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F-contractive type mappings in b-metric spaces and some related fixed point results

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Abstract

In this paper, we define F-contractive type mappings in b-metric spaces and prove some fixed point results with suitable examples. F-expanding type mappings are also defined and a fixed point result is obtained.

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Keywords: b-metric space; F-contraction; Kannan contraction; fixed point

1 Introduction

In 1922, Banach [7] proved a fixed point theorem for metric spaces, which later on came to be known as the famous "Banach contraction principle". Since then generalizations of the contraction principle in different directions as well as many new fixed point results with applications have been established by different researchers (refer to [6, 8, 12, 14–18, 22, 29, 32–35, 37, 38]).

It is a well-known fact that every Banach contraction is continuous. In 1968, Kannan [27] proved the following independent result without assuming the condition of continuity of the mapping.

Theorem 1.1 ([27]) *Let (X,d) be a complete metric space and $T : X \longrightarrow X$ be a mapping such that*

$$d(Tx,Ty) \le p\{d(x,Tx) + d(y,Ty)\}$$

for all $x,y \in X$ and $0 \le p < \frac{1}{2}$. Then T has a unique fixed point $z \in X$, and for any $x \in X$ the sequence of iterates $\{T^n x\}$ converges to z.

This result shows that there exist contractive mappings with a unique fixed point which is not necessarily continuous. Incidentally, the result of Kannan also gave a characterization of the metric space (X,d) in terms of the fixed point of T. This was shown by Subrahmanyam [44] in 1975, by proving that a metric space is complete if and only if every Kannan mapping has a unique fixed point.

In 1989, Bakhtin [6] introduced the notion of b-metric spaces, which was formally defined by Czerwik [14] in 1993 with a view of generalizing Banach contraction principle. There are many authors who have worked on the generalization of fixed point theorems in b-metric spaces (refer to [24, 26, 28, 43] and the references therein). However, unlike the normal metric the b-metric d is not continuous in the topology generated by it (for instance, refer to Example 2.6 of [36]). The problems that arise in proving fixed point results due to the possible discontinuity of the b-metric can be fortunately managed with the following lemma.

Lemma 1.2 ([30]) *Let (X, d, s) be a b-metric space and $\{x_n\}$ be a convergent sequence in X with $\lim_{n\to\infty} x_n = x$. Then for all $y \in X$*

$$s^{-1} d(x, y) \leq \lim_{n\to\infty} \inf d(x_n, y) \leq \lim_{n\to\infty} \sup d(x_n, y) \leq s d(x, y).$$

In 2012, Wardowski [46] introduced a new type of contraction called F-contraction (also called *Wardowski contraction* [21]) and proved a fixed point theorem concerning F-contractions. Since then much work has been done on the fixed point theory of F-contraction mappings and their extensions (refer to [25, 30, 31, 39, 42, 45, 47]).

In this paper, we present some results on fixed point theory in b-metric spaces considering a new type of mapping which is a combination of F-contraction by Wardowski [46] as well as Kannan contraction [27] mappings. We also try to develop a fixed point existence results for such type of expanding mappings on b-metric spaces.

We start by defining some of the terms used in this paper.

Definition 1.3 ([6, 14]) Let X be a non-empty set and $s \geq 1$ be a given real number. A function $d : X \times X \longrightarrow [0, \infty)$ is called b-metric if it satisfies the following properties:

1. $d(x, y) = 0$ if and only if $x = y$;
2. $d(x, y) = d(y, x)$; and
3. $d(x, z) \leq s[d(x, y) + d(y, z)]$, for all $x, y, z \in X$.

The triplet (X, d, s) is called a b-metric space with coefficient s.

Example 1.4 For some classical examples of b-metric spaces, one may refer to [8, 40]. There follow two other examples:

1. Let $X = [0, 2]$ and $d : X \times X \longrightarrow [0, \infty)$ be defined by

$$d(x, y) = \begin{cases} (x - y)^2, & x, y \in [0, 1], \\ \left|\frac{1}{x^2} - \frac{1}{y^2}\right|, & x, y \in [1, 2], \\ |x - y|, & \text{otherwise.} \end{cases}$$

   It can easily be seen that d is a b-metric on X and so, (X, d, s) is a b-metric space with $s = 2$.

2. Let $X = \{1, 2, 3, 4\}$ and define $d : X \times X \longrightarrow [0, \infty)$ as follows:

$$d(n, n) = 0, \quad n = 1, 2, 3, 4;$$

$$d(1, 2) = d(2, 1) = 2;$$

$$d(2, 3) = d(3, 2) = \frac{1}{2};$$

$$d(1, 3) = d(3, 1) = 1;$$

$$d(1,4) = d(4,1) = \frac{3}{2};$$
$$d(2,4) = d(4,2) = d(3,4) = d(4,3) = 3.$$

Then d is a b-metric with $s = 2$.

The class of b-metric spaces is larger than that of metric spaces as there are b-metric spaces which are not a metric space (refer to the examples in [24, 26, 43]), and a metric space is a b-metric space with coefficient $s = 1$. Moreover, the notion of convergent sequence, Cauchy sequence, completeness, etc. can as well be defined accordingly in b-metric spaces.

Definition 1.5 ([46]) Let $F : (0,\infty) \longrightarrow \mathbb{R}$ be a map satisfying the following conditions:
(F1) F is strictly increasing.
(F2) For each sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive numbers $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$.
(F3) There exists $k \in (0,1)$ such that $\lim_{\alpha\to 0^+} \alpha^k F(\alpha) = 0$.
For a metric space (X,d), a mapping $T : X \longrightarrow X$ is said to be a Wardowski F-contraction if there exists $\tau > 0$ such that $d(Tx,Ty) > 0$ implies

$$\tau + F\bigl(d(Tx,Ty)\bigr) \le F\bigl(d(x,y)\bigr)$$

for all $x, y \in X$.

In 2015, Cosentino et al. [13] introduced the following condition in Definition 1.5 to obtain some fixed point results in b-metric spaces. In this paper, we also use this extended definition with the following condition added to Definition 1.5:
(F4) Let $s \ge 1$ be a real number. For each sequence $\{\alpha_n\}_{n\in\mathbb{N}}$ of positive numbers such that $\tau + F(s\alpha_n) \le F(\alpha_{n-1})$ for all $n \in \mathbb{N}$ and some $\tau > 0$, then

$$\tau + F\bigl(s^n\alpha_n\bigr) \le F\bigl(s^{n-1}\alpha_{n-1}\bigr) \quad \text{for all } n \in \mathbb{N}.$$

In [3], Alsulami et al. defined a generalized F-Suzuki type contractions in a b-metric space (X,d,s) as a mapping $T : X \longrightarrow X$ such that there exists $\tau > 0$ and for all $x, y \in X$ with $x \ne y$,

$$\frac{1}{2s} d(x,Tx) < d(x,y)$$

implies

$$\tau + F\bigl(d(Tx,Ty)\bigr) \le \alpha F\bigl(d(x,y)\bigr) + \beta F\bigl(d(x,Tx)\bigr) + \gamma F\bigl(d(y,Ty)\bigr),$$

where $\alpha, \beta \in [0,1]$, $\gamma \in [0,1)$ with $\alpha + \beta + \gamma = 1$ and F satisfy conditions F1 and F2.

Motivated by these ideas, here we define a new type of F-contractive mappings with F satisfying conditions F1, F2, F3 and F4.

Definition 1.6 For a b-metric space (X,d,s), a mapping $T:X\longrightarrow X$ is said to be an F-*contractive type mapping* if there exists $\tau>0$ such that $d(x,Tx)d(y,Ty)\neq 0$ implies

$$\tau+F\bigl(sd(Tx,Ty)\bigr)\leq\frac{1}{3}\bigl\{F\bigl(d(x,y)\bigr)+F\bigl(d(x,Tx)\bigr)+F\bigl(d(y,Ty)\bigr)\bigr\}\tag{1}$$

and $d(x,Tx)d(y,Ty)=0$ implies

$$\tau+F\bigl(sd(Tx,Ty)\bigr)\leq\frac{1}{3}\bigl\{F\bigl(d(x,y)\bigr)+F\bigl(d(x,Ty)\bigr)+F\bigl(d(y,Tx)\bigr)\bigr\}\tag{2}$$

for all $x,y\in X$.

Example 1.7 We consider an arbitrary b-metric space (X,d,s). Corresponding to different types of mappings F, we obtain different classes of F-contractive type mappings on (X,d,s). Some of the mappings $F:(0,\infty)\longrightarrow\mathbb{R}$ satisfying (F1)–(F4) are $F_1(x)=\log x$, $F_2(x)=x+\log x$ and $F_3(x)=\log(x^2+x)$ (refer to [13, 30]).

For the class of F-contractive type mappings with $F(x)=\log x$, the conditions (1) and (2) reduce to the following conditions:

$d(x,Tx)d(y,Ty)\neq 0$ implies

$$s^3d(Tx,Ty)^3\leq e^{-3\tau}d(x,y)d(x,Tx)d(y,Ty)\tag{3}$$

and $d(x,Tx)d(y,Ty)=0$ implies

$$s^3d(Tx,Ty)^3\leq e^{-3\tau}d(x,y)d(x,Ty)d(y,Tx)\tag{4}$$

for all $x,y\in X$.

Since F is an increasing function, it is easily seen that every F-contraction mapping is a contraction mapping and hence continuous (refer to [46]). However, every F-contractive type mapping need not be continuous as is shown in the following example.

Example 1.8 Let $X=[0,\infty)$ with the b-metric

$$d(x,y)=\begin{cases}x+y, & x\neq y,\\ 0, & x=y.\end{cases}$$

Define $T:X\longrightarrow X$ by

$$T(x)=\begin{cases}1, & 0\leq x\leq 2,\\ \frac{1}{x}, & x>2.\end{cases}$$

Then T is discontinuous at $x=2$. Let $F(x)=\log x$.

For $x,y\in[0,2]$ with $x\neq y$, $d(Tx,Ty)=0$ and (3) holds trivially.

For $x \neq y$ and $x, y > 2$,

$$d(Tx, Ty)^3 = \left(\frac{1}{x} + \frac{1}{y}\right)^3 < 1,$$

$$d(x,y)d(x,Tx)d(y,Ty) = (x+y)\left(x + \frac{1}{x}\right)\left(y + \frac{1}{y}\right) > 16.$$

For $1 \neq x \in [0,2]$ and $y > 2$

$$d(Tx, Ty)^3 = \left(1 + \frac{1}{y}\right)^3 \leq \left(1 + \frac{1}{2}\right)^3 = \frac{27}{8},$$

$$d(x,y)d(x,Tx)d(y,Ty) = (x+y)(x+1)\left(y + \frac{1}{y}\right) \geq 2 \times 1 \times 2 = 4.$$

Thus, if $d(x,Tx)d(y,Ty) \neq 0$, (3) holds for $e^{-3\tau} = \frac{1}{16}$ or $\tau = \frac{4\ln 2}{3}$.

On the other hand, if $x > 2$,

$$d(Tx, T1)^3 = \left(\frac{1}{x} + 1\right)^3 < \frac{27}{8},$$

$$d(x,1)d(x,T1)d(1,Tx) = (x+1)^2\left(1 + \frac{1}{x}\right) > 3^2 \times 1 = 9.$$

Thus, (3) and (4) holds for $e^{-3\tau} = \frac{1}{2}$ or $\tau = \frac{\ln 2}{3}$ and $1 > p^3 \geq \frac{7}{9}$.

Hence T is an F-contractive type mapping which is not continuous.

In a b-metric space (X,d,s), a mapping $T : X \longrightarrow X$ is said to be a *Picard operator* [41] if it has a unique fixed point $z \in X$ and the Picard iteration $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = Tx_n, \quad n = 0, 1, 2, \ldots,$$

converges to z for any $x_0 \in X$.

2 Main results

In [46], Wardowski proved the following fixed point result for F-contraction mappings in complete metric spaces.

Theorem 2.1 ([46]) *Let (X,d) be a complete metric space and let $T : X \longrightarrow X$ be an F-contraction. Then T has a unique fixed point $x^* \in X$ and for every $x_0 \in X$ the sequence $\{T^n x_0\}$ converges to x^*.*

Here we establish a similar type of result for F-contractive type mappings in a b-metric space by appealing to a lemma given in [30] which is also found in [13]. For the following results, the b-metric need not be continuous.

Theorem 2.2 *Let (X,d,s) be a complete b-metric space and let $T : X \longrightarrow X$ be an F-contractive type mapping. Then T is a Picard operator.*

Proof Let $x_0 \in X$ be an arbitrary (but fixed) and consider the sequence $\{x_n\}$, where $x_n = Tx_n$, $n \in \mathbb{N}$. Denote $d(x_n, x_{n+1})$ by μ_n and suppose that $\mu_n > 0$ for all $n \in \mathbb{N}$. Then since T is an F-contractive type mapping and $Tx_n \neq x_n$ for all $n \in \mathbb{N}$, we have

$$F(s\mu_n) \leq \frac{1}{3}\bigl\{F\bigl(d(x_{n-1}, x_n)\bigr) + F\bigl(d(x_{n-1}, x_n)\bigr) + F\bigl(d(x_n, x_{n+1})\bigr)\bigr\} - \tau$$

or

$$F(s\mu_n) \leq F(\mu_{n-1}) - \frac{3}{2}\tau.$$

By condition (F4), we have

$$F\bigl(s^n\mu_n\bigr) \leq F\bigl(s^{n-1}\mu_{n-1}\bigr) - \frac{3}{2}\tau$$

and hence, by induction,

$$F\bigl(s^n\mu_n\bigr) \leq F\bigl(s^{n-1}\mu_{n-1}\bigr) - \frac{3}{2}\tau \leq \cdots \leq F(\mu_0) - \frac{3}{2}n. \tag{5}$$

In the limit as $n \to \infty$, we get

$$\lim_{n\to\infty} F\bigl(s^n\mu_n\bigr) = -\infty$$

so that

$$\lim_{n\to\infty} s^n\mu_n = 0.$$

From condition (F3), there exists $k \in (0, 1)$ such that

$$\lim_{n\to\infty} \bigl(s^n\mu_n\bigr)^k F\bigl(s^n\mu_n\bigr) = 0.$$

Multiplication of (5) with $(s^n\mu_n)^k$ yields

$$0 \leq \bigl(s^n\mu_n\bigr)^k F\bigl(s^n\mu_n\bigr) + \frac{3}{2}n\bigl(s^n\mu_n\bigr)^k\tau \leq \bigl(s^n\mu_n\bigr)^k F(\mu_0).$$

Taking the limit as $n \to \infty$, we get

$$\lim_{n\to\infty} n\bigl(s^n\mu_n\bigr)^k = 0.$$

Now, following the proof of Theorem 3.2 in [30], we can show that $\{x_n\}$ is a Cauchy sequence. Since (X, d, s) is complete, there exists $z \in X$ such that

$$\lim_{n\to\infty} x_n = z.$$

Applying Lemma 1.2, we get

$$\lim_{n\to\infty} d(z, x_n) = \limsup d(z, x_n) \leq sd(z, z) = 0.$$

Also, using (1), we have for all $n \in \mathbb{N}$

$$\tau + F\bigl(sd(Tz, Tx_n)\bigr) \leq \frac{1}{3}\bigl\{F\bigl(d(z,x_n)\bigr) + F\bigl(d(z, Tz)\bigr) + F\bigl(d(x_n, x_{n+1})\bigr)\bigr\}.$$

Hence, in the limit as $n \to \infty$, we get (since $d(z,x_n) \to 0$)

$$\tau + \lim_{n\to\infty} F\bigl(sd(Tz, Tx_n)\bigr) \leq -\infty.$$

This implies

$$\lim_{n\to\infty} d(Tz, x_{n+1}) = \lim_{n\to\infty} d(Tz, Tx_n) = 0.$$

Since the convergent sequence $\{x_n\}$ converges to both z and Tz, it must be the case that $Tz = z$.

To show the uniqueness of the fixed point, let, if possible, z' be another fixed point of T with $z \neq z'$. Then from (2)

$$\tau + F\bigl(sd\bigl(Tz, Tz'\bigr)\bigr) \leq \frac{1}{3}\bigl\{F\bigl(d\bigl(z,z'\bigr)\bigr) + F\bigl(d\bigl(z, Tz'\bigr)\bigr) + F\bigl(d\bigl(Tz,z'\bigr)\bigr)\bigr\},$$

or

$$F\bigl(sd\bigl(z,z'\bigr)\bigr) < F\bigl(d\bigl(z,z'\bigr)\bigr),$$

which is a contradiction. This proves the result. □

Example 2.3 Let $X = \mathbb{R}$ which is complete with respect to the b-metric $d(x,y) = |x-y|^2$ for all $x, y \in X$. For some $k \in \mathbb{R}$, consider the constant function $Tx = k$ for all $x \in \mathbb{R}$. Then it is easy to check that T is an F-contractive type mapping for $0 \leq p < 1$. So, by Theorem 2.2, T has a unique fixed point. Clearly, k is the unique fixed point for T here.

Also, we have seen that the function T given in Example 1.8 is an F-contractive type mapping and hence by Theorem 2.2, T has a unique fixed point, which is $x = 1$ in this case.

Example 2.4 The b-metric space (X, d, s), where

$$X = [0,1] \cup [2,\infty) \quad \text{and} \quad d(x,y) = \begin{cases} \min\{x+y, 2\}, & x \neq y, \\ 0, & x = y, \end{cases}$$

is complete with $s = 1$. Let the mapping $T : X \longrightarrow X$ be defined by

$$Tx = \begin{cases} \frac{1}{2}, & 0 \leq x < 1, \\ 0, & x = 1, \\ \frac{1}{2} - \frac{1}{x}, & x \geq 2. \end{cases}$$

It is easily seen that T is not continuous at $x = 1$.

Now, for $x, y \geq 2$ and $x \neq y$, we have

$$d(Tx, Ty)^3 = \min\left\{1 - \frac{1}{x} - \frac{1}{y}, 2\right\}^3 = \left\{1 - \frac{1}{x} - \frac{1}{y}\right\}^3 < 1$$

and

$$d(x,y)d(x, Tx)d(y, Ty) = 2\min\left\{x + \frac{1}{2} - \frac{1}{x}, 2\right\} \min\left\{y + \frac{1}{2} - \frac{1}{y}, 2\right\} = 8.$$

Again, if $x \in [0,1)$ and $y \geq 2$ (or conversely)

$$d(Tx, Ty)^3 = d\left(\frac{1}{2}, \frac{1}{2} - \frac{1}{y}\right)^3 = \min\left\{1 - \frac{1}{y}, 2\right\}^3 = \left\{1 - \frac{1}{y}\right\}^3 < 1$$

and

$$\begin{aligned} d(x,y)d(x, Tx)d(y, Ty) &= \min\{x + y, 2\} \min\left\{x + \frac{1}{2}, 2\right\} \min\left\{y + \frac{1}{2} - \frac{1}{y}, 2\right\} \\ &= 2\left(x + \frac{1}{2}\right)2 \geq 2. \end{aligned}$$

Also, for $x \in [0,1)$,

$$\begin{aligned} &d(Tx, T1)^3 = d\left(\frac{1}{2}, 0\right)^3 = \min\left\{\frac{1}{2}, 2\right\}^3 = \frac{1}{8}, \\ &d(x,1)d(x, Tx)d(1, T1) = \min\{x + 1, 2\} \min\left\{x + \frac{1}{2}, 2\right\} \min\{1 + 0, 2\} \geq \frac{1}{2}. \end{aligned}$$

And, for $x \geq 2$,

$$\begin{aligned} &d(Tx, T1)^3 = d\left(\frac{1}{2} - \frac{1}{x}, 0\right)^3 = \min\left\{\frac{1}{2} - \frac{1}{x}, 2\right\}^3 \leq \frac{1}{8}, \\ &d(x,1)d(x, Tx)d(1, T1) = \min\{x + 1, 2\} \min\left\{x + \frac{1}{2} - \frac{1}{x}, 2\right\} \min\{1, 2\} \geq 2. \end{aligned}$$

Thus, if $d(x, Tx)d(y, Ty) \neq 0$, (3) is satisfied for $e^{-3\tau} = \frac{1}{16}$ or $\tau = \frac{4\ln 2}{3}$.

Further,

$$d\left(T1, T\frac{1}{2}\right)^3 = \min\left\{0 + \frac{1}{2}, 2\right\}^3 = \frac{1}{8}$$

and

$$d\left(1, \frac{1}{2}\right)d\left(1, T\frac{1}{2}\right)d\left(T1, \frac{1}{2}\right) = \frac{9}{8}.$$

And finally, for $x \geq 2$

$$d\left(Tx, T\frac{1}{2}\right)^3 = \min\left\{1 - \frac{1}{x}, 2\right\}^3 < 1$$

and

$$d\left(x, \frac{1}{2}\right) d\left(x, T\frac{1}{2}\right) d\left(Tx, \frac{1}{2}\right) = \frac{5}{2} \times \frac{5}{2}\left(1 - \frac{1}{x}\right) \geq \frac{25}{8}.$$

Since we have cases when $d(Tx, Ty) = 0$ trivially holds, we see that if $d(x, Tx)d(y, Ty) = 0$, (4) is satisfied for $e^{-3\tau} = \frac{1}{3^2}$ or $\tau = \frac{2\ln 3}{3}$.

Thus T is an F-contractive type mapping and, by inspection, we see that T has a unique fixed point $x = \frac{1}{2}$.

Corollary 2.5 *Let (X, d, s) be a complete b-metric space and $T : X \longrightarrow X$ be a mapping such that, for some $\tau > 0$, $d(x, Tx)d(y, Ty) \neq 0$ implies*

$$\tau + F\left(sd\left(T^n x, T^n y\right)\right) \leq \frac{1}{3}\left\{F\left(d(x, y)\right) + F\left(d\left(x, T^n x\right)\right) + F\left(d\left(y, T^n y\right)\right)\right\}$$

and $d(x, Tx)d(y, Ty) = 0$ implies

$$\tau + F\left(sd\left(T^n x, T^n y\right)\right) \leq \frac{1}{3}\left\{F\left(d(x, y)\right) + F\left(d\left(x, T^n y\right)\right) + F\left(d\left(y, T^n x\right)\right)\right\}$$

where n is a positive integer. Then there exists a unique fixed point of T.

Proof Applying Theorem 2.2 to the self-mapping $S = T^n$, we see that S has a unique fixed point, say z, so that $T^n z = Sz = z$. Since $T^{n+1} z = Tz$,

$$STz = T^n(Tz) = T^{n+1} z = Tz,$$

and so Tz is a fixed point of S. By the uniqueness of the fixed point of S, we get $Tz = z$. □

Example 2.6 We consider the same b-metric space (X, d, s) as in Example 2.4, which is complete. Consider the function $T : X \longrightarrow X$ defined by

$$Tx = \begin{cases} \frac{1}{2} - \frac{1}{x}, & x \geq 2, \\ \frac{1}{2}, & 0 \leq x \leq 1. \end{cases}$$

It is easily seen that T is continuous.

By a simple computation (refer Example 2.4), we see that T is an F-contractive type mapping with $F(x) = \log x$. So, by Theorem 2.2, T has a unique fixed point in X, which is clearly $x = \frac{1}{2}$ here.

Remark 2.7 It is interesting to note that the function T as given in Example 2.6 ceases to be an F-contractive type mapping with $F(x) = \log x$ if the metric is replaced by the b-metric

$$d(x, y) = \begin{cases} \min\{x + y, 2\}^2, & x \neq y, \\ 0, & x = y, \end{cases}$$

in which case the b-metric space (X, d, s) has coefficient $s = 2$.

Example 2.8 Consider the b-metric space (X, d, s), where

$$X = [0,1] \cup [3,\infty) \quad \text{and} \quad d(x,y) = \begin{cases} \min\{x+y, 3\}^2, & x \neq y, \\ 0, & x = y, \end{cases}$$

which is complete with $s = 2$. Consider the function $T : X \longrightarrow X$ defined by

$$Tx = \begin{cases} \frac{1}{2} - \frac{1}{x}, & x \geq 2, \\ \frac{1}{2}, & 0 \leq x \leq 1. \end{cases}$$

Then, by a similar computation to Example 2.4, it is seen that T is an F-contractive type mapping and, by Theorem 2.2, it has a unique fixed point, which is $x = \frac{1}{2}$.

In [12], Cobzas outlined the proof of the first characterization of completeness in terms of contraction by Hu [23]. In the following we present a similar result for F-contractive type mappings in b-metric spaces. It may also be mentioned here that the same result also holds true for *Kannan F-contractive type mappings*, which will be defined later.

Theorem 2.9 *For a b-metric space* (X, d, s), *if for every closed subset* Y *of* X *every F-contractive type mapping* $T : Y \longrightarrow Y$ *has a fixed point, then* X *is complete.*

Proof Consider a Cauchy sequence in X. If it has a convergent subsequence, then it is convergent. Suppose that this is not the case, then

$$\beta(x_n) := \inf\{d(x_n, x_m) : m > n\} > 0 \quad \forall n \in \mathbb{N}.$$

Here, we note that $\beta(x_n) \leq \beta(x_m)$ for $m \geq n$. For a given α with $0 < \alpha < 1$, we construct inductively a subsequence $\{x_{n_k}\}$ such that

$$sd(x_i, x_j) < \alpha\beta(x_{n_{k-1}}) \quad \forall i, j \geq n_k.$$

Then $Y = \{x_{n_k} : k \in \mathbb{N}\}$ is a closed subset of X. Define $T : Y \longrightarrow Y$ by

$$Tx_{n_k} = x_{n_{k+1}} \quad \forall k \in \mathbb{N}.$$

Then it is clear that T is fixed point free. Now,

$$sd(Tx, Ty) = sd(Tx_{n_k}, Tx_{n_{k+i}}) = sd(x_{n_{k+1}}, x_{n_{k+i+1}}) < \alpha\beta(x_{n_k}). \tag{6}$$

By definition,

$$\begin{aligned} \beta(x_{n_k}) &\leq d(x_{n_k}, x_{n_{k+i}}) = d(x, y) & (7)\\ &\leq d(x_{n_k}, x_{n_{k+1}}) = d(x, Tx) & (8)\\ &\leq \beta(x_{n_{k+i}}) \leq d(x_{n_{k+i}}, x_{n_{k+i+1}}) = d(y, Ty). & (9) \end{aligned}$$

From the above inequalities, we get

$$\tau + F\big(sd(Tx, Ty)\big) \leq \frac{1}{3}\big\{F\big(d(x,y)\big) + F\big(d(x, Tx)\big) + F\big(d(y, Ty)\big)\big\}$$

for some $\tau > 0$, showing that T is an F-contractive type mapping on a closed subset of X which is fixed point free. This is a contradiction. □

Now, we define Kannan F-contractive type mappings and prove some fixed point results for the same.

Definition 2.10 Let (X, d, s) be a b-metric space. A mapping $T : X \longrightarrow X$ is said to be a *Kannan F-contractive type mapping* if there exists $\tau > 0$ such that $d(x, Tx)d(y, Ty) \neq 0$ implies

$$\tau + F\big(sd(Tx, Ty)\big) \leq \frac{1}{2}\big\{F\big(d(x, Tx)\big) + F\big(d(y, Ty)\big)\big\} \tag{10}$$

and $d(x, Tx)d(y, Ty) = 0$ implies

$$\tau + F\big(sd(Tx, Ty)\big) \leq \frac{1}{2}\big\{F\big(d(x, Ty)\big) + F\big(d(y, Tx)\big)\big\}. \tag{11}$$

for all $x, y \in X$.

Example 2.11 The functions defined in Examples 2.4 and 2.6 are also Kannan F-contractive type mappings, while the mapping given in Example 1.8 is not a Kannan F-contractive type mapping.

Theorem 2.12 *Let (X, d, s) be a complete b-metric space and let $T : X \longrightarrow X$ be a Kannan F-contractive type mapping. Then T is a Picard operator.*

Proof The result follows, following the proof of Theorem 2.2. □

Example 2.13 Let $X = [0, 1] \cup [2, 3]$ with the b-metric $d(x, y) = \max\{x, y\}^2$ for all $x, y \in X$. Consider the function $T : X \longrightarrow X$ given by

$$Tx = \begin{cases} 1, & x \in [0,1], \\ 1 - \frac{1}{x}, & x \in [2,3]. \end{cases}$$

Now, for $x \in [0, 1)$ and $y \in [2, 3]$, we have

$$d(Tx, Ty)^2 = d\left(1, 1 - \frac{1}{y}\right)^2 = 1$$

and

$$d(x, Tx)d(y, Ty) = d(x, 1)d\left(y, 1 - \frac{1}{y}\right) = 1.y^2 \geq 2^2.$$

Also, for $x, y \in [2,3]$ with $x < y$, we have

$$d(Tx,Ty)^2 = d\left(1-\frac{1}{x}, 1-\frac{1}{y}\right)^2 = \left(1-\frac{1}{y}\right)^4 < 1$$

and

$$d(x,Tx)d(y,Ty) = d\left(x, 1-\frac{1}{x}\right)d\left(y, 1-\frac{1}{y}\right) = x^2.y^2 \geq 2^4.$$

Thus, if $d(x,Tx)d(y,Ty) \neq 0$, (10) is satisfied for $e^{-2\tau} = \frac{1}{2^4}$ or $\tau = \frac{4\ln 2}{3}$.

On the other hand, for $x \geq 2$

$$d(Tx,T1)^2 = d\left(1-\frac{1}{x}, 1\right)^2 = 1$$

and

$$d(x,T1)d(1,Tx) = \max\{x,1\}^2 \max\left\{1, 1-\frac{1}{x}\right\}^2 = x^2 \times 1 \geq 2^2.$$

Thus, if $d(x,Tx)d(y,Ty) = 0$, (11) is satisfied for $e^{-2\tau} = \frac{1}{2^2}$ or $\tau = \frac{2\ln 2}{3}$ and hence T is a Kannan F-contractive type mapping.

Hence, by the above theorem, T has a unique fixed point, the fixed point here being $x = 1$.

Incidentally, further calculation reveals that T is also an F-contractive type mapping with $F(x) = \log x$.

In [19], Garai et al. defined boundedly compact metric spaces as metric spaces in which every bounded sequence has a convergent subsequence. The same definition may be extended to b-metric spaces as well. It may be noted that this condition is slightly weaker than the condition of sequential compactness as the set of real numbers $\mathbb{R}$ with the usual metric is boundedly compact but not sequentially compact. It may also be noted that the completeness condition in the above result can be replaced by boundedly compactness condition of b-metric spaces.

Theorem 2.14 *Let (X,d,s) be a boundedly compact b-metric space and $T : X \longrightarrow X$ be a Kannan F-contractive type mapping. Then T is a Picard operator.*

Proof Let x_0 be an arbitrary point of X. Consider the iterated sequence $\{x_n\}$, where $x_n = T^n x_0$ for every $n \in \mathbb{N}$. We denote $d(x_n, x_{n+1})$ by μ_n and suppose that $\mu_n > 0$ for all $n \in \mathbb{N}$. Then we have

$$\begin{aligned}\tau + F(s\mu_n) &= F\left(sd\left(T^n x_0, T^{n+1}x_0\right)\right) = F\left(sd\left(T\left(T^{n-1}x_0\right), T\left(T^n x_0\right)\right)\right) \\ &\leq \frac{1}{2}\left\{F\left(d\left(T^{n-1}x_0, T^n x_0\right)\right) + F\left(d\left(T^n x_0, T^{n+1}x_0\right)\right)\right\} \\ &= \frac{1}{2}\left\{F(p\mu_{n-1}) + F(p\mu_n)\right\} \leq \frac{1}{2}F(p\mu_{n-1}) + \frac{1}{2}F(\mu_n).\end{aligned}$$

This implies

$$\tau' + F(s\mu_n) \leq F(\mu_{n-1}) \quad \forall n \in \mathbb{N}.$$

The rest of the proof is similar to that of Theorem 2.12, using the fact that a Cauchy sequence is bounded. The uniqueness of the fixed point is derived from (11). □

Example 2.15 Consider the b-metric space (X, d, s) given in Example 1.8. It is clear that every bounded sequence in X has a convergent sequence. Define $T : X \longrightarrow X$ by

$$T(x) = \begin{cases} \frac{1}{2}, & 0 \leq x \leq 2, \\ \frac{1}{x}, & x > 2. \end{cases}$$

Then T is a continuous Kannan F-contractive type mapping for $e^{-3\tau} = \frac{1}{4}$ and hence by Theorem 2.14, T has a unique fixed point, which is $x = \frac{1}{2}$, here.

The proofs given for Theorems 2.2 and 2.12 do not hold if we replace the constants $\frac{1}{3}$ and $\frac{1}{2}$ by p with $p \neq \frac{1}{3}$ and $p \neq \frac{1}{2}$, respectively. Here, we try to increase the value of $p = \frac{1}{2}$ in Definition 2.10 and derive an existence result by assuming T to be an asymptotically regular mapping. For a metric space (X, d), a mapping $T : X \longrightarrow X$ is called *asymptotically regular* [10] if

$$\lim_{n\to\infty} d\big(T^n x, T^{n+1} x\big) = 0 \quad \text{for all } x \in X.$$

For further details in asymptotic regular mappings we refer to [5, 11] and the references therein.

Theorem 2.16 *Let (X, d, s) be a complete b-metric space and $T : X \longrightarrow X$ be an asymptotically regular mapping such that, for some $\tau > 0$, $d(x, Tx)d(y, Ty) \neq 0$ implies*

$$\tau + F\big(sd(Tx, Ty)\big) \leq F\big(d(x, Tx)\big) + F\big(d(y, Ty)\big) \tag{12}$$

and $d(x, Tx)d(y, Ty) = 0$ implies

$$\tau + F\big(sd(Tx, Ty)\big) \leq F\big(d(x, Ty)\big) + F\big(d(y, Tx)\big) \tag{13}$$

for all $x, y \in X$. Then T has a fixed point $z \in X$.

Proof Let $x_0 \in X$ be an arbitrary point (but fixed) and consider the sequence $\{x_n\}$, where $x_n = T^n x_0$, $n \in \mathbb{N}$. Denote $d(x_n, x_{n+1})$ by μ_n and suppose that $\mu_n > 0$ for all $n \in \mathbb{N}$. Since T is asymptotically regular, we have

$$\lim_{n\to\infty} \mu_n = 0.$$

Now, since $Tx_n \neq x_n$ for all $n \in \mathbb{N}$, we have for $n < m \in \mathbb{N}$

$$\begin{aligned} \tau + F\big(sd(x_{n+1}, x_{m+1})\big) &\leq F\big(d\big(T^n x_0, T^{n+1} x_0\big)\big) + F\big(d\big(T^m x_0, T^{m+1} x_0\big)\big) \\ &= F(\mu_n) + F(\mu_m). \end{aligned}$$

In the limit as $n \to \infty$ we get

$$\lim_{n\to\infty} F\big(sd(x_{n+1}, x_{m+1})\big) = -\infty$$

or

$$\lim_{n\to\infty} sd(x_{n+1}, x_{m+1}) = 0,$$

showing that $\{x_n\}$ is a Cauchy sequence. The completeness of X ensures the existence of $z \in X$ such that

$$\lim_{n\to\infty} x_n = z.$$

By Lemma 1.2, we get $\lim_{n\to\infty} d(x_n, z) = 0$. Also, from (12) we have for all $n \in \mathbb{N}$

$$\tau + F\big(sd(Tz, Tx_n)\big) \leq F\big(sd(z, Tz)\big) + F\big(d(x_n, Tx_n)\big).$$

Hence in the limit as $n \to \infty$, we get (since $\lim_{n\to\infty} d(x_n, Tx_n) = 0$)

$$\tau + \lim_{n\to\infty} F\big(sd(Tz, Tx_n)\big) \leq -\infty,$$

that is, $\lim_{n\to\infty} d(Tz, x_{n+1}) = 0$.

Since the convergent sequence $\{x_n\}$ converges to both z and Tz, it must be the case that $Tz = z$, as required. □

Remark 2.17 From the given examples of F-contractive type mappings and Kannan F-contractive type mappings, we note that the mapping given in Example 1.8 is an F-contractive type mapping but not a Kannan F-contractive type mapping. The F-contractive type mappings given in Examples 2.4 and 2.6 are also Kannan F-contractive type mappings. And finally, the Kannan F-contractive type mappings given in Examples 2.13 and 2.15 are also F-contractive type mappings, with $F(x) = \log x$.

F-expanding type mappings

In [20] Górnicki defined F-expanding mappings and proved a fixed point theorem using the result in [46]. In a similar manner, we define new types of F-expanding mapping and prove a fixed point result in b-metric spaces.

Definition 2.18 A mapping $T : X \longrightarrow X$ is said to be an F-expanding type mapping if there exists $t > 0$ such that $d(x, Tx)d(y, Ty) \neq 0$ implies

$$t + F\big(sd(x, y)\big) \leq \frac{1}{3}\big\{F\big(d(Tx, Ty)\big) + F\big(d(x, Tx)\big) + F\big(d(y, Ty)\big)\big\} \tag{14}$$

and $d(x, Tx)d(y, Ty) = 0$ implies

$$t + F\big(sd(x, y)\big) \leq \frac{1}{3}\big\{F\big(d(Tx, Ty)\big) + F\big(d(x, Ty)\big) + F\big(d(y, Tx)\big)\big\} \tag{15}$$

for all $x, y \in X$.

As in Example 1.7, F-expanding type conditions for F_1, F_2 and F_3 may similarly be derived.

The idea behind the following results is from the paper of Górnicki [20], in which he used the following lemma for metric spaces. It may be restated for b-metric spaces.

Lemma 2.19 ([20]) *Let (X,d,s) be a b-metric space and $T : X \longrightarrow X$ be surjective. Then there exists a mapping $T^* : X \longrightarrow X$ such that $T \circ T^*$ is the identity map on X.*

Theorem 2.20 *Let (X,d,s) be a complete b-metric space and $T : X \longrightarrow X$ be surjective and an F-expanding type mapping. Then T has a unique fixed point $z \in X$.*

Proof By Lemma 2.19, there exists a mapping $T^* : X \longrightarrow X$ such that $T \circ T^*$ is the identity map on X. Let x and y be arbitrary points of X such that $x \neq y$, and let $u = T^*x$ and $v = T^*y$. It is obvious that $u \neq v$. Applying (14) on u and v, we have, for $d(u, Tu)d(v, Tv) \neq 0$,

$$\tau + F\bigl(sd(u,v)\bigr) \leq \frac{1}{3}\bigl\{F\bigl(d(Tu,Tv)\bigr) + F\bigl(d(u,Tu)\bigr) + F\bigl(d(v,Tv)\bigr)\bigr\}$$

and, for $d(x, Tx)d(y, Ty) = 0$,

$$\tau + F\bigl(sd(u,v)\bigr) \leq \frac{1}{3}\bigl\{F\bigl(d(Tu,Tv)\bigr) + F\bigl(d(u,Tv)\bigr) + F\bigl(d(v,Tu)\bigr)\bigr\}.$$

Since $Tu = T(T^*(x)) = x$ and $Tv = T(T^*y) = y$, we get

$$\tau + F\bigl(sd\bigl(T^*x,T^*y\bigr)\bigr) \leq \frac{1}{3}\bigl\{F\bigl(d(x,y)\bigr) + F\bigl(d\bigl(x,T^*x\bigr)\bigr) + F\bigl(d\bigl(y,T^*y\bigr)\bigr)\bigr\}$$

for $d(x, Tx)d(y, Ty) \neq 0$ and

$$\tau + F\bigl(sd\bigl(T^*x,T^*y\bigr)\bigr) \leq \frac{1}{3}\bigl\{F\bigl(d(x,y)\bigr) + F\bigl(d\bigl(x,T^*y\bigr)\bigr) + F\bigl(d\bigl(y,T^*x\bigr)\bigr)\bigr\}$$

for $d(x, Tx)d(y, Ty) = 0$, showing that T^* is an F-contractive type mapping.

By Theorem 2.2, T^* has a unique fixed point $z \in X$ and for every $x_0 \in X$ the sequence $\{T^{*n}x_0\}$ converges to z. In particular, z is also a fixed point of T since $T^*z = z$ implies that $Tz = T(T^*z) = z$.

Finally, if $w = Tw$ is another fixed point, then from (15)

$$\tau + F\bigl(sd(z,w)\bigr) \leq \frac{1}{3}\bigl\{F\bigl(d(Tz,Tw)\bigr) + F\bigl(d(z,Tw)\bigr) + F\bigl(d(w,Tz)\bigr)\bigr\}$$

or

$$\tau + \frac{2}{3}F\bigl(sd(z,w)\bigr) \leq \frac{2}{3}F\bigl(d(z,w)\bigr),$$

which is not possible, and hence the fixed point of T is unique. □

In a similar manner, we can define a *Kannan F-expanding type mapping* by the relation

$$t + F\bigl(sd(x,y)\bigr) \leq \frac{1}{2}\bigl\{F\bigl(d(x,Tx)\bigr) + F\bigl(d(y,Ty)\bigr)\bigr\}$$

if $d(x,Tx)d(y,Ty) \neq 0$, and

$$t + F\big(sd(x,y)\big) \leq \frac{1}{2}\big\{F\big(d(x,Tx)\big) + F\big(d(y,Ty)\big)\big\}$$

if $d(x,Tx)d(y,Ty) = 0$, and prove the following.

Theorem 2.21 *Let (X,d,s) be a complete b-metric space and $T : X \longrightarrow X$ be surjective and a Kannan F-expanding type mapping. Then T has a unique fixed point $z \in X$.*

Remark 2.22 The above results may also be extended to other generalized metric spaces. There are many applications of b-metric spaces in different directions (one may refer to [1, 2, 4, 9]). In view of this, applications of the results obtained in this paper may also be investigated as regards different aspects.

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References

1. Afshari, H., Aydi, H., Karapinar, E.: Existence of fixed points of set-valued mappings in *b*-metric spaces. East Asian Math. J. **32**(3), 319–332 (2016). https://doi.org/10.7858/eamj.2016.024
2. Aksoy, U., Karapinar, E., Erhan, I.M.: Fixed points of generalized α-admissible contractions on *b*-metric spaces with an application to boundary value problems. J. Nonlinear Convex Anal. **17**(6), 1095–1108 (2016)
3. Alsulami, H.H., Karapinar, E., Piri, H.: Fixed points of generalised F-Suzuki type contraction in complete *b*-metric spaces. Discrete Dyn. Nat. Soc. **2015**, Article ID 969726 (2015)
4. Aydi, H., Bota, M.-F., Karapinar, E., Moradi, S.: A common fixed point for weak-ϕ-contractions on *b*-metric spaces. Fixed Point Theory **13**(2), 337–346 (2012)
5. Baillon, J.B., Bruck, R.E., Reich, S.: On the asymptotic behaviour of non-expansive mappings and semi-groups in Banach spaces. Houst. J. Math. **4**, 1–9 (1978)
6. Bakhtin, I.A.: The contraction mapping principle in almost metric spaces. Funct. Anal. **30**, 26–37 (1989)
7. Banach, S.: Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales. Fundam. Math. **3**, 133–181 (1922)
8. Boriceanu, M.: Fixed point theory for multivalued generalized contraction on a set with two *b*-metric. Stud. Univ. Babeş–Bolyai, Math. **3**, 1–14 (2009)
9. Bota, M.-F., Karapinar, E.: A note on "Some results on multi-valued weakly Jungck mappings in b-metric space". Cent. Eur. J. Math. **11**(9), 1711–1712 (2013). https://doi.org/10.2478/s11533-013-0272-2
10. Browder, F.E., Petryshyn, W.V.: The solution by iteration of non-linear functional equations in Banach spaces. Bull. Am. Math. Soc. **72**, 571–575 (1966)

11. Bruck, R.E., Reich, S.: Non-expansive projections and resolvents of accretive operators in Banach spaces. Houst. J. Math. **3**, 459–470 (1977)
12. Cobzas, S.: Fixed points and completeness in metric and in generalized metric spaces (2016). arXiv:1508.05173v4 [math.FA]
13. Cosentino, M., Jleli, M., Samet, B., Vetro, C.: Solvability of integrodifferential problems via fixed point theory in *b*-metric spaces. Fixed Point Theory Appl. **2015**, 70 (2015). https://doi.org/10.1186/s13663-015-0317-2
14. Czerwik, S.: Contraction mappings in *b*-metric spaces. Acta Math. Inform. Univ. Ostrav. **1**, 5–11 (1993)
15. Das, D., Goswami, N.: Some fixed point theorems on the sum and product of operators in tensor product spaces. Int. J. Pure Appl. Math. **109**, 651–663 (2016)
16. Das, D., Goswami, N., Mishra, V.N.: Some results on fixed point theorems in Banach algebras. Int. J. Anal. Appl. **13**, 32–40 (2017)
17. Das, D., Goswami, N., Mishra, V.N.: Some results on the projective cone normed tensor product spaces over Banach algebras. Bol. Soc. Parana. Mat. **38**(1), 197–221 (2020) (online available). https://doi.org/10.5269/bspm.v38i1.36450
18. Deepmala: A study on fixed point theorems for nonlinear contractions and its applications. Ph.D. Thesis, Pt. Ravishankar Shukla University, Raipur (2014)
19. Garai, H., Senapati, T., Dey, L.K.: A study on Kannan type contractive mappings (2017). arXiv:1707.06383v1 [math.FA]
20. Górnicki, J.: Fixed point theorems for *F*-expanding mappings. Fixed Point Theory Appl. **2017**, 9 (2017). https://doi.org/10.1186/s13663-017-0602-3
21. Górnicki, J.: Remarks on contractive type mappings. Fixed Point Theory Appl. **2017**, 8 (2017). https://doi.org/10.1186/s13663-017-0601-4
22. Haokip, N., Goswami, N.: Some fixed point theorems for generalized Kannan type mappings in *b*-metric spaces. Proyecciones J. Math. **38**(4) (2019, in press)
23. Hu, T.K.: On a fixed-point theorem for metric spaces. Am. Math. Mon. **74**, 436–437 (1967)
24. Hussain, N., Dorić, D., Kadelburg, Z., Radenović, S.: Suzuki-type fixed point results in metric type spaces. Fixed Point Theory Appl. **2012**, 126 (2012)
25. Hussain, N., Salimi, P.: Suzuki–Wardowski type fixed point theorems for α-GF-contractions. Taiwan. J. Math. **18**(6), 1879–1895 (2014). https://doi.org/10.11650/tjm.18.2014.4462
26. Jovanović, M., Kadelburg, Z., Radenović, S.: Common fixed point results in metric-type spaces. Fixed Point Theory Appl. **2010**, Article ID 978121 (2010)
27. Kannan, R.: Some results on fixed points. Bull. Calcutta Math. Soc. **60**, 71–76 (1968)
28. Khamsi, M.A.: Remarks on cone metric spaces and fixed point theorems of contractive mappings. Fixed Point Theory Appl. **2010**, Article ID 315398 (2010)
29. Liu, X., Zhou, M., Mishra, L.N., Mishra, V.N., Damjanović, B.: Common fixed point theorem of six self-mappings in Menger spaces using (CLR_{ST}) property. Open Math. **16**, 1423–1434 (2018)
30. Lukács, A., Kajántó, S.: Fixed point theorems for various types of *F*-contractions in complete *b*-metric spaces. Fixed Point Theory **19**(1), 321–334 (2018). https://doi.org/10.24193/fpt-ro.2018.1.25
31. Mınak, G., Helvacı, A., Altun, I.: Ćirić type generalized F-contractions on complete metric spaces and fixed point results. Filomat **28**(6), 1143–1151 (2014). https://doi.org/10.2298/FIL1406143M
32. Mishra, L.N.: On existence and behavior of solutions to some nonlinear integral equations with applications. Ph.D. Thesis, National Institute of Technology, Silchar (2017)
33. Mishra, L.N., Jyoti, K., Rani Vandana, A.: Fixed point theorems with digital contractions image processing. Nonlinear Sci. Lett. A **9**(2), 104–115 (2018)
34. Mishra, L.N., Tiwari, S.K., Mishra, V.N.: Fixed point theorems for generalized weakly S-contractive mappings in partial metric spaces. J. Appl. Anal. Comput. **5**(4), 600–612 (2015). https://doi.org/10.11948/2015047
35. Mishra, L.N., Tiwari, S.K., Mishra, V.N., Khan, I.A.: Unique fixed point theorems for generalized contractive mappings in partial metric spaces. J. Funct. Spaces **2015**, Article ID 960827 (2015)
36. Mohanta, S.K.: Coincidence points and common fixed points for expansive type mappings in *b*-metric spaces. Iran. J. Math. Sci. Inform. **11**(1), 101–113 (2016). https://doi.org/10.7508/ijmsi.2016.01.009
37. Pathak, H.K., Deepmala: Common fixed point theorems for PD-operator pairs under relaxed conditions with applications. J. Comput. Appl. Math. **239**, 103–113 (2013)
38. Patir, B., Goswami, N., Mishra, V.N.: Some results on fixed point theory for a class of generalized nonexpansive mappings. Fixed Point Theory Appl. **2018**, 19 (2018). https://doi.org/10.1186/s13663-018-0644-1
39. Piri, H., Kumam, P.: Some fixed point theorems concerning F-contraction in complete metric spaces. Fixed Point Theory Appl. **2014**, 210 (2014). https://doi.org/10.1186/1687-1812-2014-210
40. Roshan, J.R., Parvaneh, V., Sedghi, S., Shobkolaei, N., Shatanawi, W.: Common fixed points of almost generalized $(\psi,\varphi)_s$-contractive mappings in ordered *b*-metric spaces. Fixed Point Theory Appl. **2013**, 159 (2013). https://doi.org/10.1186/1687-1812-2013-159
41. Rus, I.A.: Picard operators and applications. Sci. Math. Jpn. **58**(1), 191–219 (2003)
42. Secelean, N.A.: Weak F-contractions and some fixed point results. Bull. Iran. Math. Soc. **42**(3), 779–798 (2016)
43. Sintunavarat, W., Plubtieng, S., Katchang, P.: Fixed point result and applications on *b*-metric space endowed with an arbitrary binary relation. Fixed Point Theory Appl. **2013**, 296 (2013)
44. Subrahmanyam, V.: Completeness and fixed points. Monatshefte Math. **80**, 325–330 (1975)
45. Udo-utun, X.: On inclusion of *F*-contractions in (δ, k)-weak contractions. Fixed Point Theory Appl. **2014**, 65 (2014). https://doi.org/10.1186/1687-1812-2014-65
46. Wardowski, D.: Fixed points of a new type of contractive mappings in complete metric spaces. Fixed Point Theory Appl. **2012**, 94 (2012) https://doi.org/10.1186/1687-1812-2012-94
47. Wardowski, D., Van Dung, N.: Fixed points of F-weak contractions on complete metric spaces. Demonstr. Math. **47**(1), 146–155 (2014). https://doi.org/10.2478/dema-2014-0012