z)$.

Theorem 4.2 *Let X_ω be a complete modular metric space and $T : X_\omega \to X_\omega$ be a self-mapping satisfying the following assertions*:

(i) *for $x \in X$ with $\omega_{\lambda/l}(Tx, T^2x) > 0$ we have*

$$\tau + F\big(\omega_{\lambda/l}(Tx, T^2x)\big) \le F\big(\omega_{\lambda/l}(x, Tx)\big),$$

where $\tau > 0$ and $F \in \Delta_F$;

(ii) *T is an orbitally continuous function.*

Then T has the property P.

Proof Define, $\alpha : X \times X \to [0, +\infty)$ by

$$\alpha(x, y) = \begin{cases} 1, & \text{if } x \in O(z), \\ 0, & \text{otherwise}, \end{cases}$$

where $z \in X$. Let $\alpha(x, y) \ge 1$, then $x, y \in O(z)$. So $Tx, Ty \in O(z)$. That is, $\alpha(Tx, Ty) \ge 1$. Therefore, T is α-admissible mapping. Since $z, Tz \in O(z)$, $\alpha(z, Tz) \ge 1$. By Remark 1.1 we conclude that T is an α-continuous mapping. If $x \in X$ with $\omega_{\lambda/l}(Tx, T^2x) > 0$, then, from (i), we have

$$\tau + F\big(\omega_{\lambda/l}(Tx, T^2x)\big) \le F\big(\omega_{\lambda/l}(x, Tx)\big).$$

Thus all conditions of Theorem 2.3 hold true and T has the property P. □

5 Fixed point results for integral type contractions

Recently, Azadifar *et al.* [29] and Razani and Moradi [30] proved common fixed point theorems for integral type contractions in modular metric spaces. In this section, we deduce more general fixed point theorems for integral type *GF*-contractions.

Theorem 5.1 *Let X_ω be a complete modular metric space. Let $T : X_\omega \to X_\omega$ be a self-mapping satisfying the following assertions*:

(i) *T is an α-admissible mapping with respect to η;*

(ii) *there exists $x_0 \in X_\omega$ such that $\alpha(x_0, Tx_0) \ge \eta(x_0, Tx_0)$;*

(iii) *T is α-η-continuous;*

(iv) *there exist $G \in \Delta_G$, $F \in \Delta_F$ such that for all $x, y \in X_\omega$ and $\lambda > 0$*

$$G\bigg(\int_0^{\omega_{\lambda/l}(x,Tx)} \rho(t)\,dt, \int_0^{\omega_{\lambda/l}(y,Ty)} \rho(t)\,dt, \int_0^{\omega_{\lambda/l}(x,Ty)} \rho(t)\,dt, \int_0^{\omega_{\lambda/l}(y,Tx)} \rho(t)\,dt\bigg)$$

$$+ F\bigg(\int_0^{\omega_{\lambda/l}(Tx,Ty)} \rho(t)\,dt\bigg) \le F\bigg(\int_0^{\omega_{\lambda/l}(x,y)} \rho(t)\,dt\bigg),$$

where $\rho : [0, \infty) \to [0, \infty)$ is a Lebesgue-integrable mapping satisfying $\int_0^\varepsilon \rho(t)\,dt > 0$ for $\varepsilon > 0$.

Then T has a fixed point. Moreover, T has a unique fixed point when $\alpha(x, y) \ge \eta(x, x)$ for all $x, y \in \text{Fix}(T)$.

Theorem 5.2 *Let X_ω be a complete modular metric space. Let $T : X_\omega \to X_\omega$ be a self-mapping satisfying the following assertions*:

(i) *T is an α-admissible mapping with respect to η;*

(ii) *there exists $x_0 \in X_\omega$ such that $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$;*

(iii) *if $\{x_n\}$ is a sequence in X_ω such that $\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$ with $x_n \to x$ as $n \to \infty$, then either*

$$\eta\left(Tx_n, T^2x_n\right) \leq \alpha(Tx_n, x) \quad or \quad \eta\left(T^2x_n, T^3x_n\right) \leq \alpha\left(T^2x_n, x\right)$$

holds for all $n \in \mathbb{N}$;

(iv) *condition (iv) of Theorem 5.1 holds.*

Then T has a fixed point. Moreover, T has a unique fixed point whenever $\alpha(x, y) \geq \eta(x, x)$ for all $x, y \in \mathrm{Fix}(T)$.

Theorem 5.3 *Let X_ω be a complete modular metric space and T be a continuous self-mapping on X_ω. Assume for $x, y \in X$ with*

$$\int_0^{\omega_{\lambda/l}(x,Tx)} \rho(t)\,dt \leq \int_0^{\omega_{\lambda/l}(x,y)} \rho(t)\,dt \quad and \quad \int_0^{\omega_{\lambda/l}(Tx,Ty)} \rho(t)\,dt > 0$$

we have

$$G\left(\int_0^{\omega_{\lambda/l}(x,Tx)} \rho(t)\,dt, \int_0^{\omega_{\lambda/l}(y,Ty)} \rho(t)\,dt, \int_0^{\omega_{\lambda/l}(x,Ty)} \rho(t)\,dt, \int_0^{\omega_{\lambda/l}(y,Tx)} \rho(t)\,dt\right)$$
$$+ F\left(\int_0^{\omega_{\lambda/l}(Tx,Ty)} \rho(t)\,dt\right) \leq F\left(\int_0^{\omega_{\lambda/l}(x,y)} \rho(t)\,dt\right),$$

where $G \in \Delta_G$, $F \in \Delta_F$, and $\rho : [0, \infty) \to [0, \infty)$ is a Lebesgue-integrable mapping satisfying $\int_0^\varepsilon \rho(t)\,dt > 0$ for $\varepsilon > 0$. Then T has a unique fixed point.

Theorem 5.4 *Let X_ω be a complete modular metric space and T be a self-mapping on X_ω. Assume that there exists $\tau > 0$ such that*

$$\frac{1}{2(1+\tau)} \int_0^{\omega_{\lambda/l}(x,Tx)} \rho(t)\,dt \leq \int_0^{\omega_{\lambda/2l}(x,y)} \rho(t)\,dt$$
$$\Rightarrow \quad \tau + F\left(\int_0^{\omega_{\lambda/l}(Tx,Ty)} \rho(t)\,dt\right) \leq F\left(\int_0^{\omega_{\lambda/l}(x,y)} \rho(t)\,dt\right)$$

for $x, y \in X$ with $\int_0^{\omega_{\lambda/l}(Tx,Ty)} \rho(t)\,dt > 0$ where $F \in \Delta_F$ and $\rho : [0, \infty) \to [0, \infty)$ is a Lebesgue-integrable mapping satisfying $\int_0^\varepsilon \rho(t)\,dt > 0$ for $\varepsilon > 0$. Then T has a unique fixed point.

Theorem 5.5 *Let X_ω be a complete modular metric space and $T : X_\omega \to X_\omega$ be a self-mapping satisfying the following assertions:*

(i) *for $x, y \in O(z)$ with $\int_0^{\omega_{\lambda/l}(Tx,Ty)} \rho(t)\,dt > 0$ we have*

$$G\left(\int_0^{\omega_{\lambda/l}(x,Tx)} \rho(t)\,dt, \int_0^{\omega_{\lambda/l}(y,Ty)} \rho(t)\,dt, \int_0^{\omega_{\lambda/l}(x,Ty)} \rho(t)\,dt, \int_0^{\omega_{\lambda/l}(y,Tx)} \rho(t)\,dt\right)$$
$$+ F\left(\int_0^{\omega_{\lambda/l}(Tx,Ty)} \rho(t)\,dt\right) \leq F\left(\int_0^{\omega_{\lambda/l}(x,y)} \rho(t)\,dt\right),$$

where $G \in \Delta_G$, $F \in \Delta_F$, and $\rho : [0, \infty) \to [0, \infty)$ is a Lebesgue-integrable mapping satisfying $\int_0^\varepsilon \rho(t)\, dt > 0$ for $\varepsilon > 0$.

(ii) *T is an orbitally continuous function*;

Then T has a fixed point. Moreover, T has a unique fixed point when $\mathrm{Fix}(T) \subseteq O(w)$.

Theorem 5.6 *Let X_ω be a complete modular metric space and $T : X_\omega \to X_\omega$ be a self-mapping satisfying the following assertions*:

(i) *for $x \in X$ with $\int_0^{\omega_{\lambda/l}(Tx, T^2 x)} \rho(t)\, dt > 0$ we have*

$$
\tau + F\left(\int_0^{\omega_{\lambda/l}(Tx, T^2 x)} \rho(t)\, dt \right) \leq F\left(\int_0^{\omega_{\lambda/l}(x, Tx)} \rho(t)\, dt \right),
$$

where $\tau > 0$ and $F \in \Delta_F$ and $\rho : [0, \infty) \to [0, \infty)$ is a Lebesgue-integrable mapping satisfying $\int_0^\varepsilon \rho(t)\, dt > 0$ for $\varepsilon > 0$.

(ii) *T is an orbitally continuous function.*

Then T has the property P.

6 Modular metric spaces to fuzzy metric spaces

In 1988, Grabiec [31] defined contractive mappings on a fuzzy metric space and extended fixed point theorems of Banach and Edelstein in such spaces. Successively, George and Veeramani [32] slightly modified the notion of a fuzzy metric space introduced by Kramosil and Michálek and then defined a Hausdorff and first countable topology on it. Then the notion of a complete fuzzy metric space presented by George and Veeramani has emerged as another characterization of completeness, and many fixed point theorems have also been proved (for more details see [33–42] and the references therein). In this section we deduce fixed point results in a triangular fuzzy metric space.

Definition 6.1 [32] The 3-tuple $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary nonempty set, $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$, $s, t > 0$,

(1) $M(x, y, t) > 0$,

(2) $M(x, y, t) = 1$ iff $x = y$,

(3) $M(x, y, t) = M(y, x, t)$,

(4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(5) $M(x, y, \cdot) : (0, \infty) \to [0, 1]$ is continuous.

The function $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t.

Definition 6.2 [34] Let $(X, M, *)$ be a fuzzy metric space. The fuzzy metric M is called triangular whenever

$$
\frac{1}{M(x, y, t)} - 1 \leq \frac{1}{M(x, z, t)} - 1 + \frac{1}{M(z, y, t)} - 1 \tag{6.1}
$$

for all $x, y, z \in X$ and all $t > 0$.

Example 6.1 Let $X = \mathbb{R}^2$. Define M on $X \times X \times (0, \infty)$ by $M((x_1, x_2), (y_1, y_2), t) = \frac{1}{1 + |x_1 - y_1| + |x_2 - y_2|}$ and $a \star b = \min\{a, b\}$. Then $(M, X, \star)$ is a triangular fuzzy metric space.

Further if $X = \mathbb{R}$ and M on $X \times X \times (0, \infty)$ is defined by $M(x, y, t) = \min\{x, y\} / \max\{x, y\}$ and $a \star b = \min\{a, b\}$. Then $(M, X, \star)$ is a non-triangular fuzzy metric space.

Lemma 6.1 [31, 33] *For all $x, y \in X$, $M(x, y, \cdot)$ is non-deceasing on $(0, \infty)$.*

Lemma 6.2 ([40], Lemma 34) *Let $(X, M, *)$ be a triangular fuzzy metric space. Define*

$$\omega_\lambda(x, y) = \frac{1}{M(x, y, \lambda)} - 1 \tag{6.2}$$

for all $x, y \in X$ and all $\lambda > 0$. Then ω_λ is a modular metric on X.

As an application of Lemma 6.2 and the results proved above we deduce the following new fixed point theorems in triangular fuzzy metric spaces.

Theorem 6.1 *Let $(X, M, *)$ be a complete triangular fuzzy metric space. Let $T : X \to X$ be a self-mapping satisfying the following assertions*:

(i) *T is an α-admissible mapping with respect to η;*
(ii) *there exists $x_0 \in X_\omega$ such that $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$;*
(iii) *T is α-η-continuous;*
(iv) *for $x, y \in X$ with $\eta(x, Tx) \leq \alpha(x, y)$, $M(Tx, Ty, \lambda/l) > 0$, we have*

$$G\left(\frac{1}{M(x, Tx, \lambda/l)} - 1, \frac{1}{M(y, Ty, \lambda/l)} - 1, \frac{1}{M(x, Ty, \lambda/l)} - 1, \right.$$
$$\left. \frac{1}{M(y, Tx, \lambda/l)} - 1 \right) + F\left(\frac{1}{M(Tx, Ty, \lambda/l)} - 1 \right) \leq F\left(\frac{1}{M(x, y, \lambda/l)} - 1 \right), \tag{6.3}$$

where $G \in \Delta_G$ and $F \in \Delta_F$.

Then T has a fixed point. Moreover, T has a unique fixed point when $\alpha(x, y) \geq \eta(x, x)$ for all $x, y \in \mathrm{Fix}(T)$.

Theorem 6.2 *Let $(X, M, *)$ be a complete triangular fuzzy metric space. Let $T : X \to X$ be a self-mapping satisfying the following assertions*:

(i) *T is an α-admissible mapping with respect to η;*
(ii) *there exists $x_0 \in X_\omega$ such that $\alpha(x_0, Tx_0) \geq \eta(x_0, Tx_0)$;*
(iii) *if $\{x_n\}$ is a sequence in X_ω such that $\alpha(x_n, x_{n+1}) \geq \eta(x_n, x_{n+1})$ with $x_n \to x$ as $n \to \infty$, then either*

$$\eta\left(Tx_n, T^2 x_n \right) \leq \alpha(Tx_n, x) \quad or \quad \eta\left(T^2 x_n, T^3 x_n \right) \leq \alpha\left(T^2 x_n, x \right)$$

holds for all $n \in \mathbb{N}$;
(iv) *condition (iv) of Theorem 6.1 holds.*

Then T has a fixed point. Moreover, T has a unique fixed point whenever $\alpha(x, y) \geq \eta(x, x)$ for all $x, y \in \mathrm{Fix}(T)$.

Theorem 6.3 *Let $(X, M, *)$ be a complete triangular fuzzy metric space and $T : X \to X$ be an α-continuous self-mapping. Assume that there exists $\tau > 0$ such that*

$$\tau + F\left(\frac{1}{M(Tx, T^2 x, \lambda/l)} - 1 \right) \leq F\left(\frac{1}{M(x, Tx, \lambda/l)} - 1 \right) \tag{6.4}$$

holds for all $x \in X$ with $M(Tx, T^2x, \lambda/l) > 0$ where $F \in \Delta_F$. If T is an α-admissible mapping and there exists $x_0 \in X_\omega$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has the property P.

Theorem 6.4 *Let $(X, M, *, \preceq)$ be a complete partially ordered triangular fuzzy metric space. Assume that the following assertions hold true:*

(i) *T is nondecreasing;*

(ii) *there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;*

(iii) *either for a given $x \in X$ and sequence $\{x_n\}$*

$$x_n \to x \quad as \ n \to \infty \quad and \quad x_n \preceq x_{n+1} \quad for \ all \ n \in \mathbb{N} \ we \ have \ Tx_n \to Tx$$

or if $\{x_n\}$ is a sequence such that $x_n \preceq x_{n+1}$ with $x_n \to x$ as $n \to \infty$, then either

$$Tx_n \preceq x \quad or \quad T^2x_n \preceq x$$

holds for all $n \in \mathbb{N}$;

(iv) *for $x, y \in X$ with $x \preceq y \ \eta(x, Tx) \leq \alpha(x, y)$, $M(Tx, Ty, \lambda/l) > 0$, we have*

$$G\left(\frac{1}{M(x, Tx, \lambda/l)} - 1, \frac{1}{M(y, Ty, \lambda/l)} - 1, \frac{1}{M(x, Ty, \lambda/l)} - 1, \right.$$
$$\left. \frac{1}{M(y, Tx, \lambda/l)} - 1\right) + F\left(\frac{1}{M(Tx, Ty, \lambda/l)} - 1\right) \leq F\left(\frac{1}{M(x, y, \lambda/l)} - 1\right), \qquad (6.5)$$

where $G \in \Delta_G$ and $F \in \Delta_F$.

Then T has a fixed point.

Theorem 6.5 *Let $(X, M, *, \preceq)$ be a complete partially ordered triangular fuzzy metric space. Assume that the following assertions hold true:*

(i) *T is nondecreasing;*

(ii) *there exists $x_0 \in X$ such that $x_0 \preceq Tx_0$;*

(iii) *T is continuous;*

(iv) *for $x, y \in X$ with $x \preceq y \ \eta(x, Tx) \leq \alpha(x, y)$, $M(Tx, Ty, \lambda/l) > 0$, we have*

$$G\left(\frac{1}{M(x, Tx, \lambda/l)} - 1, \frac{1}{M(y, Ty, \lambda/l)} - 1, \frac{1}{M(x, Ty, \lambda/l)} - 1, \right.$$
$$\left. \frac{1}{M(y, Tx, \lambda/l)} - 1\right) + F\left(\frac{1}{M(Tx, Ty, \lambda/l)} - 1\right) \leq F\left(\frac{1}{M(x, y, \lambda/l)} - 1\right), \qquad (6.6)$$

where $G \in \Delta_G$ and $F \in \Delta_F$.

Then T has a fixed point.

Theorem 6.6 *Let $(X, M, *)$ be a complete triangular fuzzy metric space and T be a continuous self-mapping on X. Assume for $x, y \in X$ with $\frac{1}{M(x, Tx, \lambda/l)} \leq \frac{1}{M(x, y, \lambda/l)}$ and $\frac{1}{M(Tx, Ty, \lambda/l)} > 0$ we have*

$$G\left(\frac{1}{M(x,Tx,\lambda/l)}-1, \frac{1}{M(y,Ty,\lambda/l)}-1, \frac{1}{M(x,Ty,\lambda/l)}-1, \frac{1}{M(y,Tx,\lambda/l)}-1\right)$$

$$+ F\left(\frac{1}{M(Tx,Ty,\lambda/l)}-1\right) \le F\left(\frac{1}{M(x,y,\lambda/l)}-1\right), \tag{6.7}$$

where $G \in \Delta_G$ and $F \in \Delta_F$. Then T has a unique fixed point.

Theorem 6.7 *Let $(X,M,*)$ be a complete triangular fuzzy metric space and T be a self-mapping on X. Assume that there exists $\tau > 0$ such that*

$$\frac{1}{2(1+\tau)}\left(\frac{1}{M(x,Tx,\lambda/l)}-1\right) \le \frac{1}{M(x,y,\lambda/2l)}-1$$

$$implies \quad \tau + F\left(\frac{1}{M(Tx,Ty,\lambda/l)}-1\right) \le F\left(\frac{1}{M(x,y,\lambda/l)}-1\right) \tag{6.8}$$

for $x,y \in X$ with $\frac{1}{M(Tx,Ty,\lambda/l)} > 0$ where $F \in \Delta_F$. Then T has a unique fixed point.

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