Pragadeeswarar and Gopi *Fixed Point Theory Algorithms Sci Eng* (2023) 2023:19
https://doi.org/10.1186/s13663-023-00757-8

Fixed Point Theory and Algorithms for Sciences and Engineering

RESEARCH Open Access

Existence and convergence of best proximity points for generalized pseudo-contractive and Lipschitzian mappings via an Ishikawa-type iterative scheme

V. Pragadeeswarar[1*] and R. Gopi[2]

*Correspondence: v_pragadeeswarar@cb.amrita.edu
[1]Department of Mathematics, Amrita School of Physical Sciences, Amrita Vishwa Vidyapeetham, Coimbatore, India
Full list of author information is available at the end of the article

Abstract

In this article, we prove the existence of the best proximity point for the class of nonself generalized pseudo-contractive and Lipschitzian mappings. Also, we approximate the best proximity point through the proposed Ishikawa's iteration process for the case of nonself-mappings. Finally, we provide an example to illustrate our main result.

Mathematics Subject Classification: Primary 41A65; secondary 47H10; 46B20; 54H25

Keywords: Pseudo-contractive mappings; Lipschitzian mappings; Best proximity points; Fixed points; *P*-property

1 Introduction

Assume that M and N are nonempty subsets of a metric space (X,d). If $M \cap N = \emptyset$, then the mapping f from M to N does not have a solution for the fixed-point equation $f(\eta) = \eta$. When the fixed-point equation does not possess a solution, then it is attempted to determine an approximate solution η such that the error $d(\eta, f\eta)$ is minimum. In this situation, the best proximity-point theorems guarantee the existence and uniqueness of such an optimization for the fixed-point equations. Naturally, the best proximity point for the nonself-mappings is defined as follows:

Definition 1.1 Let M, N be two nonempty and disjoint subsets of a metric space (X,d). A mapping $\Gamma : M \to N$ is said to have a best proximity point if there exist $\eta^* \in M$ such that $d(\eta^*, \Gamma\eta^*) = d(M,N)$.

Many researchers have proved the existence results on the best proximity points for various kinds of contractions. For such results, one may refer to [2, 4, 6–8, 12, 13, 15–18]. Recently, researchers have shown an interest in approximating the best proximity points through well-known iterative processes that may be seen in [1, 3, 9–11, 14, 19, 20].

On the other hand, numerous research articles have been published on the convergence of fixed points for the class of self- and nonself-contractive-type mappings in metric spaces, Hilbert spaces, and several classes of Banach spaces. For further exploration of this topic, we refer to the monograph [5] and the references cited therein.

A fundamental result in metric fixed-point theory is the following theorem, which uses the Picard iteration method.

Theorem 1.2 [5] *Let (X,d) be a complete metric space and $\Gamma : X \to X$ be a contraction, that is an operator satisfying*

$$d(\Gamma\eta,\Gamma\omega) \le ad(\eta,\omega), \quad \text{for any } \eta,\omega \in X,$$

with $a \in [0,1)$ fixed. Then, Γ has a unique fixed point.

One of the effective methods for approaching the fixed point of a mapping $\Gamma : X \to X$ is the Ishikawa iteration scheme, starting with any $\eta_0 \in X$ and for $n \ge 0$ defined by

$$\eta_{n+1} = (1-\gamma_n)\eta_n + \gamma_n\Gamma\big((1-\delta_n)\eta_n + \delta_n\Gamma\eta_n\big),$$

where $\gamma_n, \delta_n \in [0,1]$. In this direction, we state the following theorem on the iterative approximation of a fixed point that was proved by Ishikawa [11], for Lipschitzian pseudo-contractive mapping.

Theorem 1.3 [11] *Let K be a convex and compact subset of a Hilbert space H and let $\Gamma : K \to K$ be Lipschitzian pseudo-contractive and let $\eta_1 \in K$. Then, the Ishikawa iteration $\{\eta_n\}$, defined by*

$$\eta_{n+1} = (1-\gamma_n)\eta_n + \gamma_n\Gamma\big[(1-\delta_n)\eta_n + \delta_n\Gamma\eta_n\big],$$

where $\{\gamma_n\}$, $\{\delta_n\}$ are sequences of positive numbers satisfying

$$(i) \quad 0 \le \gamma_n \le \delta_n \le 1, \quad n \ge 1; \qquad (ii) \quad \lim_{n\to\infty} \delta_n = 0; \qquad (iii) \quad \sum_{n=1}^{\infty} \gamma_n\delta_n = \infty,$$

converges strongly to a fixed point of Γ.

The next result gives sufficient conditions to obtain a fixed point without assuming the Lipschitzian condition.

Theorem 1.4 [5] *Let K be a closed, bounded, and convex subset of a real uniformly convex Banach space H. Let $\Gamma : K \to K$ a strongly pseudo-contractive that has at least a fixed point η^*. Let $\eta_1 \in K$, then the Ishikawa iteration $\{\eta_n\}$, defined by*

$$\eta_{n+1} = (1-\gamma_n)\eta_n + \gamma_n\Gamma\big[(1-\delta_n)\eta_n + \delta_n\Gamma\eta_n\big],$$

where $\{\gamma_n\}$, $\{\delta_n\}$ *are sequences of positive numbers satisfying*

$$(i) \quad 0 \le \gamma_n, \delta_n < 1, \quad n \ge 1; \qquad (ii) \quad \lim_{n\to\infty} \gamma_n = 0, \qquad \lim_{n\to\infty} \delta_n = 0;$$

$$(iii) \quad \sum_{n=1}^{\infty} \gamma_n = \infty,$$

converges strongly to a fixed point of Γ.

Motivated by Theorems 1.3 and 1.4, a natural question arises: how can one construct the Ishikawa iteration for nonself-mappings that approximate the best proximity point of such mappings? In this context, we will initiate the construction of the Ishikawa iteration process for nonself-mappings and investigate the convergence results for the best proximity point.

Before presenting the iterative approximation for the best proximity point, let us establish the existence of a best proximity point. To do so, we will recall some basic notions and definitions:

Let M and N be two subsets of a Hilbert space H with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$:

$$dist(M,N) = d(M,N) = inf\big\{\|\eta - \omega\| : \eta \in M, \omega \in N\big\};$$

$$P_M(\eta) = \big\{\omega \in M : \|\eta - \omega\| = d(\eta, M)\big\};$$

$$M_0 = \big\{\eta \in M : \|\eta - \omega'\| = d(M,N) \text{ for some } \omega' \in N\big\};$$

$$N_0 = \big\{\omega \in N : \|\eta' - \omega\| = d(M,N) \text{ for some } \eta' \in M\big\}.$$

In [13], Kirk et al. proved the following lemma that guarantees the nonemptiness of M_0 and N_0.

Lemma 1.5 *Let* X *be a reflexive Banach space and* M *be a nonempty, closed, bounded, and convex subset of* X, *and* N *be a nonempty, closed, and convex subset of* X. *Then,* M_0 *and* N_0 *are nonempty and satisfy* $P_N(M_0) \subseteq N_0$, $P_M(N_0) \subseteq M_0$.

Definition 1.6 Let H be a Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. An operator $\Gamma : H \to H$ is said to be Lipschitzian if there exists a constant $s > 0$ such that, for all η, ω in H,

$$\|\Gamma\eta - \Gamma\omega\| \le s\|\eta - \omega\|.$$

Definition 1.7 Let H be a Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. An operator $\Gamma : H \to H$ is said to be a generalized pseudo-contraction if there exists a constant $r > 0$ such that, for all η, ω in H,

$$\|\Gamma\eta - \Gamma\omega\|^2 \le r^2\|\eta - \omega\|^2 + \big\|\Gamma\eta - \Gamma\omega - r(\eta - \omega)\big\|^2. \tag{1}$$

Remark 1.8

1. The condition (1), is equivalent to $\langle \Gamma\eta - \Gamma\omega, \eta - \omega \rangle \le r\|\eta - \omega\|^2$.
2. If $r = 1$, then a generalized pseudo-contraction reduces to a pseudo-contraction.

Definition 1.9 Let H be a Banach space with norm $\|\cdot\|$. An operator $\Gamma : H \to H$ is said to be strongly pseudo-contraction if there exists a constant $t > 1$ such that

$$\|\eta - \omega\| \leq \left\|(1+c)(\eta - \omega) - ct(\Gamma\eta - \Gamma\omega)\right\|$$

holds for all η, ω in H and $c > 0$.

In this work, we begin by providing a set of sufficient conditions for the existence of a best proximity point for nonself-Lipschitzian, generalized pseudo-contractive mappings. Subsequently, we construct the Ishikawa iteration for nonself-mappings and establish convergence results for the best proximity point of Lipschitzian pseudo-contractive nonself-mappings. To support our main result, we present an illustrative example.

Furthermore, we delve into the convergence of the best proximity point for strongly pseudo-contractive mappings without imposing the Lipschitzian condition. This discussion expands the scope of our findings and highlights the applicability of our results in a broader class of mappings.

2 Main results

Let us prove the existence result of the best proximity point for nonself-generalized pseudo-contractive and Lipschitzian mapping in the Hilbert space settings.

Theorem 2.1 *Let M, N be two closed and convex subsets of a real Hilbert space H assume M to be bounded. Let $\Gamma : M \to N$ be a generalized, pseudo-contractive, and Lipschitzian mapping with corresponding constants r and s such that $0 < r < 1$, $s > 1$. If $\Gamma(M_0) \subseteq N_0$, then Γ has a unique best proximity point.*

Proof Let $\lambda \in (0,1)$ satisfying, $0 < \lambda < \frac{2(1-r)}{(1-2r+s^2)}$. We consider a projection operator on M_0, that is, $P_{M_0} : \Gamma(M_0) \to M_0$. Also, we define an averaged operator $F : M_0 \to M_0$, associated with $P_{M_0}\Gamma$,

$$F(\eta) = (1-\lambda)\eta + \lambda P_{M_0}\Gamma\eta, \quad \text{for } \eta \in M_0. \tag{2}$$

Since Γ is generalized, pseudo-contractive, and Lipschitzian, we have

$$\begin{aligned}
\|F\eta - F\omega\|^2 &= \left\|(1-\lambda)\eta + \lambda P_{M_0}\Gamma\eta - (1-\lambda)\omega - \lambda P_{M_0}\Gamma\omega\right\|^2 \\
&= \left\|(1-\lambda)(\eta - \omega) + \lambda(P_{M_0}\Gamma\eta - P_{M_0}\Gamma\omega)\right\|^2 \\
&= (1-\lambda)^2\|\eta - \omega\|^2 + 2\lambda(1-\lambda)\langle P_{M_0}\Gamma\eta - P_{M_0}\Gamma\omega, \eta - \omega\rangle \\
&\quad + \lambda^2\|P_{M_0}\Gamma\eta - P_{M_0}\Gamma\omega\|^2.
\end{aligned} \tag{3}$$

Let us assume $u = \Gamma\eta - P_{M_0}\Gamma\eta$ and $v = \Gamma\omega - P_{M_0}\Gamma\omega$. Now, we claim that $u = v$. Suppose $u \neq v$, then by the strict convexity of H, we have

$$\begin{aligned}
\left\|\frac{\Gamma\eta + \Gamma\omega}{2} - \frac{P_{M_0}\Gamma\eta + P_{M_0}\Gamma\omega}{2}\right\| &= \left\|\frac{u+v}{2}\right\| \\
&< max\left\{\|u\|, \|v\|\right\}
\end{aligned}$$

$$= d(M,N),$$

which is a contradiction. Therefore, $u = v$. This implies that, $\Gamma\eta - \Gamma\omega = P_{M_0}\Gamma\eta - P_{M_0}\Gamma\omega$. Therefore, from (3), we obtain

$$\begin{aligned}
\|F\eta - F\omega\|^2 &= (1-\lambda)^2\|\eta-\omega\|^2 + 2\lambda(1-\lambda)\langle\Gamma\eta-\Gamma\omega,\eta-\omega\rangle + \lambda^2\|\Gamma\eta-\Gamma\omega\|^2 \\
&\le (1-\lambda)^2\|\eta-\omega\|^2 + 2\lambda(1-\lambda)r\|\eta-\omega\|^2 + \lambda^2 s^2\|\eta-\omega\|^2 \\
&= \left((1-\lambda)^2 + 2\lambda(1-\lambda)r + \lambda^2 s^2\right)\|\eta-\omega\|^2.
\end{aligned}$$

Then, $\|F\eta - F\omega\| \le ((1-\lambda)^2 + 2\lambda(1-\lambda)r + \lambda^2 s^2)^{1/2}\|\eta-\omega\|$.

Now, from $0 < \lambda < \frac{2(1-r)}{(1-2r+s^2)}$, we obtain

$$\begin{aligned}
&\lambda^2\left(1-2r+s^2\right) < 2\lambda(1-r) = 2\lambda(1-r) + 1 - 1, \\
&1 + \lambda^2 - 2\lambda + 2\lambda r - 2\lambda^2 r + \lambda^2 s^2 < 1, \\
&\left((1-\lambda)^2 + 2\lambda(1-\lambda)r + \lambda^2 s^2\right)^{1/2} < 1.
\end{aligned}$$

This implies that F is contraction. By Theorem 1.2, F has a unique fixed point $p^* \in M_0$. Then, $P_{M_0}\Gamma p^* = p^*$. This implies that $d(p^*,\Gamma p^*) = d(M,N)$. □

Remark 2.2

1. If $0 < s < 1$, then Γ is a contraction nonself-mapping and the result follows from [15].
2. If $s = 1$, then Γ is a nonexpansive nonself-mapping and the result follows from [18].

Now, we define a construction of Ishikawa iteration for the case of nonself-mapping:

Let M, N be two convex subsets of a Hilbert space H. Let us define $\Gamma : M \to N$ and assume $\Gamma(M_0) \subseteq N_0$. Consider the projective operator $P_{M_0}\Gamma : M_0 \to M_0$. Let $\eta_1 \in M_0$, then the Ishikawa iteration $\{\eta_n\}$, is defined by

$$\eta_{n+1} = (1-\gamma_n)\eta_n + \gamma_n P_{M_0}\Gamma\left[(1-\delta_n)\eta_n + \delta_n P_{M_0}\Gamma\eta_n\right], \quad n = 1,2,3,\ldots, \tag{4}$$

where $\gamma_n, \delta_n \in [0,1]$.

Next, we extend the convergence result of Theorem 1.3, for the case of nonself-mappings, by using the proposed Ishikawa iteration for nonself-mappings.

Theorem 2.3 *Let M, N be two closed and convex subsets of a Hilbert space H and assume M to be compact. Let $\Gamma : M \to N$ be a pseudo-contractive and Lipschitzian mapping with $\Gamma(M_0) \subseteq N_0$. Let $\eta_1 \in M_0$, then the Ishikawa iteration $\{\eta_n\}$, defined in (4), with $\{\gamma_n\}$, $\{\delta_n\}$ are sequences of positive numbers satisfying*

$$(i) \quad 0 \le \gamma_n \le \delta_n \le 1, \quad n \ge 1; \qquad (ii) \quad \lim_{n\to\infty}\delta_n = 0; \qquad (iii) \quad \sum_{n=1}^{\infty}\gamma_n\delta_n = \infty,$$

converges strongly to a best proximity point of Γ.

Proof First, we prove that the mapping $P_{M_0}\Gamma : M_0 \to M_0$ is pseudo-contractive. It is enough to show that $\langle P_{M_0}\Gamma\eta - P_{M_0}\Gamma\omega, \eta - \omega\rangle \leq \|\eta - \omega\|^2$, for all $\eta, \omega \in M_0$. Now, we assume $x = \Gamma\eta - P_{M_0}\Gamma\eta$ and $y = \Gamma\omega - P_{M_0}\Gamma\omega$. Now, we claim that $x = y$. Suppose $x \neq y$, then by the strict convexity of H, we have

$$\begin{aligned} \left\| \frac{\Gamma\eta + \Gamma\omega}{2} - \frac{P_{M_0}\Gamma\eta + P_{M_0}\Gamma\omega}{2} \right\| &= \left\| \frac{x+y}{2} \right\| \\ &< max\big\{ \|x\|, \|y\| \big\} \\ &= d(M,N), \end{aligned}$$

which is a contradiction. Therefore, $x = y$. This implies that, $\Gamma\eta - \Gamma\omega = P_{M_0}\Gamma\eta - P_{M_0}\Gamma\omega$. Since Γ is pseudo-contractive, we obtain

$$\langle P_{M_0}\Gamma\eta - P_{M_0}\Gamma\omega, \eta - \omega\rangle = \langle \Gamma\eta - \Gamma\omega, \eta - \omega\rangle \leq \|\eta - \omega\|^2.$$

Now, using that Γ is a Lipschitzian mapping, there exist $s > 0$, we obtain

$$\|P_{M_0}\Gamma\eta - P_{M_0}\Gamma\omega\| = \|\Gamma\eta - \Gamma\omega\| \leq s\|\eta - \omega\|,$$

which implies that the mapping $P_{M_0}\Gamma : M_0 \to M_0$ is a Lipschitzian operator. Moreover, M_0 satisfies all the requirements of Theorem 1.3. This implies that the sequence $\{\eta_n\}$ converges to a fixed point p^* of $P_{M_0}\Gamma$. Then, $P_{M_0}\Gamma p^* = p^*$. This implies that $d(p^*, \Gamma p^*) = d(M,N)$, that is, p^* is a best proximity point of Γ. This completes the proof. □

The following example illustrates Theorem 2.2.

Example 2.4 Let $H = \mathbb{R}^2$ be a Hilbert space with the Euclidean inner product and norm. Assume $M = \{(0,\eta) : 1/2 \leq \eta \leq 2\}$, $N = \{(1,\eta) : 1/2 \leq \eta \leq 2\}$. Clearly, $M_0 = M$, $N_0 = N$. Now, we define $\Gamma : M \to N$ by $\Gamma(0,\eta) = (1, 1/\eta)$. Then, one can easily verify that Γ is pseudo-contractive and Lipschitzian. Assume $\eta_0 = 0.5$, $\gamma_n = \delta_n = \frac{1}{\sqrt{n}}$ for all $n \geq 0$. Then,

$$\begin{aligned} (0,\eta_{n+1}) &= (1-\gamma_n)(0,\eta_n) + \gamma_n P_{M_0}\Gamma\big[(1-\delta_n)(0,\eta_n) + \delta_n P_{M_0}\Gamma(0,\eta_n)\big] \\ &= \left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)(0,\eta_n) + \frac{1}{\sqrt{n}} P_{M_0}\Gamma\left[\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)(0,\eta_n) + \frac{1}{\sqrt{n}} P_{M_0}\left(1, \frac{1}{\eta_n}\right)\right] \\ &= \left(0, \left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)\eta_n\right) + \frac{1}{\sqrt{n}} P_{M_0}\Gamma\left[\left(0, \left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)\eta_n\right) + \frac{1}{\sqrt{n}}\left(0, \frac{1}{\eta_n}\right)\right] \\ &= \left(0, \left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)\eta_n\right) + \frac{1}{\sqrt{n}} P_{M_0}\Gamma\left[\left(0, \left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)\eta_n + \frac{1}{\sqrt{n}\eta_n}\right)\right] \\ &= \left(0, \left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)\eta_n\right) + \frac{1}{\sqrt{n}} P_{M_0}\Gamma\left[\left(0, \frac{(\sqrt{n}-1)\eta_n^2 + 1}{\sqrt{n}\eta_n}\right)\right] \\ &= \left(0, \left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)\eta_n\right) + \frac{1}{\sqrt{n}} P_{M_0}\Gamma\left[\left(0, \frac{(\sqrt{n}-1)\eta_n^2 + 1}{\sqrt{n}\eta_n}\right)\right] \\ &= \left(0, \left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)\eta_n\right) + \frac{1}{\sqrt{n}} P_{M_0}\left[\left(1, \frac{\sqrt{n}\eta_n}{(\sqrt{n}-1)\eta_n^2 + 1}\right)\right] \end{aligned}$$

$$= \left(0, \left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)\eta_n\right) + \left(0, \frac{\eta_n}{(\sqrt{n}-1)\eta_n^2+1}\right)$$
$$= \left(0, \frac{\sqrt{n}-1}{\sqrt{n}}\eta_n + \frac{\eta_n}{\sqrt{n}\eta_n^2-\eta_n^2+1}\right).$$

As $n \to \infty$, the Ishikawa iteration $(0, \eta_{n+1}) \to (0, 1)$, in particular, at $(0, \eta_{118}) = (0, 1)$, reaches the best proximity point of Γ. This result is achieved by simple Matlab coding.

Finally, we approximate the best proximity point for strongly pseudo-contractive nonself-mappings without Lipschitzian. This is an extended version of Theorem 1.4, for the case of nonself-mappings.

Theorem 2.5 *Let M, N be two closed, bounded, and convex subsets of a real uniformly convex Banach space H. Let $\Gamma : M \to N$ be a strongly pseudo-contractive that has at least a best proximity point η^* and assume that $\Gamma(M_0) \subseteq N_0$. Let $\eta_1 \in M_0$, then the Ishikawa iteration $\{\eta_n\}$, defined in* (4), *with $\{\gamma_n\}$, $\{\delta_n\}$ being sequences of positive numbers satisfying*

$$(i)\quad 0 \le \gamma_n, \delta_n < 1, \quad n \ge 1; \qquad (ii)\quad \lim_{n\to\infty} \gamma_n = 0, \qquad \lim_{n\to\infty} \delta_n = 0;$$

$$(iii)\quad \sum_{n=1}^{\infty} \gamma_n = \infty,$$

converges strongly to a best proximity point of Γ.

Proof One can easily verify that the mapping $P_{M_0}\Gamma : M_0 \to M_0$ is strongly pseudo-contractive and the result follows by Theorem 1.4. □

Acknowledgements
The authors are grateful to the editor and the referees for their valuable comments and suggestions.

Funding
Not applicable.

Data availability
No data were used to support this study.

Declarations

Ethics approval and consent to participate
Not applicable.

Consent for publication
Not applicable.

Competing interests
The authors declare no competing interests.

Author contributions
All authors reviewed the manuscript.

Author details
[1]Department of Mathematics, Amrita School of Physical Sciences, Amrita Vishwa Vidyapeetham, Coimbatore, India.
[2]Department of Mathematics, School of Engineering, Presidency University, Bengaluru, Karnataka, India.

Received: 19 May 2023 Accepted: 29 November 2023 Published online: 18 December 2023

References

1. Abkar, A., Gabeleh, M.: Results on the existence and convergence of best proximity points. Fixed Point Theory Appl. **2010**, 386037 (2010)
2. Al-Thagafi, M.A., Shahzad, N.: Convergence and existence results for best proximity points. Nonlinear Anal. **70**(10), 3665–3671 (2009)
3. Anthony Eldred, A., Praveen, A.: Convergence of Mann's iteration for relatively nonexpansive mappings. Fixed Point Theory **18**(2), 545–554 (2017)
4. Anthony Eldred, A., Veeramani, P.: Existence and convergence of best proximity points. J. Math. Anal. Appl. **323**, 1001–1006 (2006)
5. Berinde, V.: Iterative Approximation of Fixed Points. Springer, Berlin (2007)
6. Gabeleh, M.: Best proximity point theorem via proximal nonself-mappings. J. Optim. Theory Appl. **164**, 565–576 (2015)
7. Gabeleh, M.: Best proximity points for weak proximal contractions. Bull. Malays. Math. Sci. Soc. **38**, 143–154 (2015)
8. Gabeleh, M., Shahzad, N.: Best proximity points, cyclic Kannan maps and geodesic metric spaces. J. Fixed Point Theory Appl. **18**, 167–188 (2016)
9. Gopi, R., Pragadeeswarar, V.: Approximating common fixed point via Ishikawa's iteration. Fixed Point Theory **22**(2), 645–662 (2021)
10. Haddadi, M.R.: Proximity point iteration for nonexpansive mapping in Banach space. J. Nonlinear Sci. Appl. **7**, 126–130 (2014)
11. Ishikawa, S.: Fixed points by a new iteration method. Proc. Am. Math. Soc. **44**, 147–150 (1974)
12. Karpagam, S., Agarwal, S.: Best proximity point theorems for *p*-cyclic Meir-Keeler contractions. Fixed Point Theory Appl. **2009**, 197308 (2009)
13. Kirk, W.A., Reich, S., Veeramani, P.: Proximal retracts and best proximity pair theorems. Numer. Funct. Anal. Optim. **24**, 851–862 (2003)
14. Pragadeeswarar, V., Gopi, R.: Iterative approximation to common best proximity points of proximally mean nonexpansive mappings in Banach spaces. Afr. Math. **32**(1–2), 289–300 (2021)
15. Raj, V.S.: A best proximity point theorem for weakly contractive non-self mappings. Nonlinear Anal. **74**, 4804–4808 (2011)
16. Sadiq Basha, S.: Best proximity points: optimal solutions. J. Optim. Theory Appl. **151**, 210–216 (2011)
17. Sadiq Basha, S.: Best proximity points: global optimal approximate solution. J. Glob. Optim. **49**, 15–21 (2011)
18. Sankar Raj, V., Anthony Eldred, A.: A characterization of strictly convex spaces and applications. J. Optim. Theory Appl. **160**, 703–710 (2014)
19. Sintunavarat, W., Kumam, P.: The existence and convergence of best proximity points for generalized proximal contraction mappings. Fixed Point Theory Appl. **2014**, 228 (2014)
20. Sun, Y., Su, Y., Zhang, J.: A new method for the research of best proximity point theorems of nonlinear mappings. Fixed Point Theory Appl. **2014**, 116 (2014)

Publisher's Note